<u>Review</u>: Comparison summary

TABLE 4.1 Rotational and linear kinematics for constant acceleration

Rotational kinematics	Linear kinematics
$\omega_f = \omega_i + \alpha\,\Delta t$	$v_{fs} = v_{is} + a_s\,\Delta t$
$\theta_f = \theta_i + \omega_i\,\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$	$v_{fs}^2 = v_{is}^2 + 2a_s\,\Delta s$

Rectilinear Motion		Rotation about a Fixed Axis	
Displacement	x	Angular displacement	θ
Velocity	$v = \frac{dx}{dt}$	Angular velocity	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	Angular acceleration	$\alpha = \frac{d\omega}{dt}$
Mass	M	Rotational inertia	I
Force	$F = Ma$	Torque	$\tau = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$\frac{1}{2}Mv^2$	Kinetic energy	$\frac{1}{2}I\omega^2$
Power	$P = Fv$	Power	$P = \tau\omega$
Linear momentum	Mv	Angular momentum	$I\omega$

Resnick & Halliday

Problem 1

A grindstone has a constant angular acceleration α of 3.0 radians/sec^2. Starting from rest a line, such as OP in Fig. 11–5, is horizontal. Find (*a*) the angular displacement of the line OP (and hence of the grindstone) and (*b*) the angular speed of the grindstone 2.0 sec later.

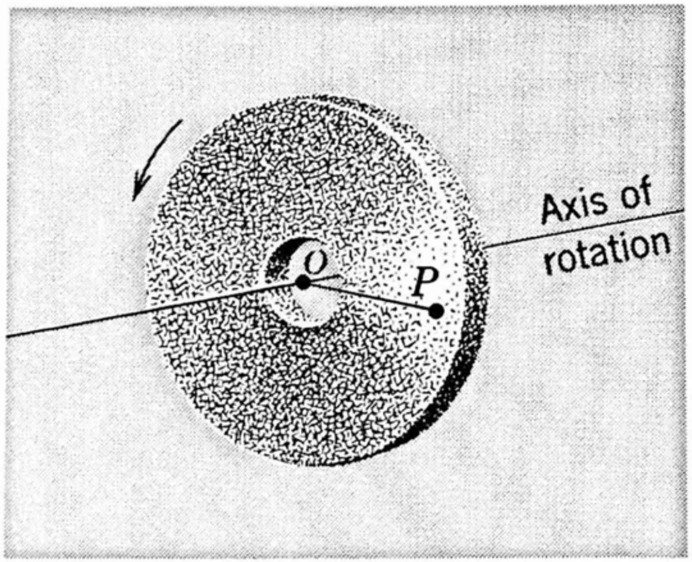

Fig. 11–5 Example 1. The line OP is attached to a grindstone rotating as shown about an axis through O that is fixed in the reference frame of the observer.

Problem 1 – Hints

TABLE 4.1 Rotational and linear kinematics for constant acceleration

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Knight (2013)

Problem 2

If the radius of the grindstone of Example 1 is 0.50 meter, calculate (*a*) the linear or tangential speed of a particle on the rim, (*b*) the tangential acceleration of a particle on the rim, and (*c*) the centripetal acceleration of a particle on the rim, at the end of 2.0 sec.

(*d*) Are the results the same for a particle halfway in from the rim, that is, at $r = 0.25$ meter?

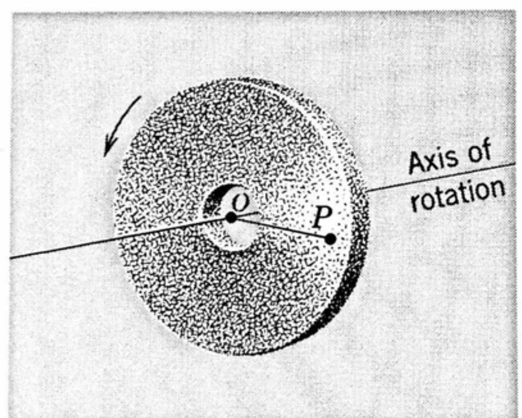


Fig. 11–5 Example 1. The line *OP* is attached to a grindstone rotating as shown about an axis through *O* that is fixed in the reference frame of the observer.

