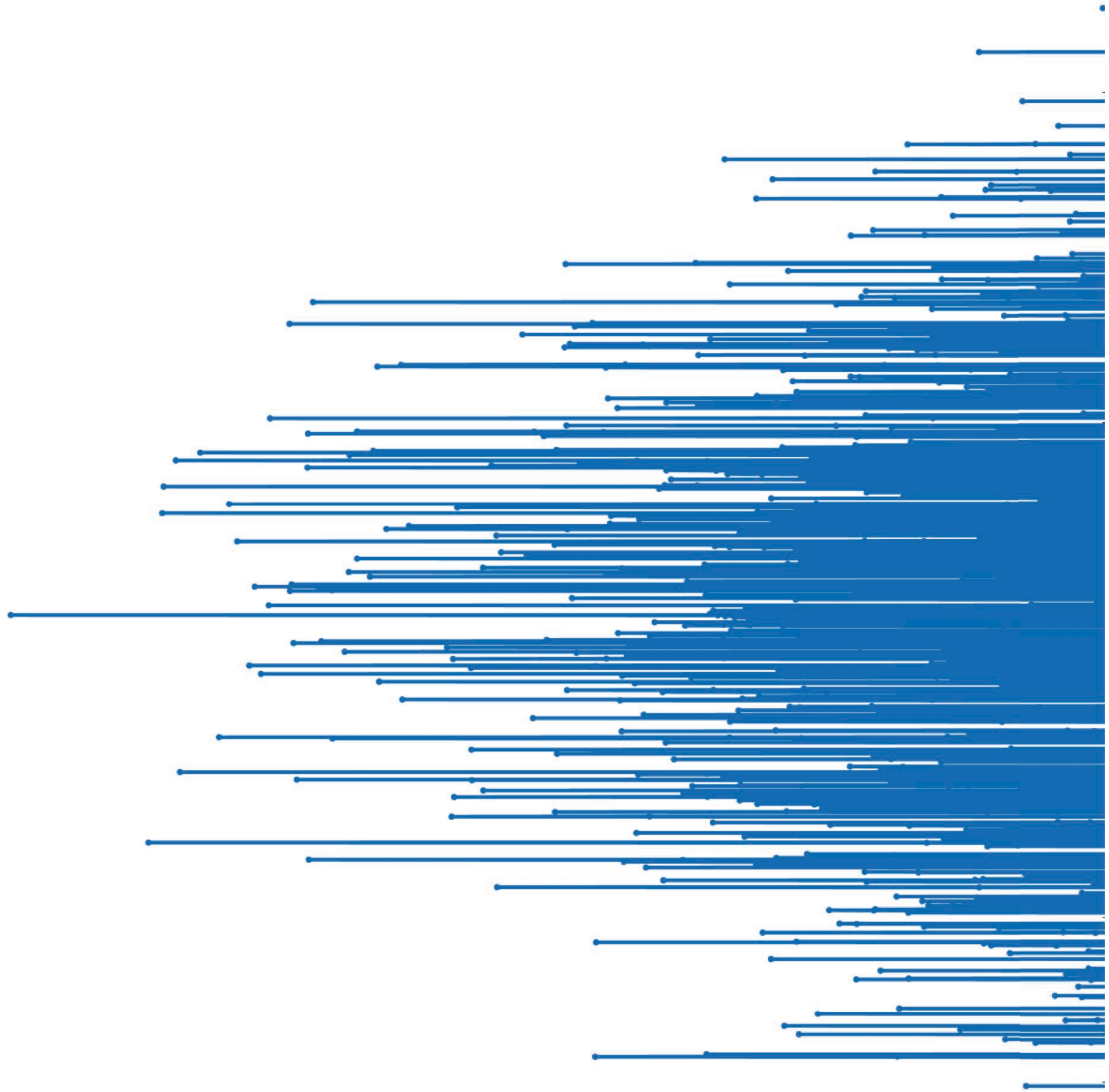
PHYS 2010 (W20)
Classical Mechanics



2020.01.14

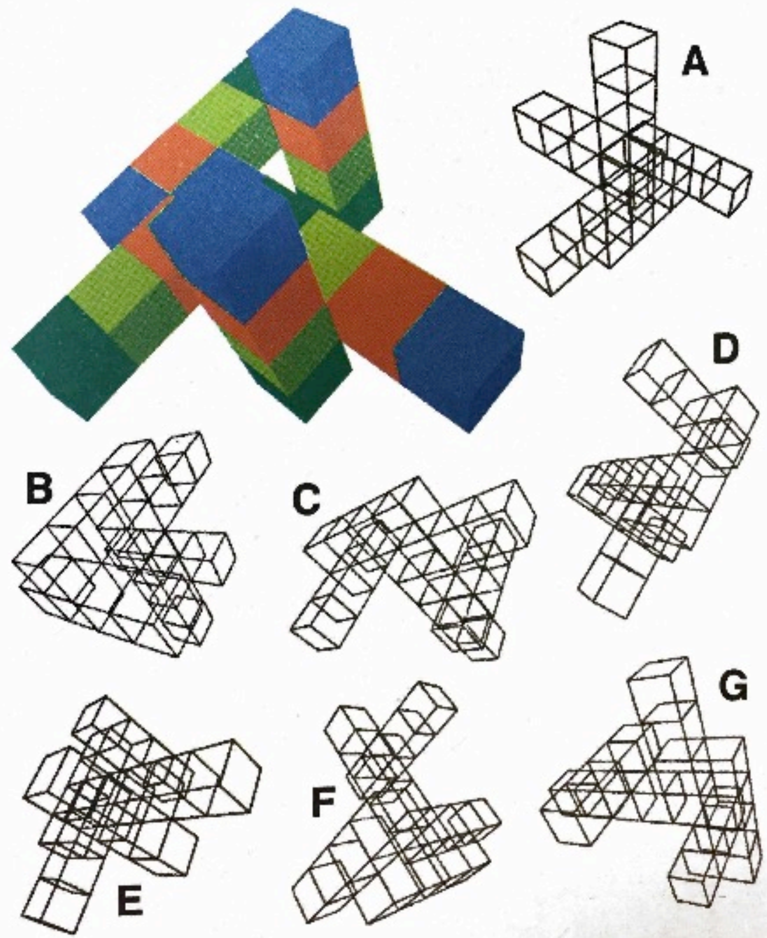
Relevant reading:
Knudsen & Hjorth: 2.1-2.2

Christopher Bergevin
York University, Dept. of Physics & Astronomy
Office: Petrie 240   Lab: Farq 103
cberge@yorku.ca

Ref. (re images):
Knudsen & Hjorth (2000), Kesten & Tauck (2012)

You've Been Framed

