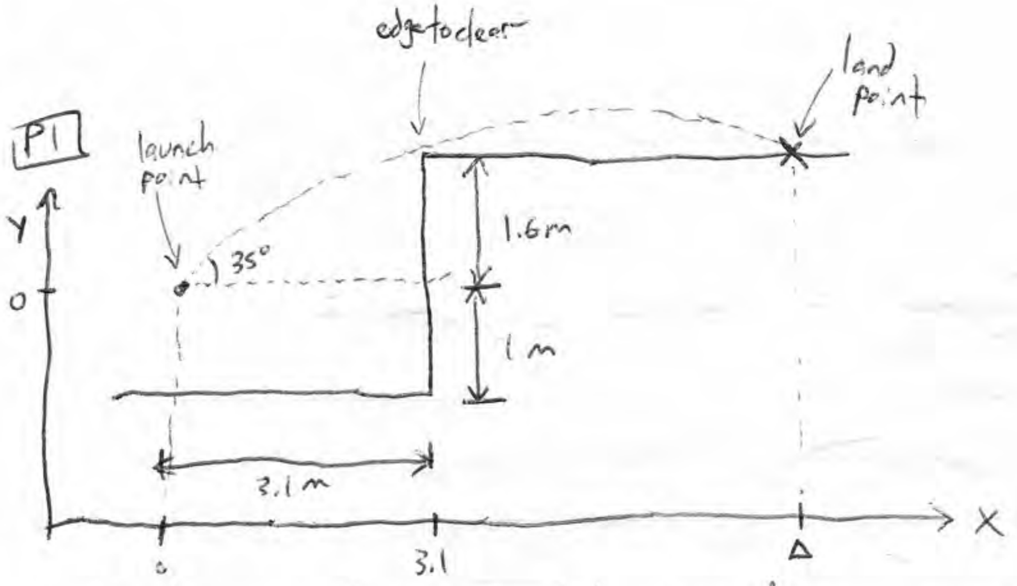
9/17/19
Tutorial sols

P1



- Hammer will take parabolic trajectory: $y = x\tan\theta_0 - \frac{g}{2V_0^2\cos^2\theta_0}x^2$

- To pass through $(x,y) = (3.1, 1.6)$ (see coord. system above), we can rearrange:

$$\frac{gx^2}{2V_0^2\cos^2\theta_0} = x\tan\theta_0 - y \rightsquigarrow V_0 = \sqrt{\frac{gx^2}{2(x\tan\theta_0 - y)\cos^2\theta_0}}$$

$$= \left[\frac{(9.8)(3.1)^2}{2\cos^2(35°)\cdot[3.1\cdot\tan(35°)-1.6]}\right]^{\frac{1}{2}} \boxed{\simeq 11.1 \text{ m/s}}$$

- Now that we know V_0, we can solve for Δ (total horizontal dist. covered) noting that this will happen when $y_\Delta = 1.6$ m

$$\frac{g}{2V_0^2\cos^2\theta_0}x^2 - \tan\theta_0 x + y_\Delta = 0 = Ax^2 + Bx + C$$

$A = \frac{g}{2V_0^2\cos^2\theta_0}$

$B = -\tan\theta_0$

$C = +y_\Delta$

(all known constants!)

$$\rightsquigarrow x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{0.7 \pm \sqrt{0.49 - 4\cdot 0.059\cdot 1.6}}{2(0.059)}$$

$$= [3.1, 8.7] \text{ m}$$ — two roots make sense (see next page)

$A = 0.059$

$B = -0.7$

$C = 1.6$

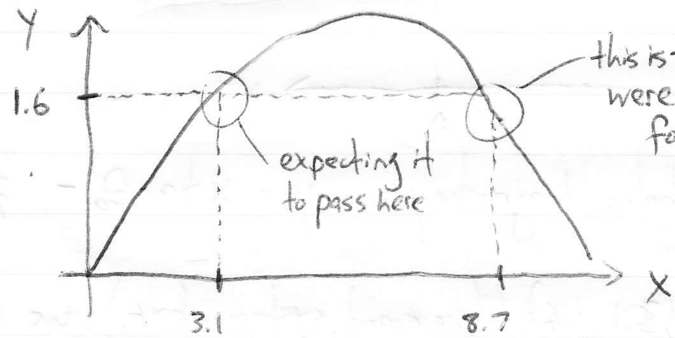
$\Longrightarrow$ $\boxed{\Delta = 8.7 \text{ m, or } 5.6 \text{ m from hole's edge}}$

cont

P1 (cont) If stuck (or need a reality check), try plotting y w/ known values!

