26. ‖ A particle's position on the x-axis is given by the function $x = (t^2 - 4t + 2)$ m, where t is in s.

a. Make a position-versus-time graph for the interval $0 \text{ s} \le t \le 5 \text{ s}$. Do this by calculating and plotting x every 0.5 s from 0 s to 5 s, then drawing a smooth curve through the points.

b. Determine the particle's velocity at $t = 1.0$ s by drawing the tangent line on your graph and measuring its slope.

c. Determine the particle's velocity at $t = 1.0$ s by evaluating the derivative at that instant. Compare this to your result from part b.

d. Are there any turning points in the particle's motion? If so, at what position or positions?

e. Where is the particle when $v_x = 4.0$ m/s?

f. Draw a motion diagram for the particle.

27. ‖ Three particles move along the x-axis, each starting with $v_{0x} = 10$ m/s at $t_0 = 0$ s. In **FIGURE P2.27**, the graph for A is a position-versus-time graph; the graph for B is a velocity-versus-time graph; the graph for C is an acceleration-versus-time graph. Find each particle's velocity at $t = 7.0$ s. Work with the geometry of the graphs, not with kinematic equations.

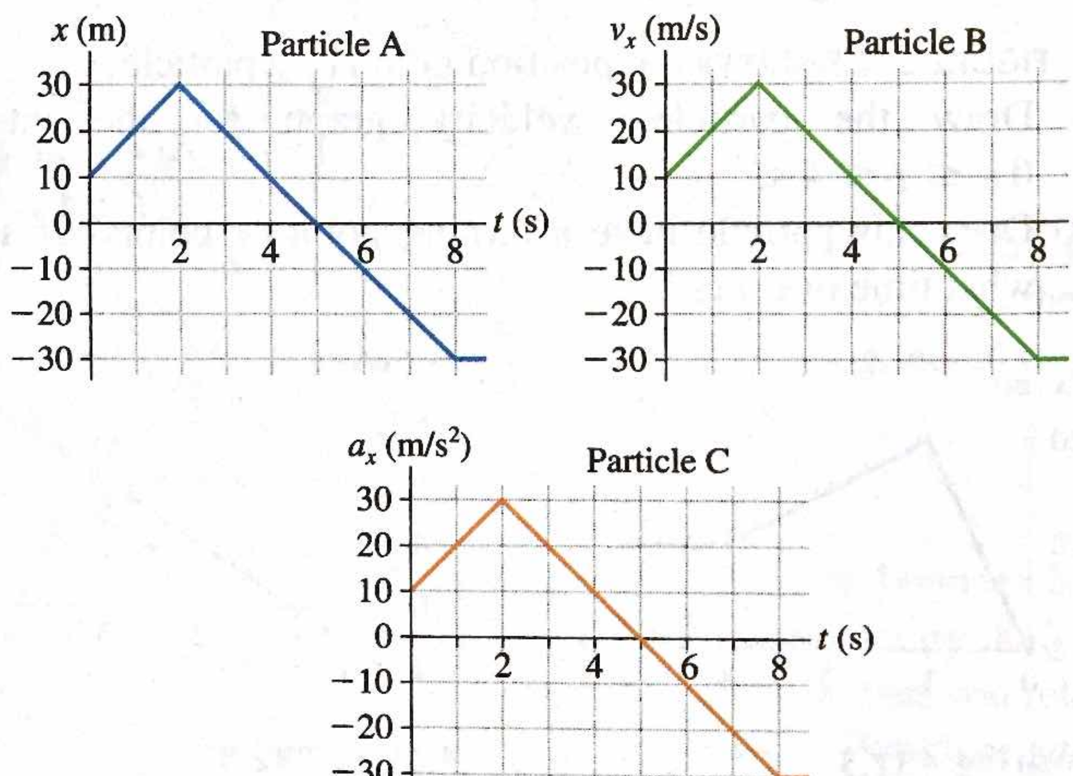


FIGURE P2.27

34. ‖ A particle's acceleration is described by the function $a_x = (10 - t)\ \text{m/s}^2$, where t is in s. Its initial conditions are $x_0 = 0$ m and $v_{0x} = 0$ m/s at $t = 0$ s.

a. At what time is the velocity again zero?

b. What is the particle's position at that time?

