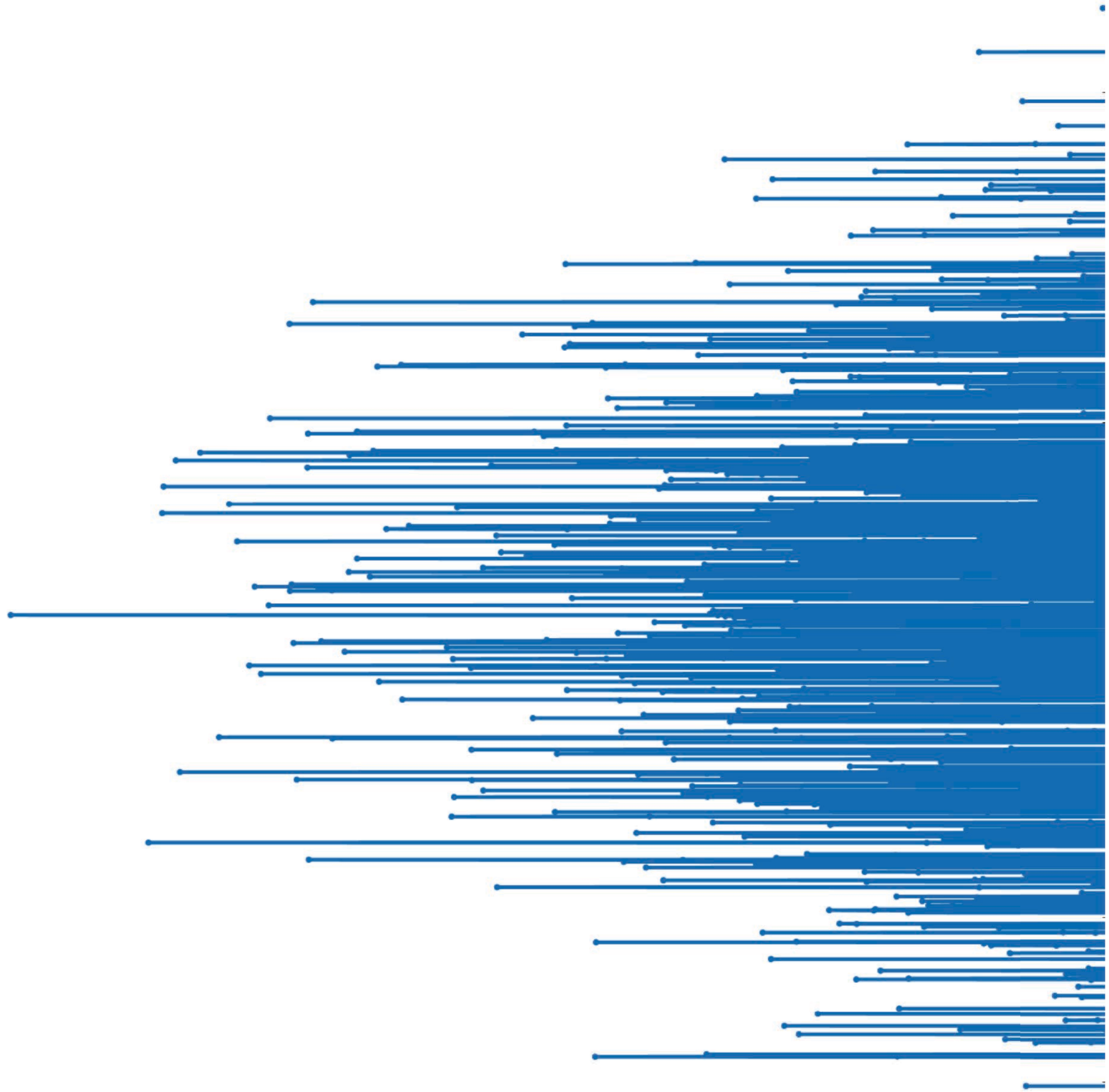
PHYS 2010 (W20)
Classical Mechanics



Due 2020.03.05
HW4

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Ref. (re images):
Knudsen & Hjorth (2000), Kesten & Tauck (2012)