Problem 2 – Hints

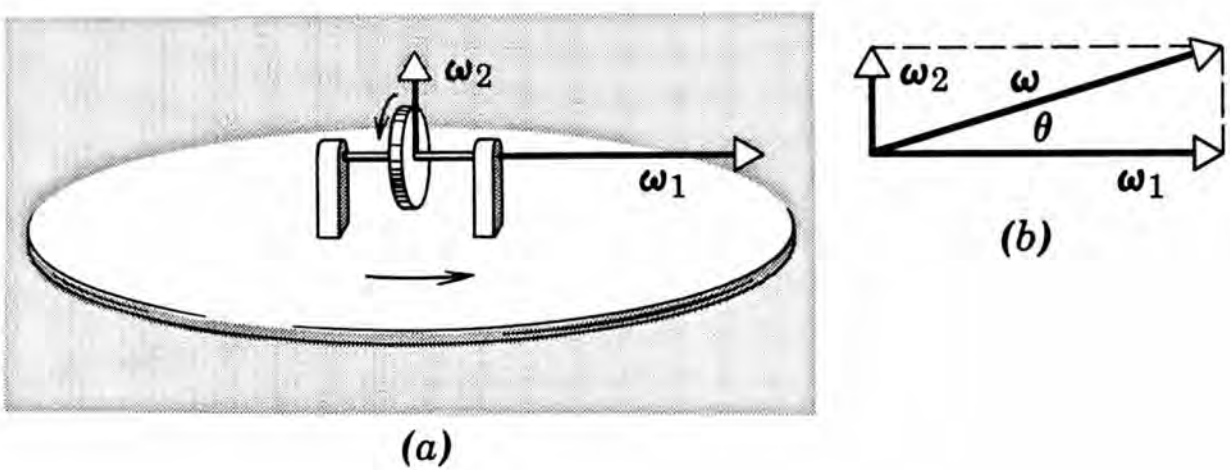
$$\mathbf{a} = \mathbf{a}_T + \mathbf{a}_R,$$

$$a_T = \alpha r$$ Tangential accel.

$$a_R = \omega^2 r = v^2/r.$$ Radial accel.

Problem 3

A disk spins on a horizontal shaft mounted in bearings, with an angular speed ω_1 of 100 radians/sec as in Fig. 11–8*a*. The entire disk and shaft assembly are placed on a turntable rotating about a vertical axis at $\omega_2 = 30.0$ radians/sec, counterclockwise as we view it from above. Describe the rotation of the disk as seen by an observer in the room.



Problem 3 – Hints

Angular velocity $\boldsymbol{\omega}$ is a vector. And you know how to add vectors....

Problem 4

A uniform disk of radius R and mass M is mounted on an axle supported in fixed frictionless bearings, as in Fig. 12–12. A light cord is wrapped around the rim of the wheel and a steady downward pull **T** is exerted on the cord. Find the angular acceleration of the wheel and the tangential acceleration of a point on the rim.

