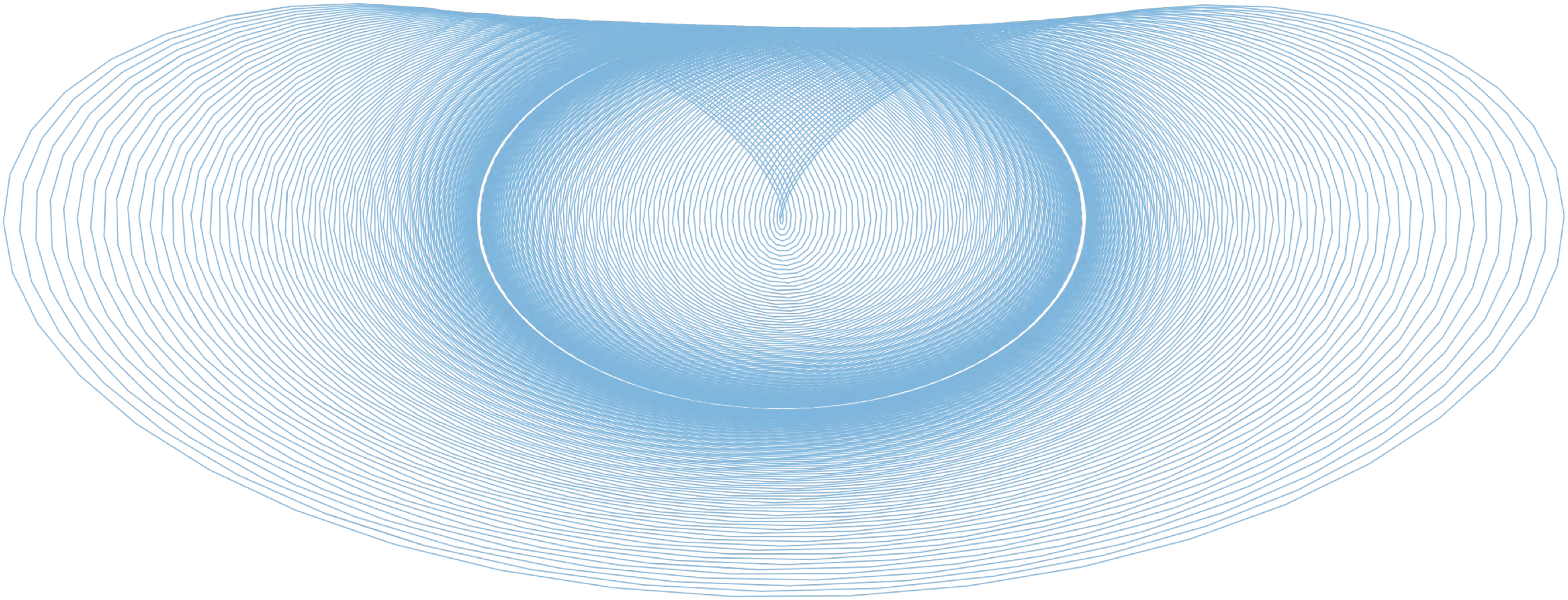
PHYS 1420 (F19)
Physics with Applications to Life Sciences



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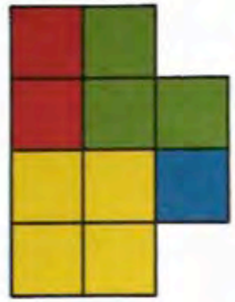
2019.11.12 (Tutorial)

16. One-to-Four

A


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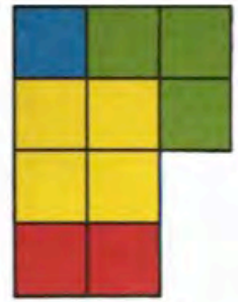