All the frame constructions below are accurate representations of the main picture, except one. Can you find the frame that doesn't belong?



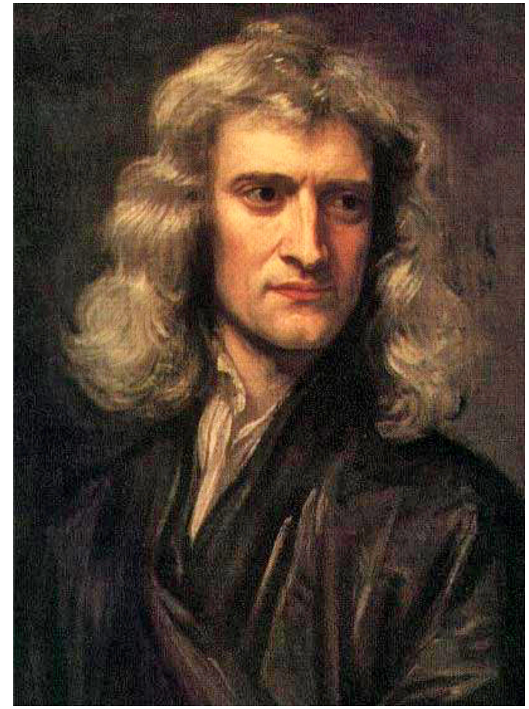
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *J S.* NEWTON, *Trin. Coll. Cantab. Soc.* Mathefeos Profeffore *Lucafiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,

Juffu *Societatis Regiæ* ac Typis *Jofephi Streater*. Proftat apud plures Bibliopolas. *Anno* MDCLXXXVII.