35. ‖ A ball rolls along the frictionless track shown in **FIGURE P2.35**. Each segment of the track is straight, and the ball passes smoothly from one segment to the next without changing speed or leaving the track. Draw three vertically stacked graphs showing position, velocity, and acceleration versus time. Each graph should have the same time axis, and the proportions of the graph should be qualitatively correct. Assume that the ball has enough speed to reach the top.

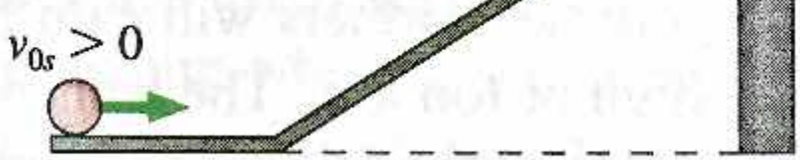


FIGURE P2.35

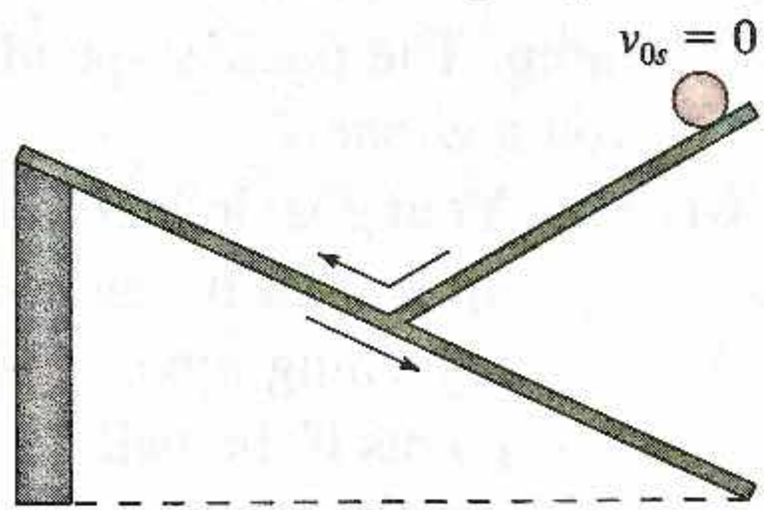


FIGURE P2.36

36. ‖ Draw position, velocity, and acceleration graphs for the ball shown in **FIGURE P2.36**. See Problem 35 for more information.

37. ‖ Draw position, velocity, and acceleration graphs for the ball shown in **FIGURE P2.37**. See Problem 35 for more information. The ball changes direction but not speed as it bounces from the reflecting wall.

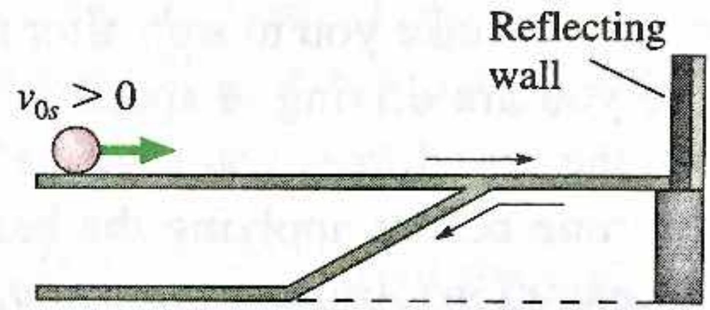


FIGURE P2.37

38. ‖ **FIGURE P2.38** shows a set of kinematic graphs for a ball rolling on a track. All segments of the track are straight lines, but some may be tilted. Draw a picture of the track and also indicate the ball's initial condition.

