Problem 1 SOL

(*a*) α and t are given; we wish to find θ. Hence, we use Eq. 11–5,

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2.$$

At $t = 0$, we have $\omega = \omega_0 = 0$ and $\alpha = 3.0$ radians/sec^2. Therefore, after 2.0 sec,

$$\theta = (0)(2.0 \text{ sec}) + \tfrac{1}{2}(3.0 \text{ radians/sec}^2)(2.0 \text{ sec})^2 = 6.0 \text{ radians} = 0.96 \text{ rev}.$$

(*b*) α and t are given; we wish to find ω. Hence we use Eq. 11–3

$$\omega = \omega_0 + \alpha t,$$

and

$$\omega = 0 + (3.0 \text{ radians/sec}^2)(2.0 \text{ sec}) = 6.0 \text{ radians/sec}.$$

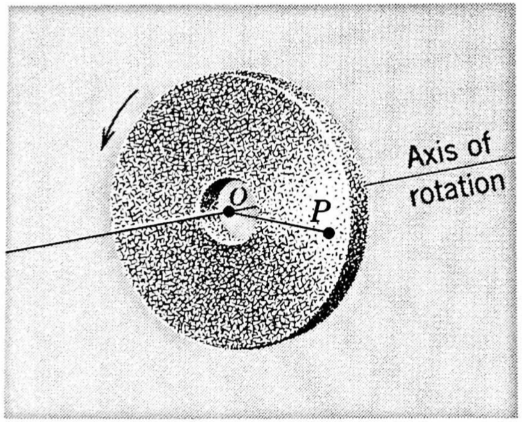


Fig. 11–5 Example 1. The line OP is attached to a grindstone rotating as shown about an axis through O that is fixed in the reference frame of the observer.

as a check, we have

$$\omega^2 = \omega_0{}^2 + 2\alpha\theta,$$

$$\omega^2 = 0 + (2)(3.0 \text{ radians/sec}^2)(6.0 \text{ radians}) = 36 \text{ radians}^2/\text{sec}^2,$$

$$\omega = 6.0 \text{ radians/sec}.$$

Problem 2 SOL

We have $\alpha = 3.0$ radians/sec^2, $\omega = 6.0$ radians/sec after 2.0 sec, and $r = 0.50$ meter. Then,

(*a*)
$$\begin{aligned} v &= \omega r \\ &= (6.0 \text{ radians/sec})(0.50 \text{ meter}) \\ &= 3.0 \text{ meter/sec} \qquad \text{(linear speed)}; \end{aligned}$$

(*b*)
$$\begin{aligned} a_T &= \alpha r \\ &= (3.0 \text{ radians/sec}^2)(0.50 \text{ meter}) \\ &= 1.5 \text{ meters/sec}^2 \qquad \text{(tangential acceleration)}; \end{aligned}$$

(*c*)
$$\begin{aligned} a_R &= v^2/r = \omega^2 r \\ &= (6.0 \text{ radians/sec})^2(0.50 \text{ meter}) \\ &= 18 \text{ meters/sec}^2 \qquad \text{(centripetal acceleration)}. \end{aligned}$$

(*d*) Are the results the same for a particle halfway in from the rim, that is, at $r = 0.25$ meter?

The *angular* variables are the same for this point as for a point on the rim. That is, once again

$$\alpha = 3.0 \text{ radians/sec}^2, \qquad \omega = 6.0 \text{ radians/sec}.$$

But now $r = 0.25$ meter, so that for this particle

$$v = 1.5 \text{ meters/sec}, \qquad a_T = 0.75 \text{ meter/sec}^2, \qquad a_R = 9.0 \text{ meters/sec}^2. \quad ◀$$

Problem 3 SOL

The disk is subject to two angular velocities simultaneously; we can describe its resultant motion by the vector sum of these vectors. The angular velocity $\boldsymbol{\omega}_1$ associated with the shaft rotation has a magnitude of 100 radians/sec and occurs about an axis that is not fixed but, as seen by an observer in the room, rotates in a horizontal plane at 30 radians/sec. The angular velocity $\boldsymbol{\omega}_2$ associated with the turntable is fixed vertically and has a magnitude of 30 radians/sec.