Facsimile of the title page of the first edition of Newton's Principia *(published 1687). It may be seen that the work was officially accepted by the Royal Society of London in July, 1686, when its president was the famous diarist Samuel Pepys (who was also Secretary to the Admiralty at the time).*

French (1971)

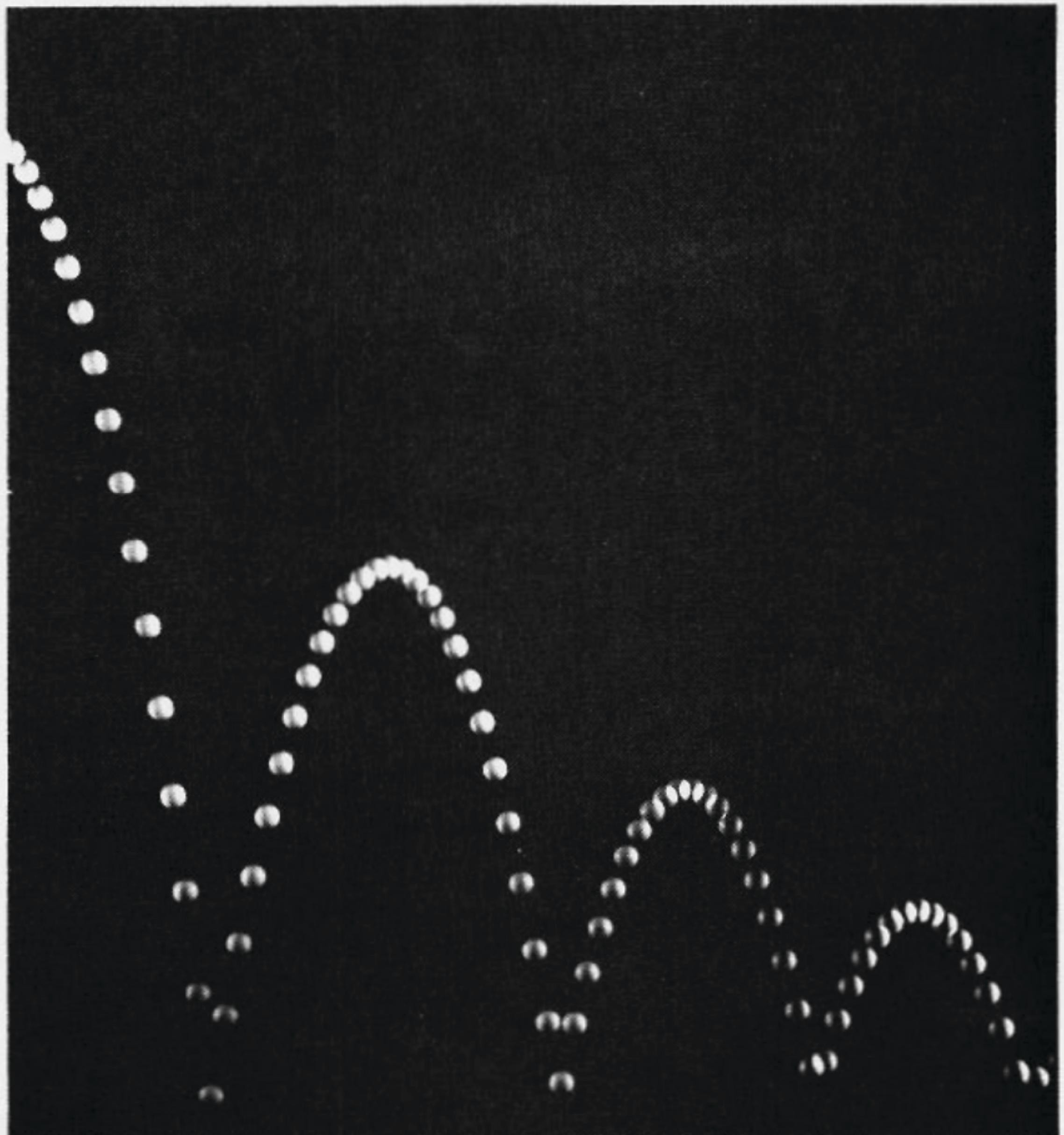


Fig. 2–1 Stroboscopic photograph of a motion. (From PSSC Physics, *D. C. Heath, Lexington, Massachusetts, 1965.)*

French (1971)

2 Motion in a Straight Line | 3 Motion in Two and Three Dimensions | **4** | 5 Using Newton's Laws | 6 Energy, Work, and Power

Force and Motion

- Consider this bit from a 1st year textbook

- The most fundamental comes right off the bat (pun!): *change*

4.1 The Wrong Question

The Right Question

Our first question—about why the spacecraft keeps moving—is the wrong question. So what's the right question? It's the second one, about why the baseball's motion *changed*. Dynamics isn't about what causes motion itself; it's about what causes *changes* in motion.