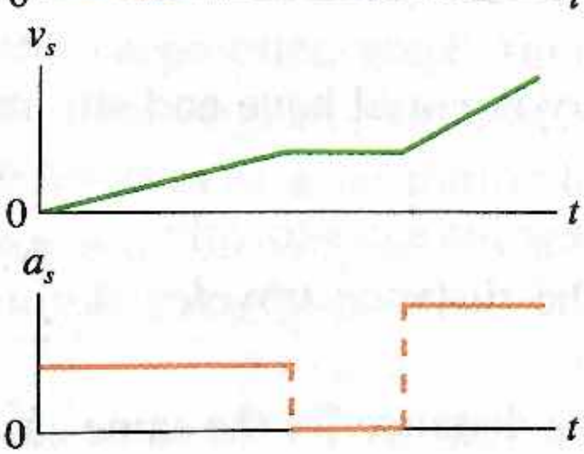


FIGURE P2.38

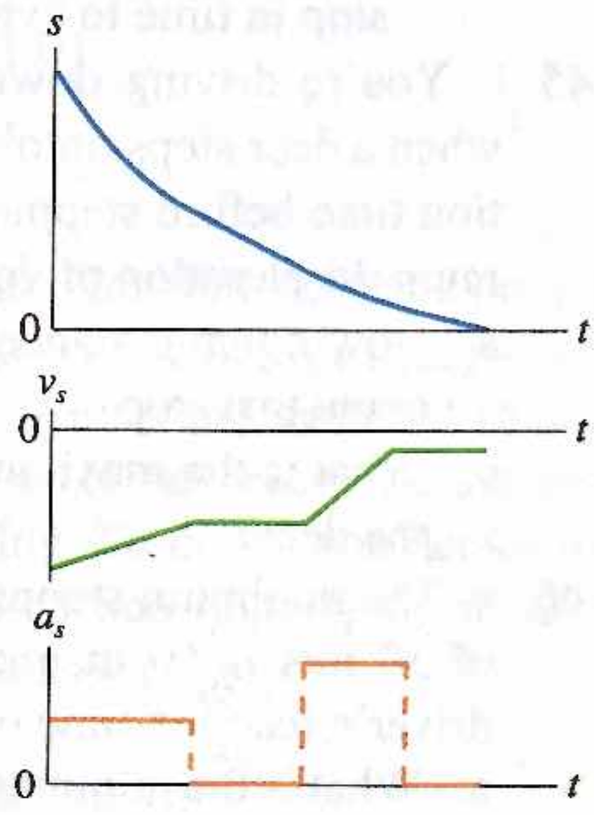
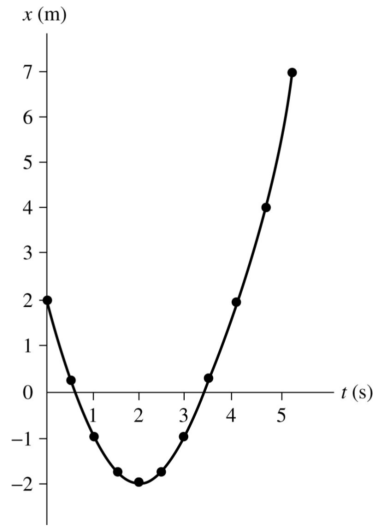


FIGURE P2.39

15. Here's a more serious, *practical* math/physics question for you to ponder. If you are making a round-trip flight from A to B and then back to A, does a steady wind blowing from A to B increase, decrease, or leave unchanged, the total travel time compared with when no wind is blowing? *Don't guess*—make a mathematical analysis (it's just high school algebra). You can find the answer at the end of Chapter 1.

2.26. Solve: (a)



(b) To be completed by student.

(c) $\dfrac{dx}{dt} = v_x = 2t - 4 \Rightarrow v_x(\text{at } t = 1\text{ s}) = [2\text{ m/s}^2(1\text{ s}) - 4\text{ m/s}] = -2\text{ m/s}$

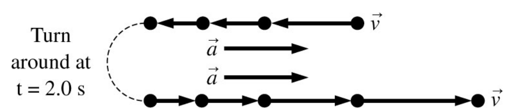
(d) There is a turning point at $t = 2$ s. At that time $x = -2$ m.

(e) Using the equation in part (c),

$$v_x = 4\text{ m/s} = (2t - 4)\text{ m/s} \Rightarrow t = 4$$

Since $x = (t^2 - 4t + 2)$ m, $x = 2$ m.

