GOT IT? 3.5 Two projectiles are launched simultaneously from the same point on a horizontal surface, one at 45° to the horizontal and the other at 60°. Their launch speeds are different and are chosen so that the two projectiles travel the same horizontal distance before landing. Which of the following statements is true? (a) A and B land at the same time; (b) B's launch speed is lower than A's and B lands sooner; (c) B's launch speed is lower than A's and B lands later; (d) B's launch speed is higher than A's and B lands sooner; or (e) B's launch speed is higher than A's and B lands later.

Find the orbital period (the time to complete one orbit) of the International Space Station in its circular orbit at altitude 400 km, where the acceleration of gravity is 89% of its surface value.

An engineer is designing a flat, horizontal road for an 80 km/h speed limit (that's 22.2 m/s). If the maximum acceleration of a vehicle on this road is 1.5 m/s^2, what's the minimum safe radius for curves in the road?

Gotit 3.5

- Recall derivation that projectile trajectory is given by:

$$y = x\tan\theta_0 - \frac{g}{2V_0^2\cos^2\theta_0}x^2$$

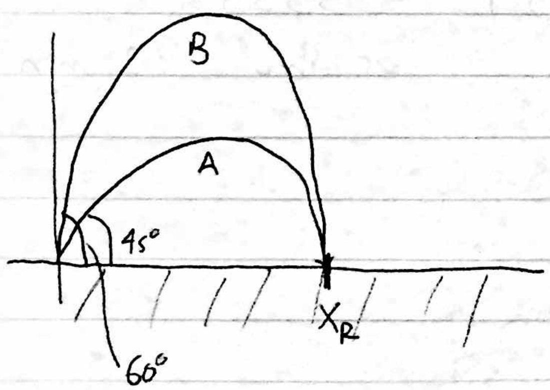
- The "range" can be found by setting $y=0$ and solving resulting quadratic eqn. to yield:

$$x = \frac{V_0^2}{g}\sin 2\theta_0$$

- To get the flight time, we note the range is just the horizontal velocity ($V_{x,0} = V_0\cos\theta_0$) times the "flight time" (t_f):

$$V_x t = V_0^2\frac{\sin 2\theta_0}{g} = V_0\cos\theta_0 \cdot t_f \longrightarrow t_f = \frac{V_0}{g}\frac{\sin 2\theta_0}{\cos\theta_0}$$

$$= \frac{V_0}{g}\frac{2\sin\theta_0\cos\theta_0}{\cos\theta_0} = \frac{2V_0\sin\theta_0}{g}$$



- For both cases:

$$V_0 = \sqrt{X_R\, g/\sin(2\theta_0)} \quad \text{and} \quad t_f = \frac{2V_0\sin\theta_0}{g}$$

$\Rightarrow$ Since $\sin(60) > \sin(45)$ and $\sin(120) < \sin(90)$, the $\theta_0 = 60°$ case means longer flight time (i.e. B lands after A) and larger V_0 (i.e. B is launched at a higher speed)

Ex. 3.7 Find orbital period (~~total~~ time to complete one orbit) of ISS if at a circular orbit at altitude of 400 km and gravitational acceleration is 89 % surface value,

- Orbital radius must be R_E (Earth radius) + altitude

$$r = R_E + 400\,km = 6.37 \times 10^6\,m + 4.00 \times 10^5\,m$$
$$= 6.77 \times 10^6\,m$$

- Orbital circumfrence is $2\pi r$ (i.e. total distance to cover one orbit) $\equiv d_o$
- Let T be the time to complete one orbit and ISS goes at velocity V_{ISS} such that $T = \frac{d_o}{V_{ISS}} = \frac{2\pi r}{V_{ISS}}$
- But $a = 0.89g = \frac{V_{ISS}^2}{r} \longrightarrow V_{ISS} = \sqrt{0.89gr}$

$$\Rightarrow T = \frac{2\pi r}{\sqrt{0.89gr}} = \frac{2\pi}{\sqrt{0.89(9.8)}}\left(6.77 \times 10^6\right)^{\frac{1}{2}} = 5536\,s$$

or about 92 min

Ex. 3.8 Minimum Safe radius

$$80\,km/h \longrightarrow 22.2\,m/s$$

$$r = \frac{v^2}{a} = \frac{(22.2)^2}{1.5} \simeq \boxed{329\,m}$$