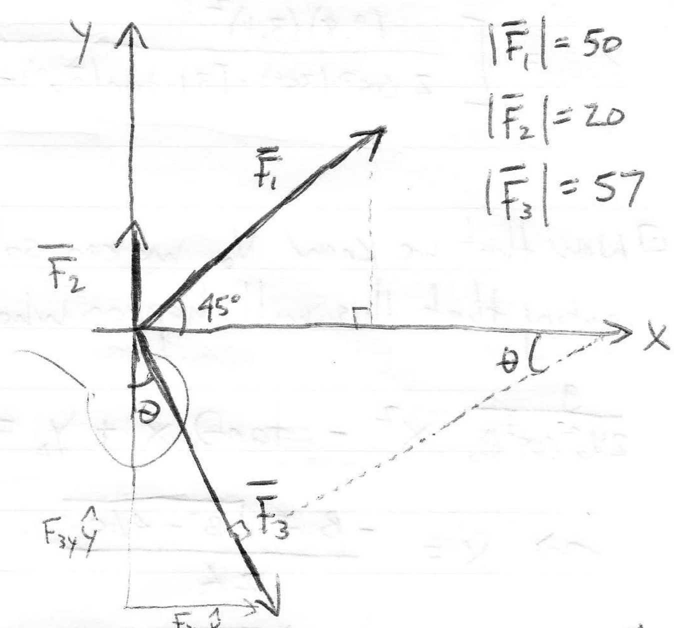
$$y = x\tan\theta_0 - \frac{g}{2V_0^2\cos^2\theta_0}x^2 = 0.7x - 0.059x^2$$

Compare this parabola to schematic on last pg.



P2 No need to worry about "forces" here per se, this is a vector problem!

- Given odd "rotated" coord system, rotate it back and determine associated vector components



NOTE: this angle is θ = 15° by inspection

- $\bar{F}_1 = \frac{50}{\sqrt{2}}\hat{x} + \frac{50}{\sqrt{2}}\hat{y}$
  $= 35.4\hat{x} + 35.4\hat{y}$

$\bar{F}_2 = 0\hat{x} + 20\hat{y}$

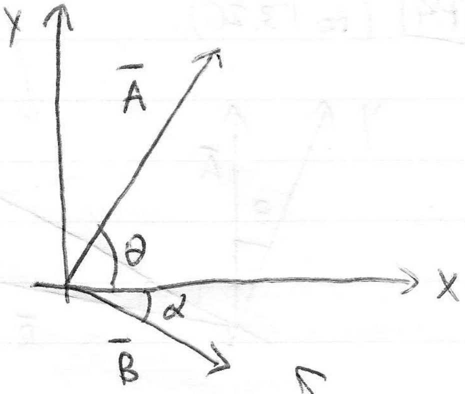
$\bar{F}_3 = 57\sin(15°)\hat{x} - 57\cos(15°)\hat{y}$
$\simeq 14.8\hat{x} - 55.1\hat{y}$

NOTE: For $\bar{F}_1$, $F_{1x} = |\bar{F}_1|\cos 45 = \frac{50}{\sqrt{2}}$ (similar arg. for F_{1y}, F_{3x}, etc...)

$$\bar{F}_T = \sum_{n=1}^{3}\bar{F}_n = \hat{x}(35.4 + 0 + 14.8) + \hat{y}(35.4 + 20 - 55.1)$$

$$\boxed{\simeq 50.2\hat{x} - 0.3\hat{y} \quad N = \bar{F}_T}$$

P3 (re "P3.25")



Given: $\theta = 40°$, $\alpha = 20°$, $|\bar{A}| = 4$, $|\bar{B}| = 2$

To Do: Find $\bar{C}$ such that $\bar{A} + \bar{B} + \bar{C} = 0$

- First express $\bar{A}$ and $\bar{B}$ in component form

$$\bar{A} = A_x \hat{x} + A_y \hat{y} = 4\cos(40°)\hat{x} + 4\sin(40°)\hat{y}$$
$$\simeq 3.06\hat{x} + 2.57\hat{y} \quad (\text{or} \sim 3.1\hat{x} + 2.6\hat{y})$$

Be careful to note "sign" for vector components appropriately!!