(f)

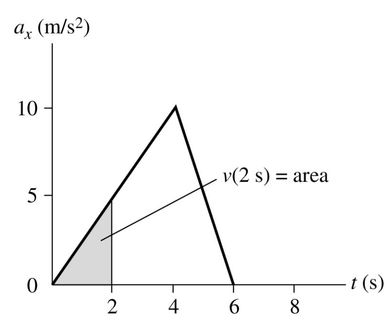


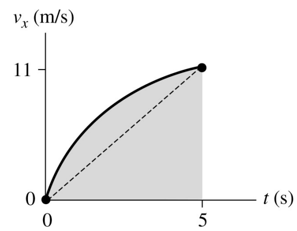
2.27. Solve: The graph for particle A is a straight line from $t = 2$ s to $t = 8$ s. The slope of this line is -10 m/s, which is the velocity at $t = 7.0$ s. The negative sign indicates motion toward lower values on the x-axis. The velocity of particle B at $t = 7.0$ s can be read directly from its graph. It is -20 m/s. The velocity of particle C can be obtained from the equation

$$v_\text{f} = v_\text{i} + \text{area under the acceleration curve between } t_\text{i} \text{ and } t_\text{f}$$

This area can be calculated by adding up three sections. The area between $t = 0$ s and $t = 2$ s is 40 m/s, the area between $t = 2$ s and $t = 5$ s is 45 m/s, and the area between $t = 5$ s and $t = 7$ s is -20 m/s. We get $(10\text{ m/s}) + (40\text{ m/s}) + (45\text{ m/s}) - (20\text{ m/s}) = 75\text{ m/s}$.

2.28. Visualize:





2.33. Solve: The position is the integral of the velocity.

$$x_1 = x_0 + \int_{t_0}^{t_1} v_x dt = x_0 + \int_0^{t_1} kt^2 dt = x_0 + \tfrac{1}{3}kt^3\Big|_0^{t_1} = x_0 + \tfrac{1}{3}kt_1^3$$

We're given that $x_0 = -9.0$ m and that the particle is at $x_1 = 9.0$ m at $t_1 = 3.0$ s. Thus

$$9.0 \text{ m} = (-9.0 \text{ m}) + \tfrac{1}{3}k(3.0 \text{ s})^3 = (-9.0 \text{ m}) + k(9.0 \text{ s}^3)$$

Solving for k gives $k = 2.0 \text{ m/s}^3$.

2.34. Solve: (a) The velocity is the integral of the acceleration.

$$v_{1x} = v_{0x} + \int_{t_0}^{t_1} a_x dt = 0 \text{ m/s} + \int_0^{t_1}(10 - t)dt = \left(10t - \tfrac{1}{2}t^2\right)\Big|_0^{t_1} = 10t_1 - \tfrac{1}{2}t_1^2$$

The velocity is zero when

$$v_{1x} = 0 \text{ m/s} = \left(10t_1 - \tfrac{1}{2}t_1^2\right) = \left(10 - \tfrac{1}{2}t_1\right) \times t_1$$

$$\Rightarrow t_1 = 0 \text{ s} \quad \text{or} \quad t_1 = 20 \text{ s}$$

The first solution is the initial condition. Thus the particle's velocity is again 0 m/s at $t_1 = 20$ s.

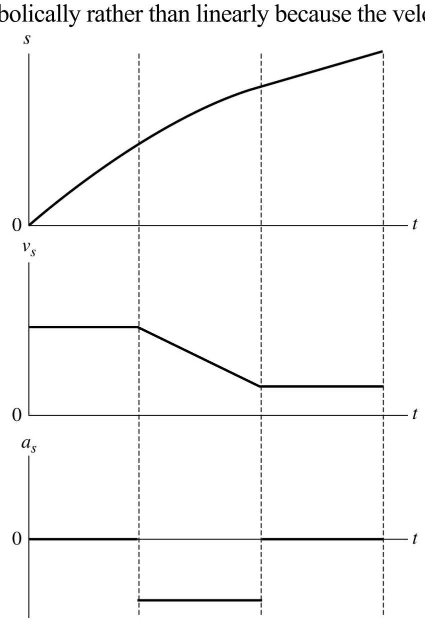
(b) Position is the integral of the velocity. At $t_1 = 20$ s, and using $x_0 = 0$ m at $t_0 = 0$ s, the position is

$$x_1 = x_0 + \int_{t_0}^{t_1} v_x dt = 0 \text{ m} + \int_0^{20}\left(10t - \tfrac{1}{2}t^2\right)dt = 5t^2\Big|_0^{20} - \tfrac{1}{6}t^3\Big|_0^{20} = 667 \text{ m}$$