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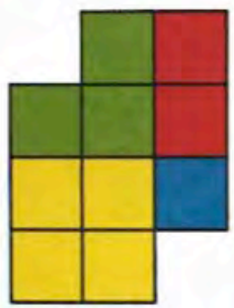
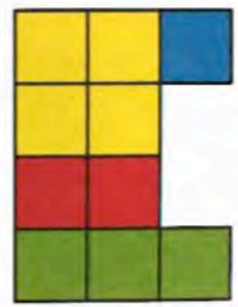
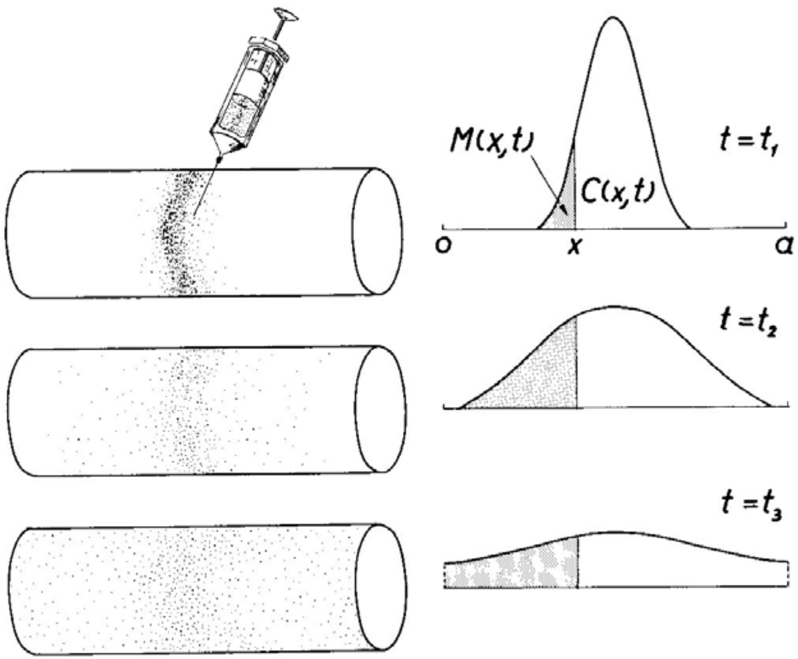


D


Which pattern does not belong?

A B C D

"Diffusion math" → Multivariable functions



Note: Concentration of a solute in a solution (c) depends upon both spatial location (x) and time (t)