The resultant angular velocity of the disk $\boldsymbol{\omega}$ is the vector sum of $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$. The magnitude of $\boldsymbol{\omega}$ is

$$\omega = \sqrt{\omega_1^2 + \omega_2^2} = \sqrt{(100 \text{ radians/sec})^2 + (30.0 \text{ radians/sec})^2}$$

$$= 104 \text{ radians/sec.}$$

The direction of $\boldsymbol{\omega}$ is not fixed in our observer's reference frame but rotates at the same angular rate as the turntable. The vector $\boldsymbol{\omega}$ does not lie in the horizontal plane but points above it by an angle θ (see Fig. 11–8*b*), where

$$\theta = \tan^{-1} \omega_2/\omega_1 = \tan^{-1} (30.0 \text{ radians/sec})/(100 \text{ radians/sec})$$

$$= \tan^{-1} 0.300 = 16.7°$$

We can describe the motion of the disk as a simple rotation about this new axis (whose direction in our observer's reference frame is changing with time as described above) at an angular rate of 104 radians/sec. How would the situation change if the direction of rotation of the disk, or of the turntable, were changed? ◀

Problem 4 SOL

The torque about the central axis is $\tau = TR$, and the rotational inertia of the disk about its central axis is $I = \frac{1}{2}MR^2$. From

$$\tau = I\alpha,$$

we have

$$TR = (\tfrac{1}{2}MR^2)\alpha,$$

or

$$\alpha = \frac{2T}{MR}.$$

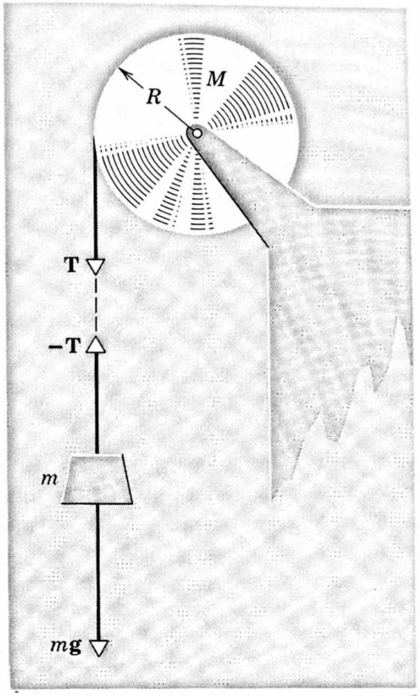


Fig. 12–12 Example 4. A steady downward force **T** produces rotation of the disk. Example 5. Here **T** is supplied by the falling mass m.

Problem 5 SOL

Now, let T be the tension in the cord. Since the suspended body will accelerate downward, the magnitude of the downward pull of gravity on it, mg, must exceed the magnitude of the upward pull of the cord on it, T. The acceleration a of the suspended body is the same as the tangential acceleration of a point on the rim of the disk. From Newton's second law

$$mg - T = ma.$$

The resultant torque on the disk is TR and its rotational inertia is $\frac{1}{2}MR^2$, so that from

$$\tau = I\alpha$$

we obtain

$$TR = \tfrac{1}{2}MR^2\alpha.$$

Using the relation $a = R\alpha$, we can write this last equation as

$$2T = Ma.$$

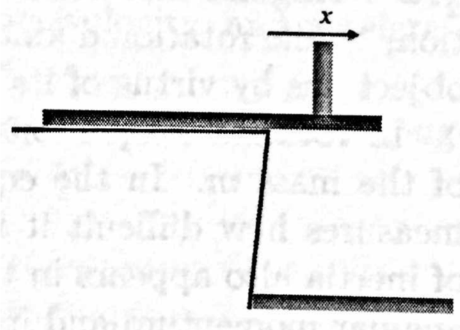
Solving the first and last equations simultaneously leads to