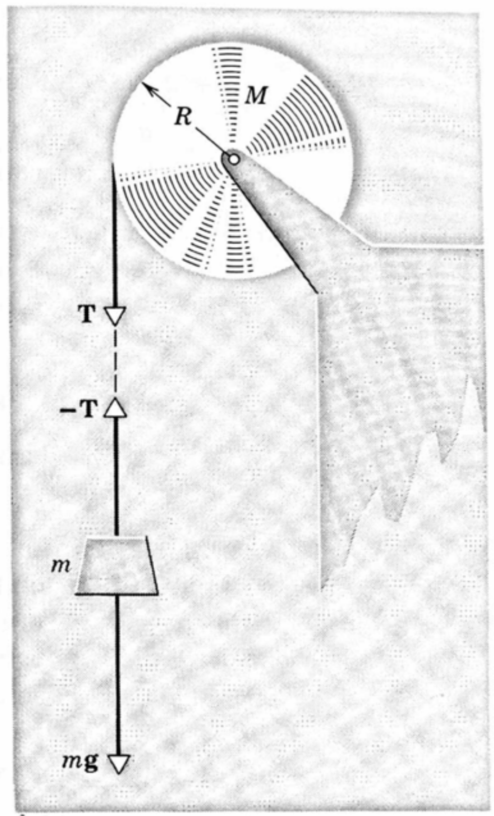


Fig. 12–12 Example 4. A steady downward force **T** produces rotation of the disk. Example 5. Here **T** is supplied by the falling mass m.

Problem 4 – Hints

$$\tau = TR,$$

$$I = \tfrac{1}{2}MR^2.$$

$$\tau = I\alpha,$$

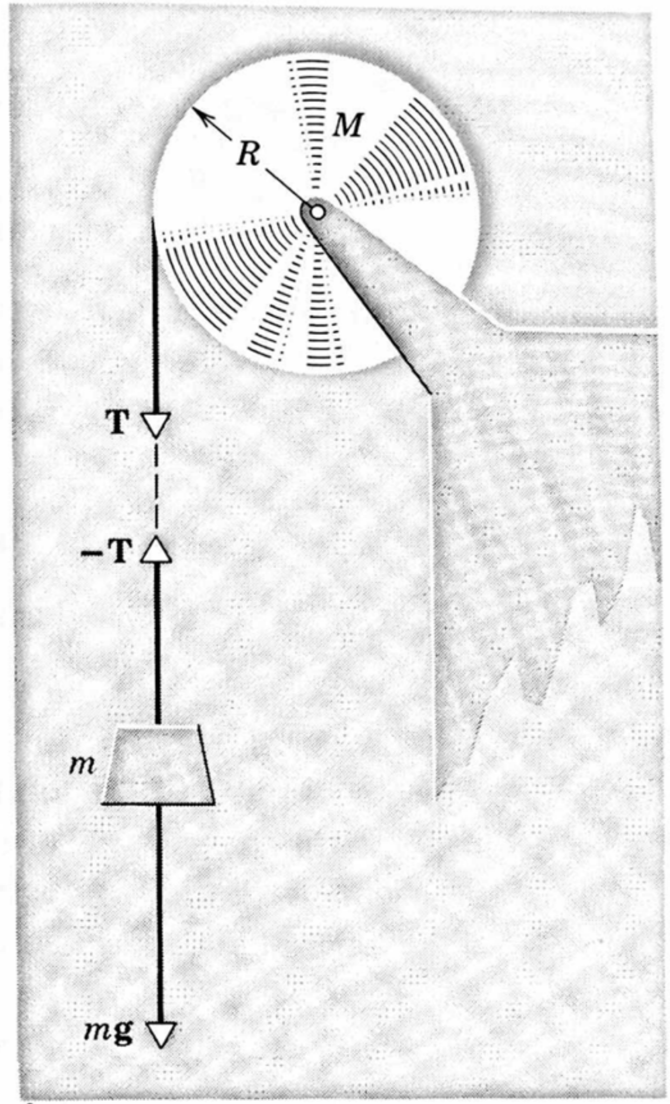


Fig. 12–12 Example 4. A steady downward force **T** produces rotation of the disk. Example 5. Here **T** is supplied by the falling mass *m*.

Problem 5

Suppose that we hang a body of mass m from the cord in the previous problem. Find the angular acceleration of the disk and the tangential acceleration of a point on the rim in this case.