$$\bar{B} = \hat{B}_x \hat{x} + B_y \hat{y}$$
$$= 2\cos(20°)\hat{x} - 2\sin(20°)\hat{y} \simeq 1.88\hat{x} - 0.68\hat{y}$$

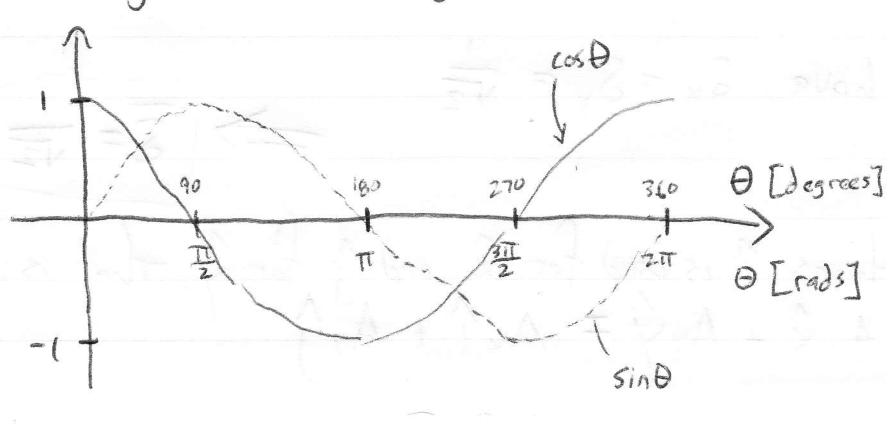
- Now put the component together and solve for the unknown:

re $\hat{x}$: $A_x + B_x + C_x = 0 = 3.06 + 1.88 + C_x \longrightarrow C_x = -4.94$

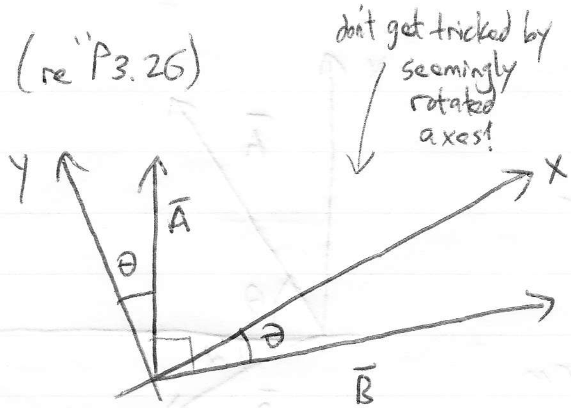
re $\hat{y}$: $A_y + B_y + C_y = 0 = 2.57 - 0.68 + C_y \longrightarrow C_y = -1.89$

$$\Rightarrow \boxed{\bar{C} = -4.94\hat{x} - 1.89\hat{y}}$$

NOTE: Cosine of $20°$ is really $\cos\left(\frac{20\pi}{180}\right) \simeq 0.94$. That is, the "argument" (i.e. the value inside the parentheses) should be in **radians**, not degrees. There are different ways to express angles (e.g. $[0, 360°]$, $[-180°, 180°]$, $[0, 2\pi]$, $[-\pi, \pi]$) that are equivalent, BUT for the sake of the trigonometric arguments, use degrees!



P4 (re "P3.26)



$|\bar{A}| = 2\ \text{m}, \quad |\bar{B}| = 4\ \text{m}, \quad \theta = 15°$

<u>Goal</u>: Find $\bar{D} = 2\bar{A} + \bar{B}$

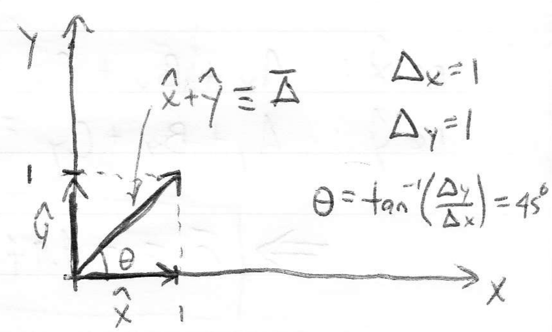
☐ First write $\bar{A}$ and $\bar{B}$ in component form:
(being careful to note where/how θ is specified!)

$$\bar{A} = A_x\hat{x} + A_y\hat{y} = 2\sin(15°)\hat{x} + 2\cos(15°)\hat{y} \simeq 0.52\hat{x} + 1.93\hat{y}$$

$$\bar{B} = B_x\hat{x} + B_y\hat{y} = 4\cos(15°)\hat{x} - 4\sin(15°)\hat{y} \approx 3.86\hat{x} - 1.04\hat{y}$$

$$\bar{D} = 2\bar{A} + \bar{B} = [2(0.52) + 3.86]\hat{x} + [2(1.93) - 1.04]\hat{y} \; \boxed{\simeq 4.9\hat{x} + 2.8\hat{y} = \bar{D}}$$

P5 ☐ Draw a picture and it is clear that
$\bar{\Delta} = \hat{x} + \hat{y}$ has an $\theta = 45°$ re x-axis.
But what is the mag.?