Multivariable function

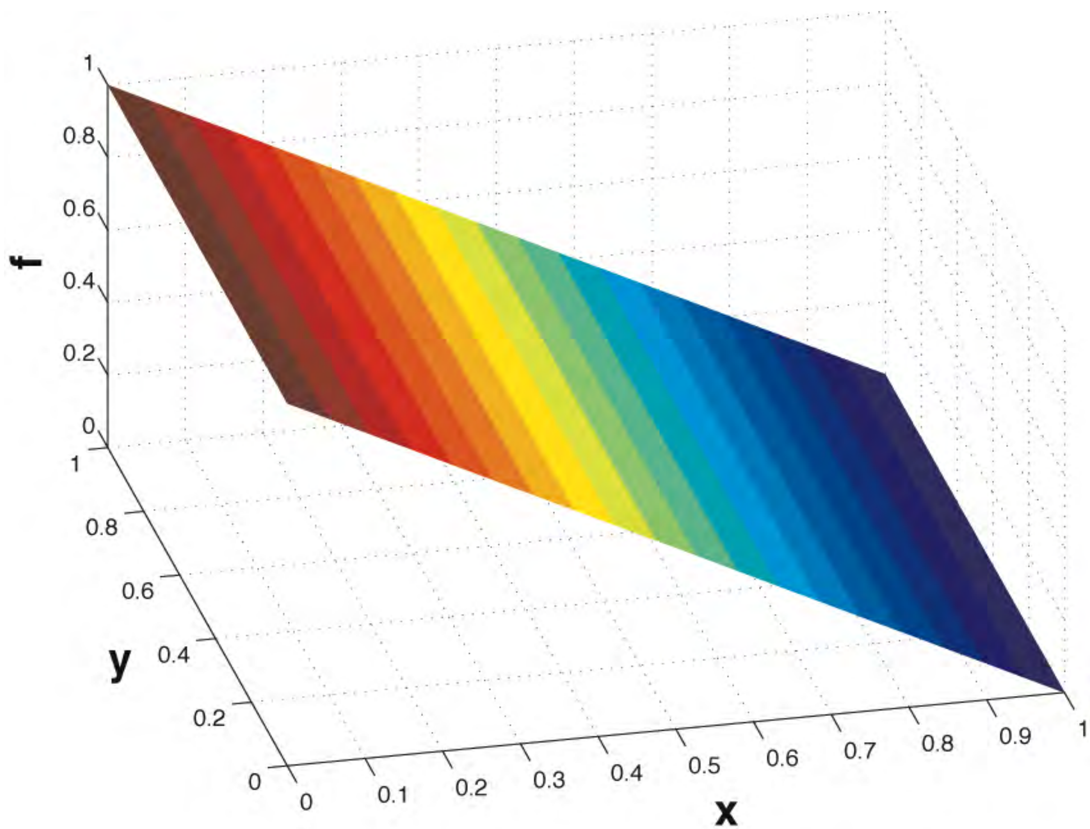
$$f = f(x, y)$$

f - dependent variable

x, y - independent variables

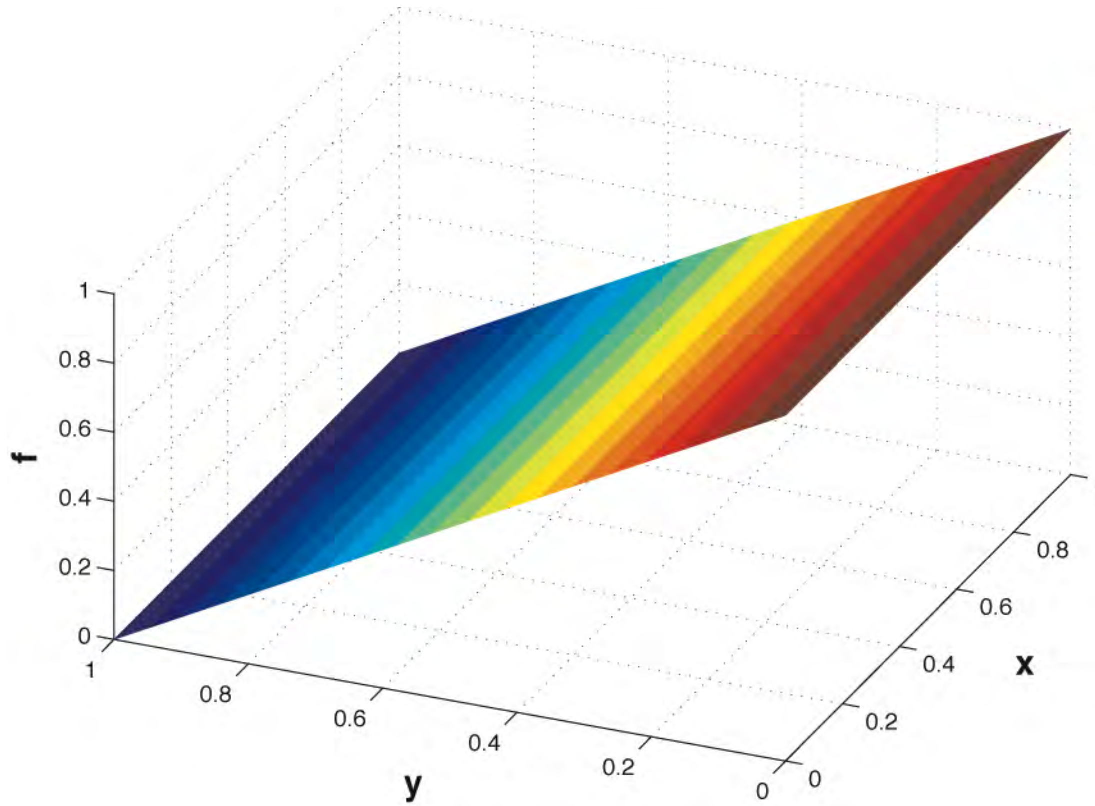
- Multivariate functions are important in many various contexts throughout science