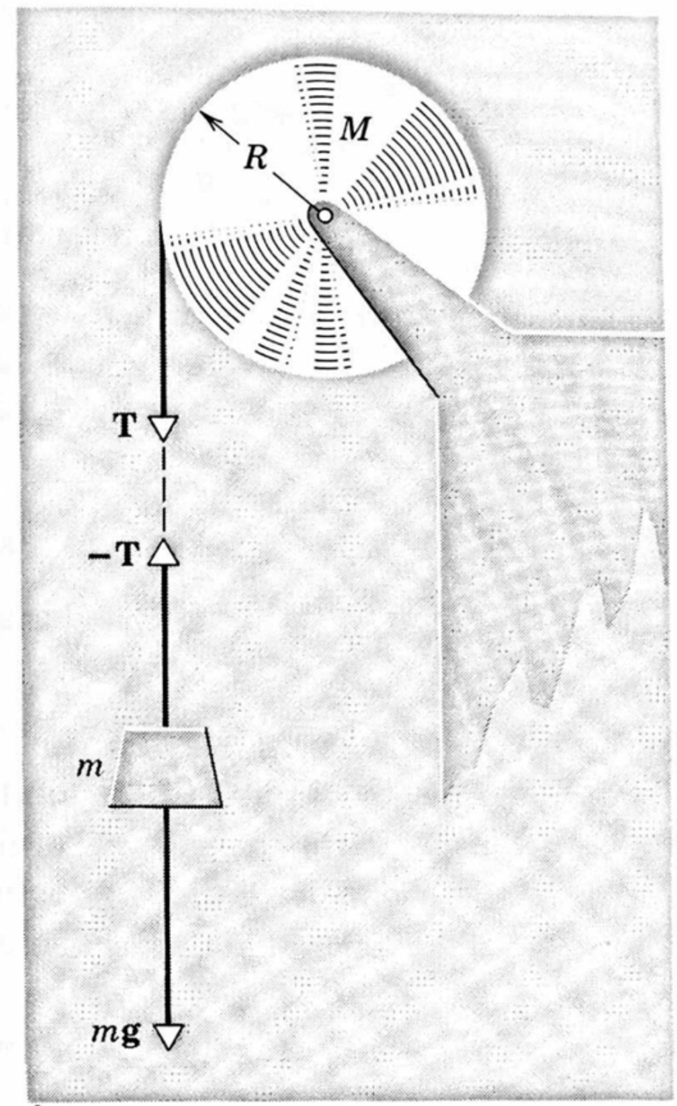


Fig. 12–12 Example 4. A steady downward force **T** produces rotation of the disk. Example 5. Here **T** is supplied by the falling mass m.

Problem 5 – Hints

$$mg - T = ma.$$

$$\tau = I\alpha, \quad \rightarrow \quad TR = \tfrac{1}{2}MR^2\alpha.$$

$$a = R\alpha, \quad \rightarrow \quad 2T = Ma.$$

Now you have two eqns. w/ two unknowns...