$$a = \left(\frac{2m}{M + 2m}\right) g,$$

and

$$T = \left(\frac{Mm}{M + 2m}\right) g.$$

Problem 6 SOL

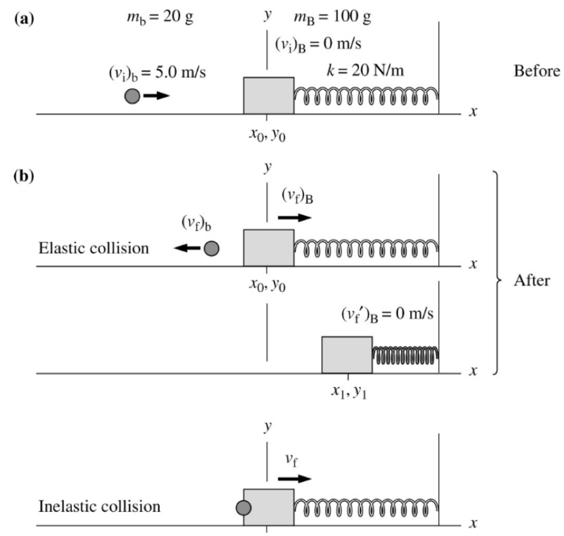


We'll use the torque equilibrium equation $\sum \mathcal{T}_E = 0$, where we calculate the torques relative to the edge of the ship, the point around which the plank will pivot. There are two torques involved: the torque produced by the plank's weight and the torque produced by the person's weight. The plank's weight acts in its centre of mass, which is located 2[m] from the edge of the ship. The torque produced by the weight of the plank is therefore given by $\mathcal{T}_2 = 120g \times 2 = 240g$[N m]. The torque produced by the person when he reaches a distance of x[m] from the edge of the ship is $\mathcal{T}_1 = -100gx$[N m]. Thus, the maximum distance the person can walk before the plank tips is $x = \frac{240g}{100g} = 2.4$[m].

Problem 7 SOL

Model: We assume the spring to be ideal and to obey Hooke's law. We also treat the block (B) and the ball (b) as particles. In the case of an elastic collision, both the momentum and kinetic energy equations apply. On the other hand, for a perfectly inelastic collision only the equation of momentum conservation is valid.

Visualize:



Place the origin of the coordinate system on the block that is attached to one end of the spring. The before-and-after pictorial representations of the elastic and perfectly inelastic collision are shown in figures (a) and (b), respectively.

Solve: (a) For an elastic collision, the ball's rebound velocity is

$$(v_f)_b = \frac{m_b - m_B}{m_b + m_B}(v_i)_b = \frac{-80 \text{ g}}{120 \text{ g}}(5.0 \text{ m/s}) = -3.33 \text{ m/s}$$

The ball's speed is 3.3 m/s.

(b) An elastic collision gives the block speed

$$(v_f)_B = \frac{2m_B}{m_b + m_B}(v_i)_b = \frac{40 \text{ g}}{120 \text{ g}}(5.0 \text{ m/s}) = 1.667 \text{ m/s}$$

To find the maximum compression of the spring, we use the conservation equation of mechanical energy for the block + spring system. That is $K_1 + U_{s1} = K_0 + U_{s0}$:

$$\frac{1}{2}m_B(v_f')_B^2 + \frac{1}{2}k(x_1 - x_0)^2 = \frac{1}{2}m_B(v_f)_B^2 + \frac{1}{2}k(x_0 - x_0)^2 \quad 0 + k(x_1 - x_0)^2 = m_B(v_f)_B^2 + 0$$

$$(x_1 - x_0) = \sqrt{(0.100 \text{ kg})(1.667 \text{ m/s})^2/(20 \text{ N/m})} = 11.8 \text{ cm}$$

(c) Momentum conservation $p_f = p_i$ for the perfectly inelastic collision means

$$(m_B + m_B)v_f = m_b(v_i)_b + m_B(v_i)_B$$

$$(0.100 \text{ kg} + 0.020 \text{ kg})v_f = (0.020 \text{ kg})(5.0 \text{ m/s}) + 0 \text{ m/v} \Rightarrow v_f = 0.833 \text{ m/s}$$

The maximum compression in this case can now be obtained using the conservation of energy equation $K_1 + U_{s1} = K_0 + U_{s0}$:

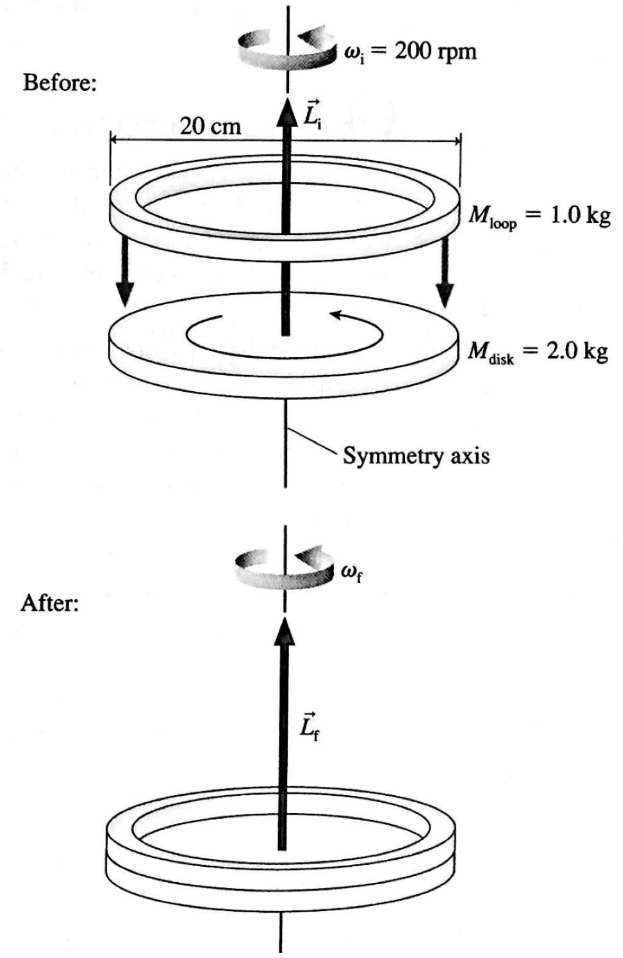
$$0 \text{ J} + (1/2)k(\Delta x)^2 = (1/2)(m_B + m_b)v_f^2 + 0 \text{ J}$$