$\Delta_x = 1$
$\Delta_y = 1$
$\theta = \tan^{-1}\left(\frac{\Delta_y}{\Delta_x}\right) = 45°$

$$|\bar{\Delta}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

☐ Now find $\bar{\delta}$ such that $|\bar{\delta}| = 1$ (ie. δ_x and δ_y = ?)

answer seems obvious, but let's be "complete"

$$\delta_x = |\delta|\cos(45°) = \frac{|\delta|}{\sqrt{2}}, \quad \delta_y = |\delta|\sin(45°) = \frac{|\delta|}{\sqrt{2}}$$

$$|\delta| = \sqrt{\delta_x^2 + \delta_y^2} = 1 \longrightarrow \sqrt{\left(\frac{|\delta|}{\sqrt{2}}\right)^2 + \left(\frac{|\delta|}{\sqrt{2}}\right)^2} = \sqrt{|\delta|^2} = |\delta| = 1$$

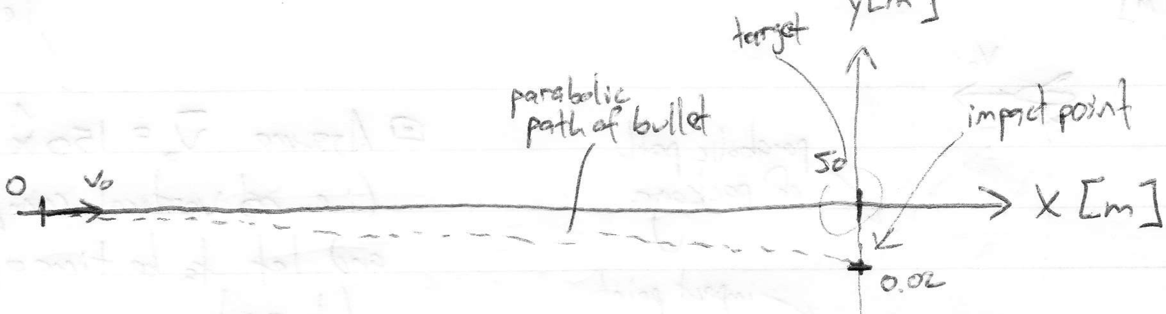
Thus must have $\delta_x = \delta_y = \frac{1}{\sqrt{2}}$

$$\Longrightarrow \boxed{\bar{\delta} = \frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}}$$

<u>NOTE</u>: Sometimes $\hat{i}$ is used for $\hat{x}$ and $\hat{j}$ for $\hat{y}$. That is:

$$A_x\hat{x} + A_y\hat{y} = A_x\hat{i} + A_y\hat{j}$$

P6



Assume no air resistance (i.e. $a_x = 0$ and $a_y = -9.8\ m/s^2$) and since bullet is fired "horizontally" that $V_{oy} = 0$ (i.e. all initial velocity is along x)

a) (Note: Essentially the same as "dropping" the bullet!)

$$y_1 = y_0 + V_{oy}(t_f - t_0) + \frac{1}{2} a_y (t_f - t_0)^2$$

t_0 = time at firing (=0)
t_f = flight time to target
y_1 = y-position of bullet at impact