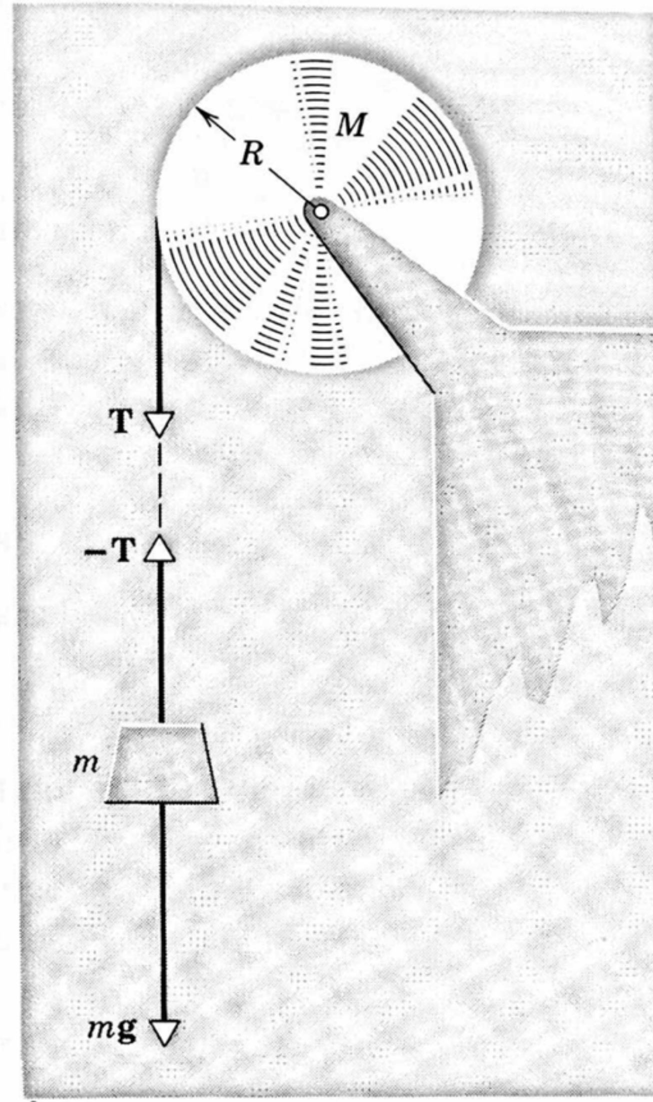
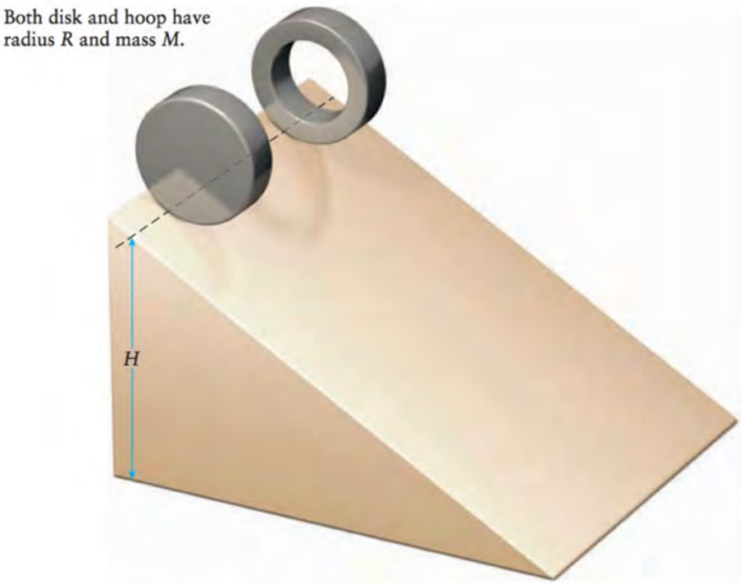


Fig. 12–12 Example 4. A steady downward force **T** produces rotation of the disk. Example 5. Here **T** is supplied by the falling mass m.

<u>Conservation of Energy</u> (Revisited)

When the smoke clears....



$$v_{\text{disk, f}} = \sqrt{\frac{4}{3}gH}$$

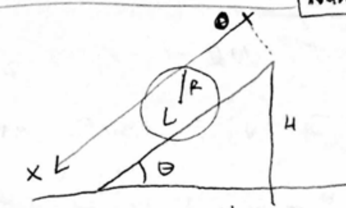
$$v_{\text{hoop, f}} = \sqrt{gH}$$

The "disk" wins the race. Perhaps not very intuitive at first....

... until you keep in mind a key principle at play here:
Energy can be stored in a variety of ways

<u>Note</u>: We will revisit this calculation during tutorial (there is a bit more here than meets the eye...)