"Diffusion math" → Multivariable functions



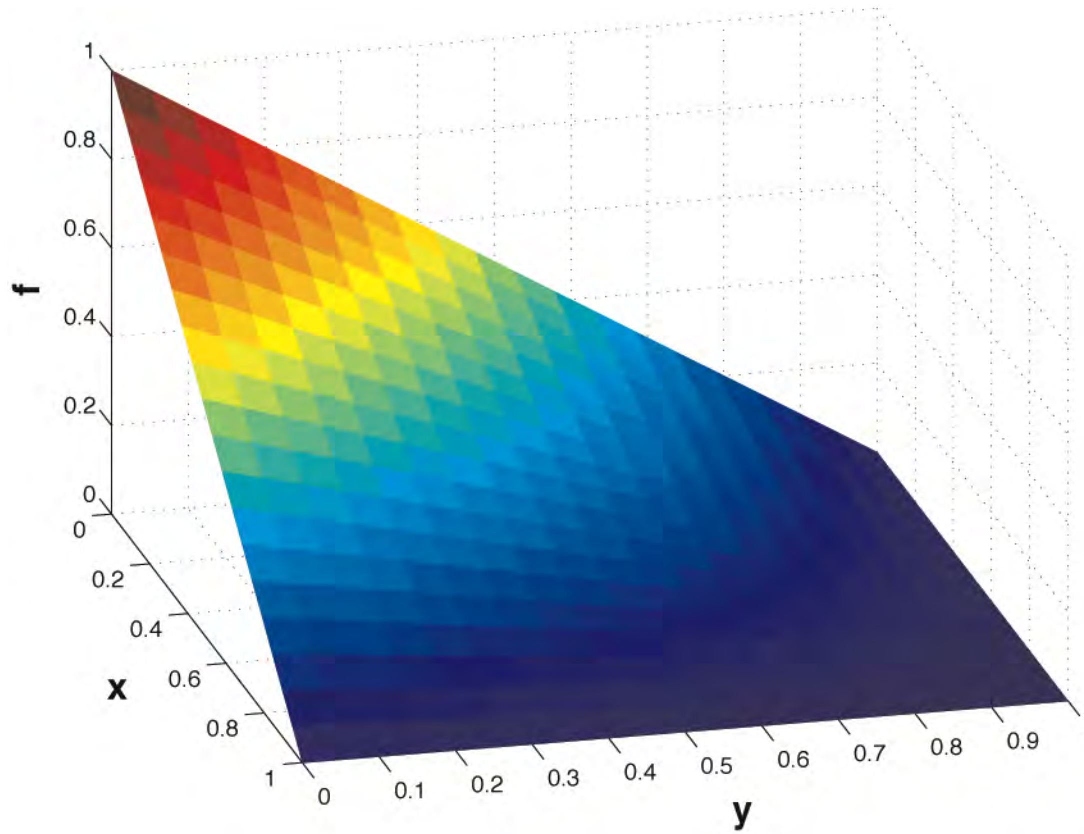
$$f(x, y) = (1 - x) = f(x)$$

"Diffusion math" → Multivariable functions



$$f(x, y) = (1 - y) = f(y)$$

“Diffusion math” → Multivariable functions



Biological Context

f - reaction rate [*mol/s*]

x, y - concentration of inhibitor agents [*mol*]

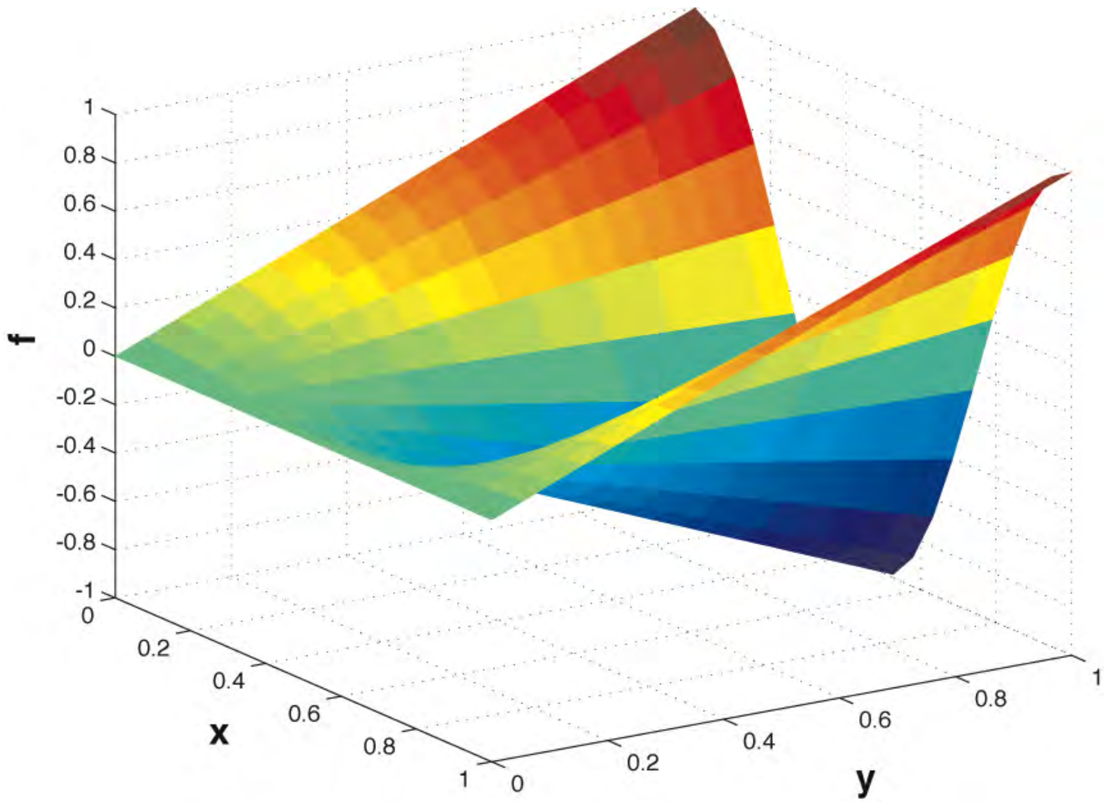
~~$$f(x,y) = (1-y)(1-x)$$~~

don't forget about units!