$$\Rightarrow \Delta x = \sqrt{\frac{m_B + m_b}{k}}v_f = \sqrt{\frac{0.120 \text{ kg}}{20 \text{ N/m}}}(0.833 \text{ m/s}) = 0.0645 \text{ m} = 6.5 \text{ cm}$$

Problem 8 SOL

MODEL The friction between the two objects creates torques that speed up the loop and slow down the disk. But these torques are internal to the combined disk + loop system, so $\tau_{net} = 0$ and the *total* angular momentum of the disk + loop system is conserved.

VISUALIZE **FIGURE 12.58** is a before-and-after pictorial representation. Initially only the disk is rotating, at angular velocity $\vec{\omega}_i$. The rotation is about an axis of symmetry, so the angular momentum $\vec{L} = I\vec{\omega}$ is parallel to $\vec{\omega}$. At the end of the problem, $\vec{\omega}_{disk} = \vec{\omega}_{loop} = \vec{\omega}_f$.



SOLVE Both angular momentum vectors point along the rotation axis. Conservation of angular momentum tells us that the magnitude of $\vec{L}$ is unchanged. Thus

$$L_f = I_{disk}\omega_f + I_{loop}\omega_f = L_i = I_{disk}\omega_i$$

Solving for ω_f gives

$$\omega_f = \frac{I_{disk}}{I_{disk} + I_{loop}}\omega_i$$

The moments of inertia for a disk and a loop can be found in Table 12.2, leading to

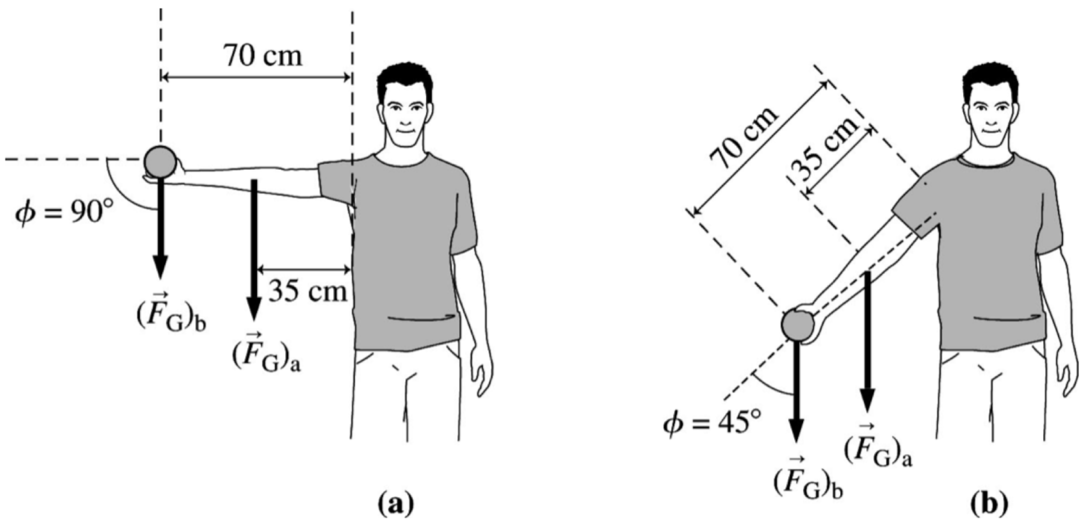
$$\omega_f = \frac{\frac{1}{2}M_{disk}R^2}{\frac{1}{2}M_{disk}R^2 + M_{loop}R^2}\omega_i = 100 \text{ rpm}$$

ASSESS What appeared to be a difficult problem turns out to be fairly easy once you recognize that the total angular momentum is conserved.