1420 Alt. derivation re Kesten + Tauck (ch. 8.4 re Fig. 8-20) | ref. Nahin



- translational KE: $E_T = \frac{1}{2}mv^2$
- rotational KE: $E_R = \frac{1}{2}I\omega^2$
- Conservation of energy requires:

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgx\sin\theta$$

$\sin\theta = \frac{H}{L}$

$L = \frac{H}{\sin\theta}$

- Let $T(x)$ be the time for one revolution of the cylinder at distance x. Then

$$\omega(x) = \frac{2\pi}{T(x)} \longrightarrow T(x) = \frac{2\pi}{\omega} \quad [s]$$

$x \equiv$ dist. a cylinder has rolled down re the top ($x \in [0, L]$)

- Cylinder has radius R and thus circumfrence $2\pi R$, so:

$$v(x) = \frac{2\pi R}{T(x)} = \frac{2\pi R}{\frac{2\pi}{\omega}} = \omega R \longrightarrow \omega(x) = \frac{v(x)}{R}$$

- Energy conserv. thus has: $\frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2} = mgx\sin(\theta)$ (true for both cylinders)

Solid cylinder (i.e. "disk")

- $I_s = \frac{1}{2}MR^2 \rightsquigarrow \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2} = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$
$= mgx\sin\theta$

or $v^2 = \frac{4gx\sin\theta}{3}$

- Now since $v = \frac{dx}{dt} \rightsquigarrow \frac{dx}{dt} = \sqrt{\frac{4g\sin\theta}{3}}\sqrt{x}$
- Now separate variable and integrate both sides (t_s = time it takes solid cylinder to reach the bottom)

$$\int_0^L \frac{dx}{\sqrt{x}} = \int_0^{t_s}\sqrt{\frac{4g\sin\theta}{3}}\,dt = 2\sqrt{x}\Big|_0^L = 2\sqrt{L}$$
$$= 2t_s\sqrt{\frac{g\sin\theta}{3}}$$

NOTE: Anti-derivative of $\frac{1}{\sqrt{x}}$ is $2\sqrt{x}$

$$\longrightarrow \boxed{t_s = \sqrt{\frac{3L}{g\sin\theta}}}$$

Hollow cylinder (i.e. "hoop")

- $I_H = MR^2 \rightsquigarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2 = mgx\sin\theta$
- So $v^2 = gx\sin\theta$ or $\frac{dx}{dt} = \sqrt{g\sin\theta}\sqrt{x}$
- Similarly as before:

$$\int_0^L \frac{dx}{\sqrt{x}} = 2\sqrt{L} = \int_0^{t_H} \sqrt{g\sin\theta}\,dt = t_H\sqrt{g\sin\theta}$$

$$\longrightarrow \boxed{t_H = \sqrt{\frac{4L}{\sin\theta}}}$$

- Now note that: $\frac{t_H}{t_S} = \sqrt{\frac{4L}{g\sin\theta}}\sqrt{\frac{g\sin\theta}{3L}} = \sqrt{\frac{4}{3}} \simeq 1.155$

- okay, for each we know the distance covered when they get to the bottom ($L = \frac{H}{\sin\theta}$) and the time it took, so we can calculate the average velocity:

$$V_S = \frac{L}{t_S} = \frac{\frac{H}{\sin\theta}}{\sqrt{\frac{3L}{g\sin\theta}}} = \frac{\frac{H\sqrt{g}}{\sin\theta}}{\sqrt{3}\sqrt{\frac{H}{\sin^2\theta}}} = \frac{\sqrt{gH}}{\sqrt{3}}$$

$$V_H = \frac{L}{t_H} = \frac{\frac{H}{\sin\theta}}{\sqrt{\frac{4L}{g\sin\theta}}} = \frac{\sqrt{gH}}{2}$$

$\Longrightarrow$ Note that this is different from K+T (they had $V_S = \sqrt{\frac{4}{3}gH}$ and and $V_H = \sqrt{gH}$), though they have a similar ratio for V_S/V_H ($\simeq 1.155$). The source of the discrepancy is ..