$$f(x,y) = k(1-x)(1-y)$$

$$[k] = \frac{1}{mol \cdot s}$$

“Diffusion math” → Multivariable functions



$$f(x, y) = y \cos(2\pi x)$$

Diffusion: Microscopic → Macroscopic

Solution to "diffusion equation"

$$c(x,t) = \frac{n_o}{\sqrt{4\pi D t}} e^{-x^2/4Dt}$$

"concentration" is a function of more than one variable!

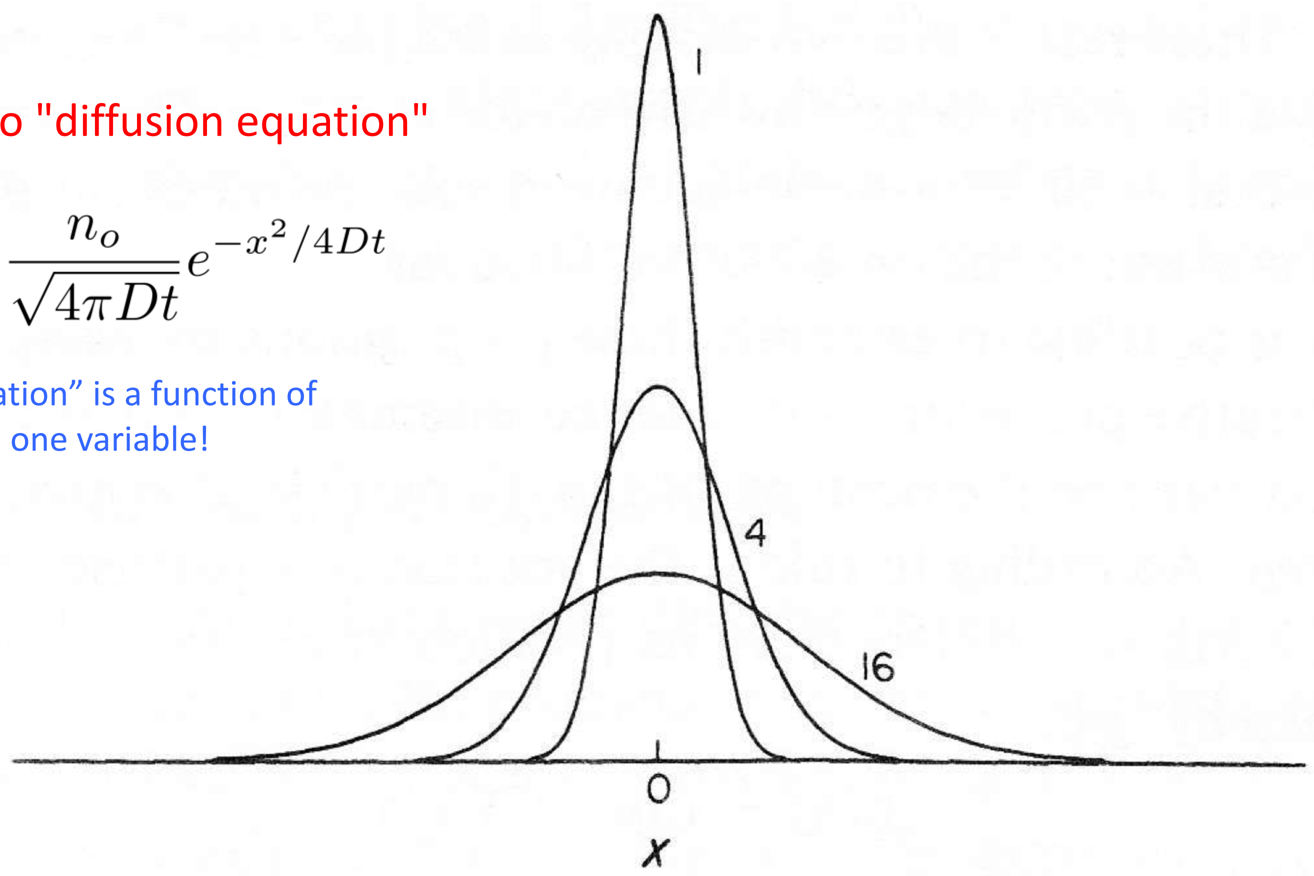