$$-0.02 = 0 + 0(t_f) + \frac{1}{2}(-9.8)t_f^2$$

$$\rightsquigarrow \boxed{t_f \doteq \sqrt{\frac{0.04}{9.8}} \simeq 0.064\ s}$$

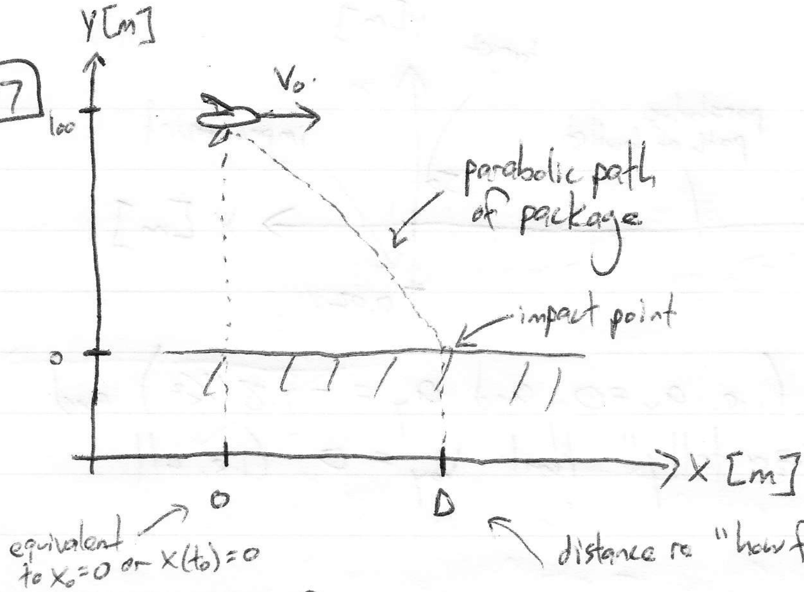
b) $X_1 = X_0 + V_{ox} t_f + \frac{1}{2} a_x t_f^2 = V_{ox} t_f$

bullet speed

$$50 = V_{ox}(0.064) \longrightarrow V_{ox} \doteq 780\ m/s$$

Since $V_{oy} = 0$, we have $\boxed{\bar{V}_0 = 780\ \hat{x}\ m/s}$ bullet velocity

P7



i.e. $V_{oy} = 0$

☐ Assume $\bar{V}_o = 150\hat{x}$ m/s (i.e. no vertical component) and let t_f be time of impact ($t_o = 0$)

Y_1 is $y(t_f)$

☐ First solve for time to fall 100 m: $y_1 = y_0 + V_{oy} t_f + \frac{1}{2} a_y t_f^2$

$$\rightsquigarrow \quad 0 = 100 + 0 - \frac{9.8}{2} t_f^2 \quad \Rightarrow \quad t_f \simeq 4.5 \text{ s}$$

☐ Now compute D given t_f and known V_{ox}

$$D = V_{ox} t_f \simeq (150)(4.5) \simeq \boxed{680 \text{ m}}$$

NOTE: with air resistance, the plane would want to drop the package closer to the target (see class notes)

P8 We are given the "constant speed" of 20 μm/s, but the average velocity (a vector!) is the net displacement (call it $\bar{D}$) divided by the total time. So we need to determine $\bar{D}$ (easy: just vector $\overline{AE}$) and the time (harder: need to find the length of each vector component and divide by 20 μm/s)

☐ First, to get $\bar{D}$: $\bar{D}$ = final position − start. position = $\overline{AE}$

$= 40\hat{x} - 20\hat{y}$ [μm] (i.e. $D_x = 40$, $D_y = -20$)

$|\bar{D}| = \sqrt{40^2 + 20^2} \simeq 45$ μm $\quad \Theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \tan^{-1}(-0.5) \simeq -0.46$ rads $\simeq -27°$

NOTE: The arctan (i.e. $\tan^{-1}$) can be a bit tricky!

P8 (cont.)

- Now find total time, which is sum over each path, i.e.

$$\Delta t = \Delta t_{AB} + \Delta t_{BC} + \Delta t_{CD} + \Delta t_{DE}$$

$$\Delta t_{AB} = D_{AB}[km] / 20 [km/s] = \frac{1}{20}\left[\sqrt{50^2+10^2}\right] \simeq 2.6 \text{ s}$$

$$\Delta t_{BC} = D_{BC}/20 = \frac{1}{20}[10] = 0.5 \text{ s}$$

$$\Delta t_{CD} = D_{CD}/20 = \frac{1}{20}\left[\sqrt{40^2+10^2}\right] \simeq 2.1 \text{ s}$$

$$\Delta t_{DE} = D_{DE}/20 = \frac{1}{20}\left[\sqrt{50^2+50^2}\right] \simeq 3.5 \text{ s}$$

$\rightsquigarrow$ $\Delta t \simeq 8.7$ s

$$\Rightarrow \boxed{|V_{net}| = \frac{|\vec{D}|}{\Delta t} = \frac{45}{8.7} \simeq 5.2 \text{ km/s}}$$ (magnitude)

$\rightsquigarrow$ As indicated prior, the "direction" is $\boxed{-27^\circ}$