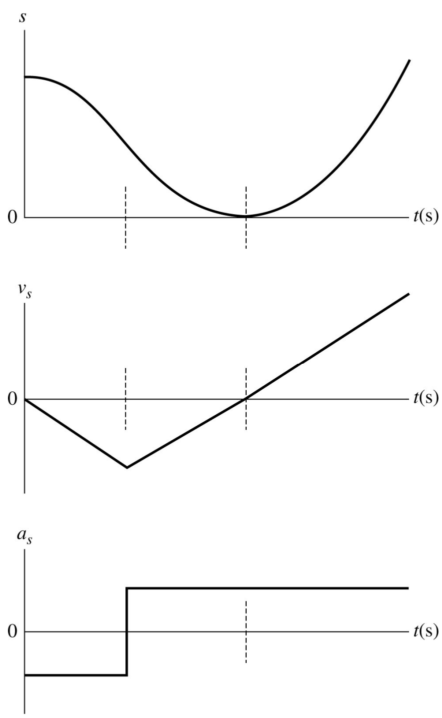
2.35. Model: Represent the ball as a particle.

Visualize: Please refer to Figure P2.35.

Solve: In the first and third segments the acceleration a_s is zero. In the second segment the acceleration is negative and constant. This means the velocity v_s will be constant in the first two segments and will decrease linearly in the third segment. Because the velocity is constant in the first and third segments, the position s will increase linearly. In the second segment, the position will increase parabolically rather than linearly because the velocity decreases linearly with time.

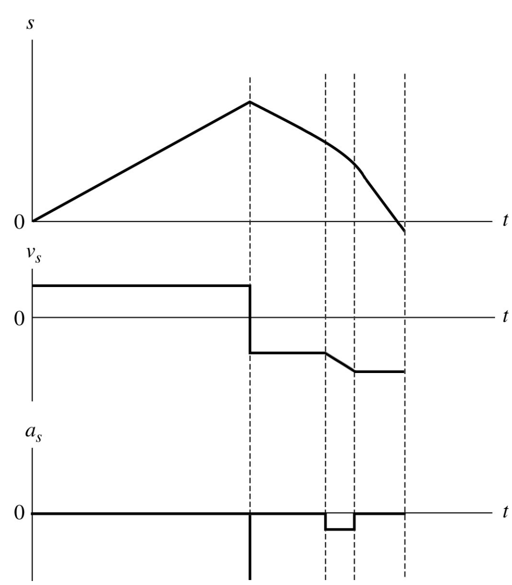


2.36. **Model:** Represent the ball as a particle.
Visualize: Please refer to Figure P2.36. The ball rolls down the first short track, then up the second short track, and then down the long track. s is the distance along the track measured from the left end (where $s = 0$). Label $t = 0$ at the beginning, that is, when the ball starts to roll down the first short track.
Solve: Because the incline angle is the same, the magnitude of the acceleration is the same on all of the tracks.



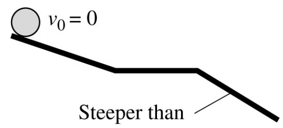
Assess: Note that the derivative of the s versus t graph yields the v_s versus t graph. And the derivative of the v_s versus t graph gives rise to the a_s versus t graph.

2.37. **Model:** Represent the ball as a particle.
Visualize: The ball moves to the right along the first track until it strikes the wall, which causes it to move to the left on a second track. The ball then descends on a third track until it reaches the fourth track, which is horizontal.
Solve:

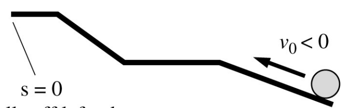


Assess: Note that the time derivative of the position graph yields the velocity graph, and the derivative of the velocity graph gives the acceleration graph.