Problem 9 SOL

Model: Model the arm as a uniform rigid rod. Its mass acts at the center of mass.

Visualize:



Solve: **(a)** The torque is due both to the gravitational force on the ball and the gravitational force on the arm:

$$\tau = \tau_{\text{ball}} + \tau_{\text{arm}} = (m_b g)r_b \sin 90° + (m_a g)r_a \sin 90°$$
$$= (3.0 \text{ kg})(9.8 \text{ m/s}^2)(0.70 \text{ m}) + (4.0 \text{ kg})(9.8 \text{ m/s}^2)(0.35 \text{ m}) = 34 \text{ N m}$$

(b) The torque is reduced because the moment arms are reduced. Both forces act at $\phi = 45°$ from the radial line, so

$$\tau = \tau_{\text{ball}} + \tau_{\text{arm}} = (m_b g)r_b \sin 45° + (m_a g)r_a \sin 45°$$
$$= (3.0 \text{ kg})(9.8 \text{ m/s}^2)(0.70 \text{ m})(0.707) + (4.0 \text{ kg})(9.8 \text{ m/s}^2)(0.35 \text{ m})(0.707) = 24 \text{ N m}$$

Problem 10 SOL

Model: Model the turntable as a rigid disk rotating on frictionless bearings. As the blocks fall from above and stick on the turntable, the turntable slows down due to increased rotational inertia of the (turntable + blocks) system. Any torques between the turntable and the blocks are internal to the system, so angular momentum of the system is conserved.

Visualize: The initial moment of inertia is I_1 and the final moment of inertia is I_2.

Solve: The initial moment of inertia is $I_1 = I_{\text{disk}} = \frac{1}{2}mR^2 = \frac{1}{2}(2.0\text{ kg})(0.10\text{ m})^2 = 0.010\text{ kg m}^2$ and the final moment of inertia is

$$I_2 = I_1 + 2mR^2 = 0.010\text{ kg m}^2 + 2(0.500\text{ kg})\times(0.10\text{ m})^2 = 0.010\text{ kg m}^2 + 0.010\text{ kg m}^2 = 0.020\text{ kg m}^2$$

Let ω_1 and ω_2 be the initial and final angular velocities. Then

$$L_f = L_i \Rightarrow \omega_2 I_2 = \omega_1 I_1 \Rightarrow \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{(0.010\text{ kg m}^2)(100\text{ rpm})}{0.020\text{ kg m}^2} = 50\text{ rpm}$$

Extra Credit SOL

Solve: The bricks are stable when the net gravitational torque on each individual brick or combination of bricks is zero. This is true as long as the center of gravity of each individual brick and any combination is over a base of support. To determine the relative positions of the bricks, work from the top down. The top brick can extend past the second brick by $L/2$. For maximum extension, their combined center of gravity will be at the edge of the third brick, and the combined center of gravity of the three upper bricks will be at the edge of the fourth brick. The combined center of gravity of all four bricks will be over the edge of the table.

Measuring from the left edge of brick 2, the center of gravity of the top two bricks is

$$(x_{12})_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m\left(\frac{L}{2}\right) + mL}{2m} = \frac{3}{4}L$$

Thus the top two bricks can extend $L/4$ past the edge of the third brick. The top three bricks have a center of mass

$$(x_{123})_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{m\left(\frac{L}{2}\right) + m\left(\frac{3L}{4}\right) + m\left(\frac{5L}{4}\right)}{3m} = \frac{5}{6}L$$

Thus the top three bricks can extend past the edge of the fourth brick by $L/6$. Finally, the four bricks have a combined center of mass at

$$(x_{1234})_{\text{com}} = \frac{m\left(\frac{L}{2}\right) + m\left(\frac{4L}{6}\right) + m\left(\frac{11L}{12}\right) + m\left(\frac{17L}{12}\right)}{4m} = \frac{7}{8}L$$

The center of gravity of all four bricks combined is $7L/8$ from the left edge of the bottom brick, so brick 4 can extend $L/8$ past the table edge. Thus the maximum distance to the right edge of the top brick from the table edge is

$$d_{\text{max}} = \frac{L}{8} + \frac{L}{6} + \frac{L}{4} + \frac{L}{2} = \frac{25}{24}L$$

Thus, yes, it is possible that no part of the top brick is directly over the table because $d_{\text{max}} > L$.

Assess: As crazy as this seems, the center of gravity of all four bricks is stably supported, so the net gravitational torque is zero, and the bricks do not fall over.