<u>Review</u>: Uniform vs Changing Motion

- Subtle but important differences at play here....

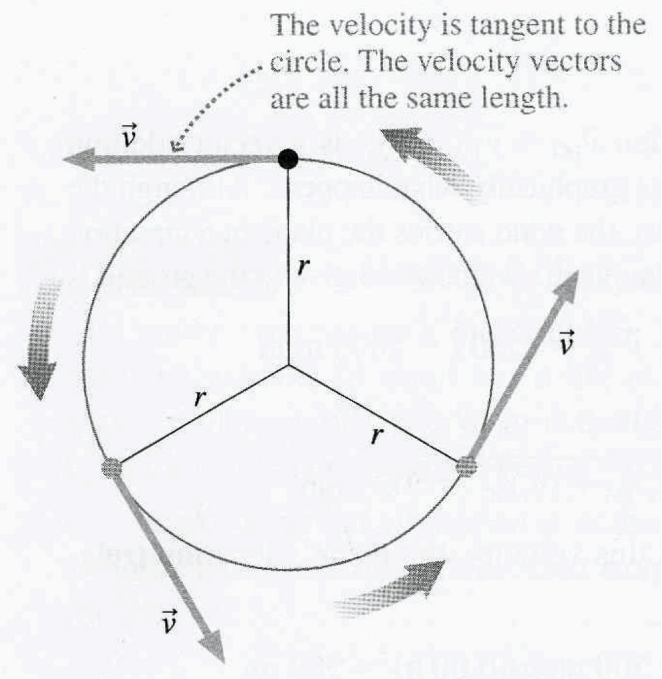
- Note that here some things are changing (e.g., θ, direction of a and v)....

- ... while others are not (e.g., speed, magnitude of a and v)

→ So in some sense, there is *changing change* (i.e., "non-uniform" motion) and *unchanging change* (i.e., "uniform" motion)

→ Such is the basis for introducing a key concept: *Force*

FIGURE 4.25 A particle in uniform circular motion.

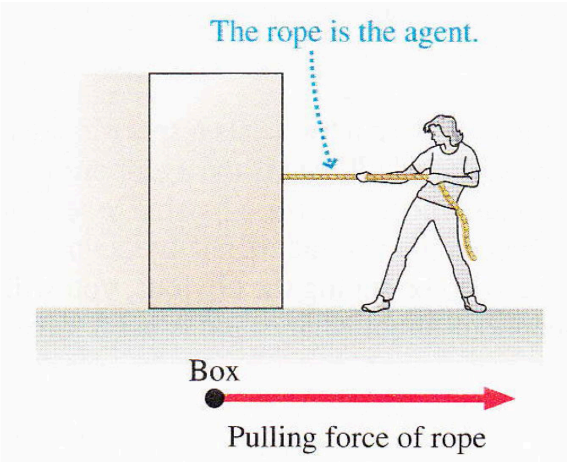


<u>Note</u>: You will see this distinction again elsewhere, though typically w/ different jargon (e.g., the notion of *steady-state* and *non-equilibrium* in chemistry/physics/biology)

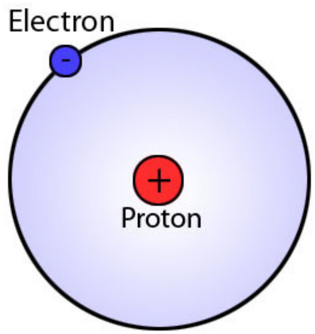
<u>Careful</u>: "Uniform motion" and "uniform circular motion" are not strictly the same thing....

Review: Force

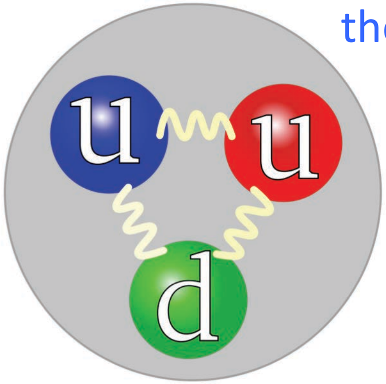
- Very fundamental concept in physics. Allows us to describe/understand how the motion of an object changes



Definition:
"Force causes change in motion"