2.38. Visualize: Please refer to Figure P2.38.
Solve:



2.39. Visualize: Please refer to Figure P2.39.
Solve:



2.40. Model: The plane is a particle and the constant-acceleration kinematic equations hold.
Solve: (a) To convert 80 m/s to mph, we calculate 80 m/s × 1 mi/1609 m × 3600 s/h = 179 mph.
(b) Using $a_s = \Delta v/\Delta t$, we have,

$$a_s(t = 0 \text{ to } t = 10 \text{ s}) = \frac{23 \text{ m/s} - 0 \text{ m/s}}{10 \text{ s} - 0 \text{ s}} = 2.3 \text{ m/s}^2 \qquad a_s(t = 20 \text{ s to } t = 30 \text{ s}) = \frac{69 \text{ m/s} - 46 \text{ m/s}}{30 \text{ s} - 20 \text{ s}} = 2.3 \text{ m/s}^2$$

For all time intervals a is 2.3 m/s^2.
(c) Using kinematics as follows:

$$v_{fs} = v_{is} + a(t_f - t_i) \Rightarrow 80 \text{ m/s} = 0 \text{ m/s} + (2.3 \text{ m/s}^2)(t_f - 0 \text{ s}) \Rightarrow t_f = 35 \text{ s}$$

(d) Using the above values, we calculate the takeoff distance as follows:

$$s_f = s_i + v_{is}(t_f - t_i) + \frac{1}{2}a_s(t_f - t_i)^2 = 0 \text{ m} + (0 \text{ m/s})(35 \text{ s}) + \frac{1}{2}(2.3 \text{ m/s}^2)(35 \text{ s})^2 = 1410 \text{ m}$$

For safety, the runway should be $3 \times 1410 \text{ m} = 4230 \text{ m}$ or 2.6 mi. This is longer than the 2.4 mi long runway, so the takeoff is not safe.

2.41. Model: Represent the car as a particle.
Solve: (a) First, we will convert units:

$$60 \frac{\text{miles}}{\text{hour}} \times \frac{1 \text{ hour}}{3600 \text{ s}} \times \frac{1610 \text{ m}}{1 \text{ mile}} = 27 \text{ m/s}$$

The motion is constant acceleration, so

$$v_1 = v_0 + a\Delta t \Rightarrow a = \frac{v_1 - v_0}{\Delta t} = \frac{(27 \text{ m/s} - 0 \text{ m/s})}{10 \text{ s}} = 2.7 \text{ m/s}^2$$

(b) The fraction is $a/g = 2.7/9.8 = 0.28$. So a is 28% of g.
(c) The distance is calculated as follows:

$$x_1 = x_0 + v_0\Delta t + \frac{1}{2}a(\Delta t)^2 = \frac{1}{2}a(\Delta t)^2 = 1.3 \times 10^2 \text{ m} = 4.3 \times 10^2 \text{ feet}$$

2.42. Model: Represent the spaceship as a particle.
Solve: (a) The known information is: $x_0 = 0$ m, $v_0 = 0$ m/s, $t_0 = 0$ s, $a = g = 9.8 \text{ m/s}^2$, and $v_1 = 3.0 \times 10^8$ m/s. Constant acceleration kinematics gives

$$v_1 = v_0 + a\Delta t \Rightarrow \Delta t = t_1 = \frac{v_1 - v_0}{a} = 3.06 \times 10^7 \text{ s}$$

Solution to the Preface Problem in Endnote 15

Let d be the distance between A and B, s the speed of the airplane in still air, and w the speed of the wind. Then, the total round-trip travel time T is the sum of the times spent traveling with, and then against, the wind:

$$T = \frac{d}{s+w} + \frac{d}{s-w} = \frac{d\,(s-w) + d\,(s+w)}{(s+w)\,(s-w)}$$

$$= \frac{2s\,d}{s^2 - w^2} = \frac{2s\,d}{s^2\left(1 - \frac{w^2}{s^2}\right)} = \frac{2d}{s}\left[\frac{1}{1 - \left(\frac{w}{s}\right)^2}\right].$$

When there is no wind ($w = 0$) then $T = \frac{2d}{s}$, and when $w > 0$ the denominator in the brackets gets smaller, and we have $T > \frac{2d}{s}$. So, a steady wind *always increases* the total travel time.

Here's a math-free way to see by inspection the special case of $w = s$. In that case the return part of the trip has the plane, with speed s, facing a headwind of the same speed. Thus, the plane *doesn't move* and so will *never* get back to A (that is, $T = \infty$ if $w = s$).