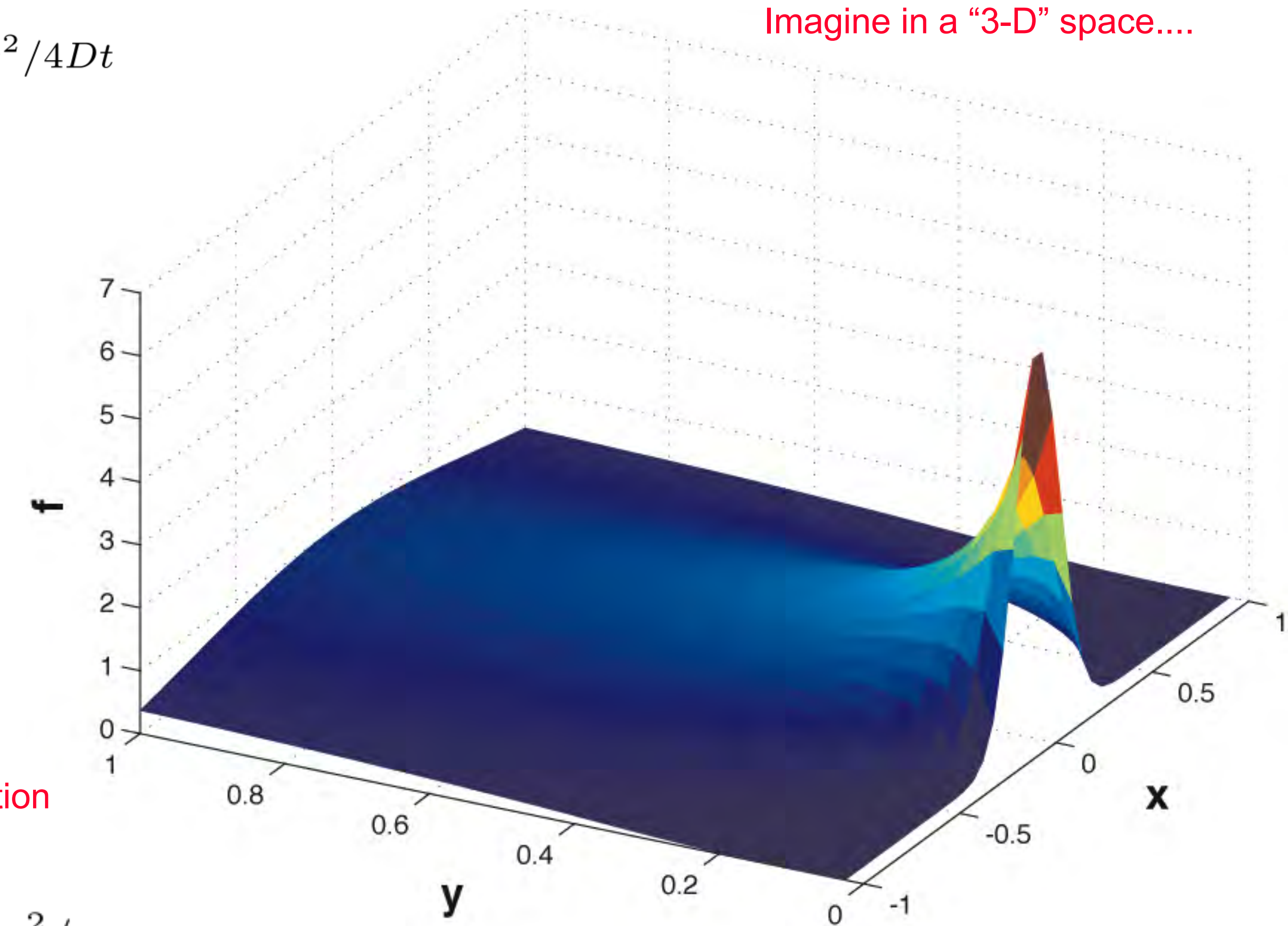
Fig. 1.3. The probability of finding particles at different points x at times $t = 1, 4,$ and 16. The particles start out at position $x = 0$ at time $t = 0$. The standard deviations (root-mean-square widths) of the distributions increase with the square-root of the time. Their peak heights decrease with the square-root of the time. See Eq. 1.22.