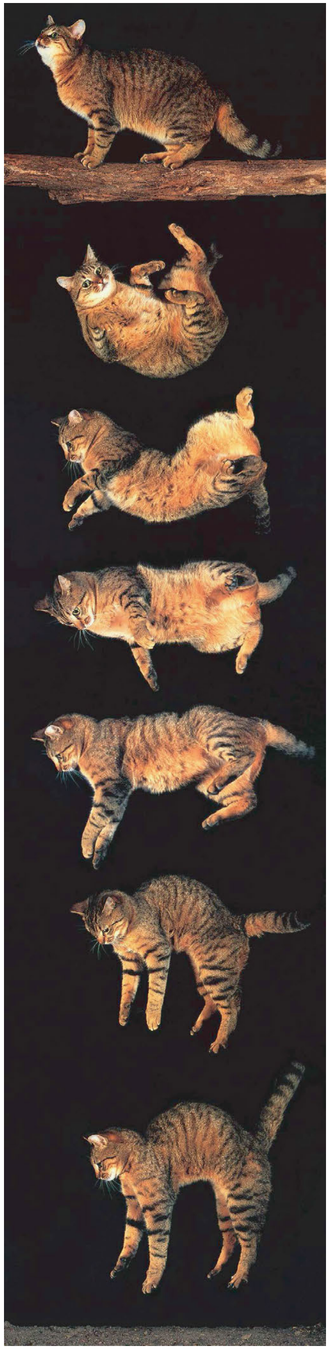


the mundane



the sub-sub-atomic

the (sub-)atomic



the Felidaes

→ Consideration of forces is key to understanding all of it!

https://sciencebasedlife.wordpress.com/2012/04/13/how-do-cats-survive-falls-from-great-heights/

Solar/galactic dynamics

→ Consideration of forces is key to understanding all of it!


LG
Direct Drive 8kg

Force (Interdisplinary point of view)