Problem 1

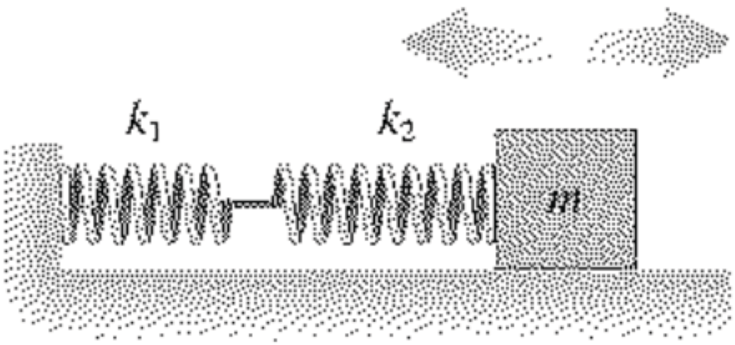
|| The 15 g head of a bobble-head doll oscillates in SHM at a frequency of 4.0 Hz.

a. What is the spring constant of the spring on which the head is mounted?

b. The amplitude of the head's oscillations decreases to 0.5 cm in 4.0 s. What is the head's damping constant?

Problem 2

|| A block on a frictionless table is connected as shown below to two springs having spring constants k_1 and k_2. Find an expression for the block's oscillation frequency f in terms of the frequencies f_1 and f_2 at which it would oscillate if attached to spring 1 or spring 2 alone.



Hint: Drawing a free-body diagram is likely useful. The mass will exert the same force on both springs, but the displacement of each will be different.

Problem 3

A particle undergoing simple harmonic motion has a velocity $\dot{x}_1$ when the displacement is x_1 and a velocity $\dot{x}_2$ when the displacement is x_2. Find the angular frequency and the amplitude of the motion in terms of the given quantities.

Hint: Since this is SHO, energy is conserved....

Problem 4

Show that the ratio of two successive maxima in the displacement of a damped harmonic oscillator is constant. (*Note:* The maxima do not occur at the points of contact of the displacement curve with the curve $Ae^{-\gamma t}$.)

Problem 5

The frequency f_d of a damped harmonic oscillator is 100 Hz, and the ratio of the amplitude of two successive maxima is one half.

(a) What is the undamped frequency f_0 of this oscillator?

(b) What is the resonant frequency f_r?

Note: Your solution to the previous problem will likely be helpful here...

Problem 6

Solve the differential equation of motion of the damped harmonic oscillator driven by a damped harmonic force:

$$F_{ext}(t) = F_0 e^{-\alpha t} \cos \omega t$$

(Hint: $e^{-\alpha t} \cos \omega t = Re(e^{-\alpha t + i\omega t}) = Re(e^{\beta t})$*, where* $\beta = -\alpha + i\omega$*. Assume a solution of the form* $Ae^{\beta t - i\phi}$*.)*

Hint: Consider the ODE to be $mx'' + cx'' + kx'' = F_{ext}$. You need to solve for A and ϕ in terms of the specified variables.

Problem 7

Would you be willing to pay 20 cents for an object valued by a mathematician at $\$j^j$? (Remember that $\cos\theta + j\sin\theta = e^{j\theta}$.)

Note: If the key bit is hard to read, for clarity it says: j^j

Problem 8

The motion of a linear oscillator may be represented by means of a graph in which x is shown as abscissa and dx/dt as ordinate. The history of the oscillator is then a curve.

(a) Show that for an undamped oscillator this curve is an ellipse.

(b) Show (at least qualitatively) that if a damping term is introduced one gets a curve spiraling into the origin.

Hint: If you are stuck, try using pplane!