Berg (1993)

Diffusion: Multivariable nature

$$c(x,t) = \frac{n_o}{\sqrt{4\pi Dt}} e^{-x^2/4Dt}$$

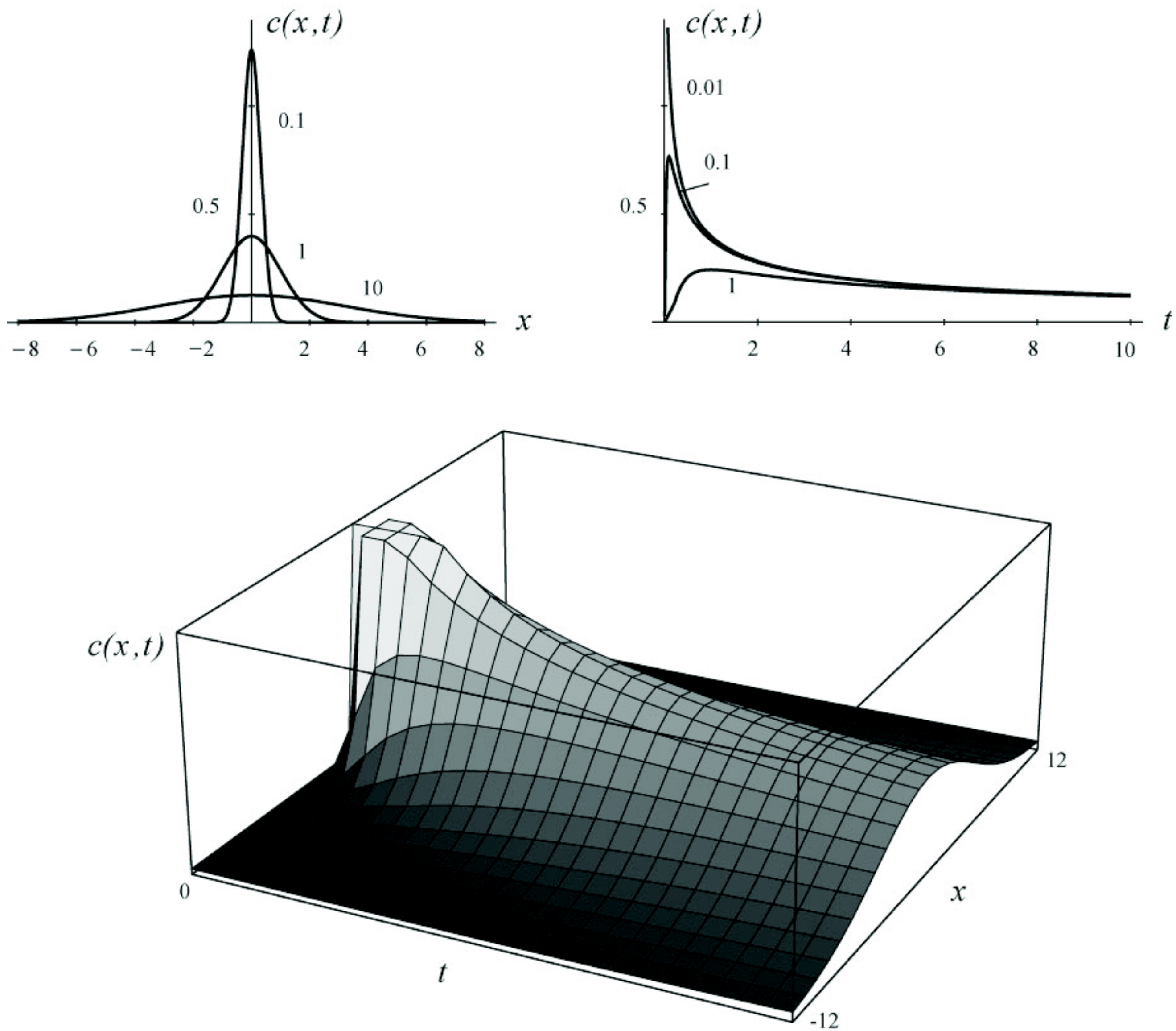
Imagine in a "3-D" space....