Membrane transport

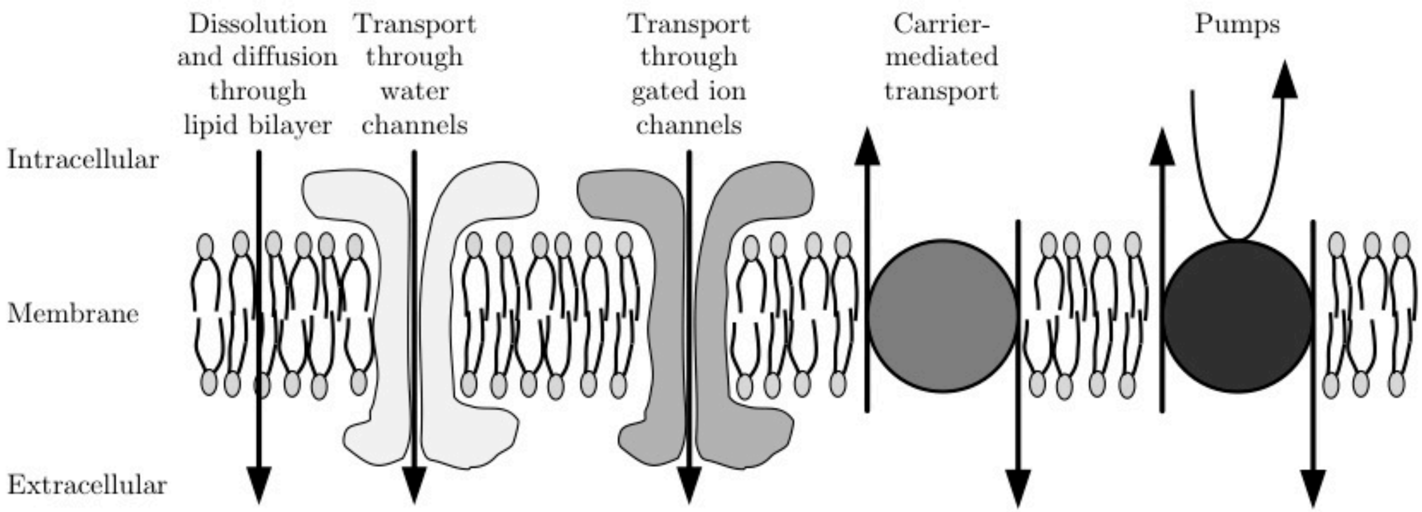
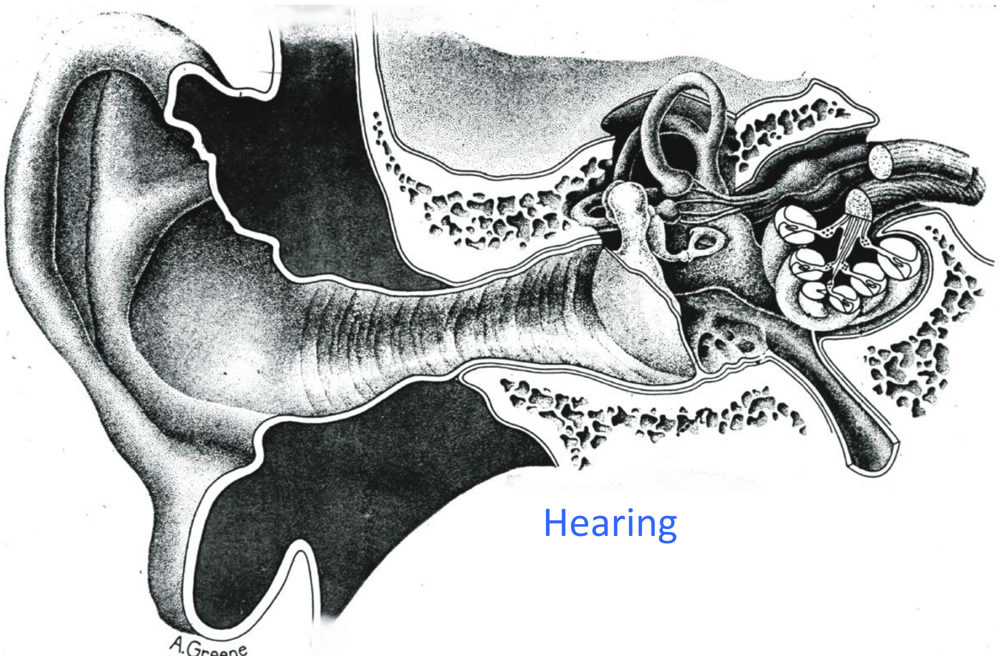
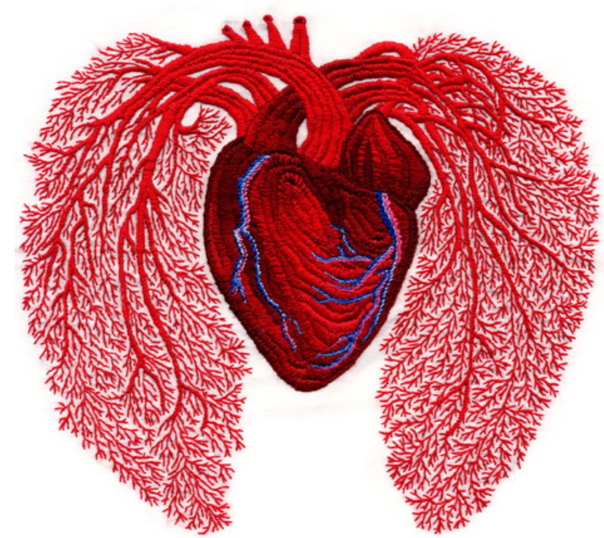


Figure 2.19

Cardiac/pulmonary dynamics



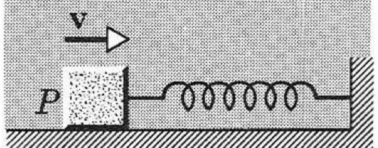
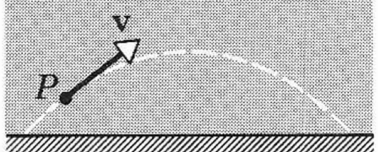
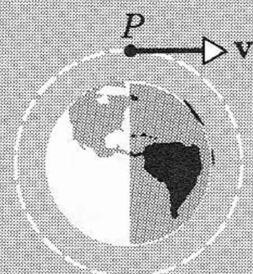
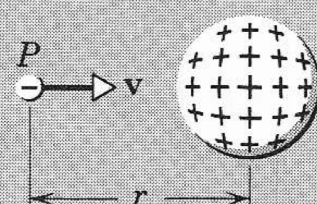
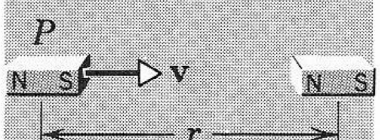
Hearing



https://vivataurelia.wordpress.com/2012/02/14/the-human-heart/

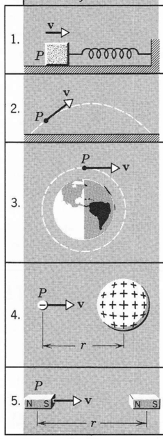
→ Consideration of forces is key to understanding all of it!