Solution to diffusion equation in a more general form:

$$f(x,y) = \frac{1}{\sqrt{y}} e^{-x^2/y}$$

Diffusion



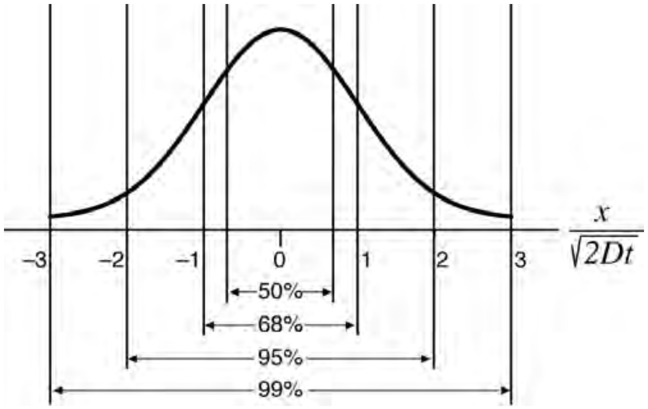
Weiss Fig.3.14 (modified)

Importance of scale

$$c(x,t) = \frac{n_o}{\sqrt{4\pi D t}} e^{-x^2/4Dt}$$

Gaussian function with zero mean and standard deviation:

$$\sigma = \sqrt{2Dt}$$



Question: How long does it take ($t_{1/2}$) for ~1/2 the solute to move at least the distance $x_{1/2}$?

$$\frac{x_{1/2}}{\sqrt{2Dt_{1/2}}} \approx \frac{2}{3} \implies t_{1/2} \approx \frac{x_{1/2}^2}{D}$$

For small solutes
(e.g. K^+ at body temperature)

$$D \approx 10^{-5}\ \frac{\mathrm{cm}^2}{\mathrm{s}}$$

	$x_{1/2}$	$t_{1/2}$
membrane sized	10 nm	$\frac{1}{10}$ µsec
cell sized	10 µm	$\frac{1}{10}$ sec
dime sized	10 mm	10^5 sec ≈ 1 day

Freeman

Tangent: Why is a cell "cell-sized"?

- Cells are typically 1-100 um or so in size. Why?

What determines cell size?

Wallace F Marshall*[1], Kevin D Young[2], Matthew Swaffer[3], Elizabeth Wood[3], Paul Nurse[3,4,5], Akatsuki Kimura[6], Joseph Frankel[7], John Wallingford[8], Virginia Walbot[9], Xian Qu[10] and Adrienne HK Roeder[11]

Marshall *et al. BMC Biology* 2012, **10**:101
http://www.biomedcentral.com/1741-7007/10/101

- Non-trivial question and likely a # of factors (e.g., optimizing volume to surface area), but….

- … limits stemming from diffusion are likely central

	$x_{1/2}$	$t_{1/2}$
membrane sized	10 nm	$\frac{1}{10}$ μsec
cell sized	10 μm	$\frac{1}{10}$ sec
dime sized	10 mm	10^5 sec ≈ 1 day

113. ••A baton is constructed by attaching two small objects that each have a mass M to the ends of a rod that has a length L and a uniform mass M. Find an expression for the moment of inertia of the baton when it is rotated around a point $3/8\ L$ from one end.

SET UP

A baton is made of a thin rod of length L and uniform mass M with a point mass M attached to each end. The total moment of inertia of the baton is the sum of the moments of inertia due to the rod and the two point masses. The baton is being twirled about a point that is (3/8) L from one end. This means we need to use the parallel-axis theorem to find the moment of inertia for the rod (only) in this case since it is the object that is being rotated about an axis parallel to its center of mass; we can still treat the two masses on the ends as point masses located at a distance $(3/8)L$ and $(5/8)L$, respectively.

SOLVE

$$I_{\text{rod}} = I_{\text{CM}} + I_{\text{from axis}} = \frac{1}{12}ML^2 + M\left(\frac{L}{8}\right)^2 = \frac{19}{192}ML^2$$

$$I_{\text{total}} = I_{\text{rod}} + I_{\text{left mass}} + I_{\text{right mass}} = \frac{19}{192}ML^2 + M\left(\frac{3}{8}L\right)^2 + M\left(\frac{5}{8}L\right)^2$$

$$= \frac{19}{192}ML^2 + \frac{9}{64}ML^2 + \frac{25}{64}ML^2 = \boxed{\frac{121}{192}ML^2}$$

REFLECT

This works out to be about $(0.63)ML^2$. The moment of inertia when twirling the baton about its center of mass is less than this value: $I_{\text{total}} = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2 = \frac{7}{12}ML^2$, or $(0.58)ML^2$, which makes sense.