<u>Forces</u> (common pedagogical examples)

	System	The "Particle"	The Environment
1.		A block	The spring; the rough surface
2.		A golf ball	The earth
3.		An artificial satellite	The earth
4.		An electron	A large uniformly charged sphere
5.		A bar magnet	A second bar magnet

Resnick & Halliday (1966)

<u>Forces</u> (common pedagogical examples)



System	The "Particle"	The Environment
1.	A block	The spring; the rough surface
2.	A golf ball	The earth
3.	An artificial satellite	The earth
4.	An electron	A large uniformly charged sphere
5.	A bar magnet	A second bar magnet

THE FORCE LAWS FOR THE SYSTEMS OF TABLE 5–1

System		Force Law
1. A block propelled by a stretched spring over a rough horizontal surface	(a)	Spring force: $F = -kx$, where x is the extension of the spring and k is a constant that describes the spring; **F** points to the right; see Chapter 15
	(b)	Friction force: $F = \mu mg$, where μ is the coefficient *of friction* and mg is the weight of the block; **F** points to the left; see Chapter 6
2. A golf ball in flight		$F = mg$; **F** points down (see Section 5–8)
3. An artificial satellite		$F = GmM/r^2$, where G is the *gravitational constant*, M the mass of the earth, and r the orbit radius; **F** points toward the center of the earth; see Chapter 16. This is *Newton's law of universal gravitation*
4. An electron near a charged sphere		$F = (1/4\pi\epsilon_0)eQ/r^2$, where ϵ_0 is a constant, e is the electron charge, Q is the charge on the sphere, and r is the distance from the electron to the center of the sphere; **F** points to the right; see Chapter 26. This is *Coulomb's law of electrostatics*
5. Two bar magnets		$F = (3\mu_0/2\pi)\mu^2/r^4$, where μ_0 is a constant, μ is the *magnetic dipole* moment of each magnet, and r is the center-to-center separation of the magnets; we assume that $r \gg l$, where l is the length of each magnet; **F** points to the right

Resnick & Halliday (1966)

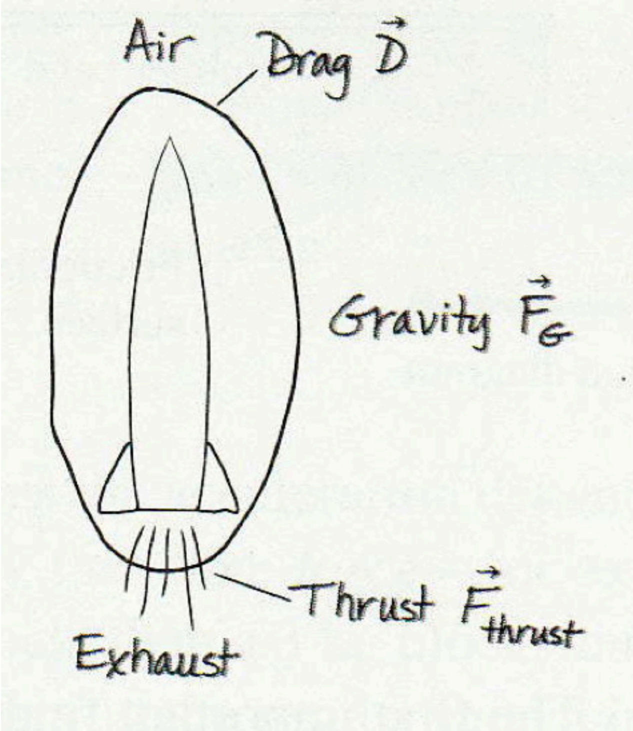
Ex.

A rocket is being launched to place a new satellite in orbit. Air resistance is not negligible. What forces are being exerted on the rocket?

Knight (2013)

Ex. (SOL)

A rocket is being launched to place a new satellite in orbit. Air resistance is not negligible. What forces are being exerted on the rocket?



→ For a "real" rocket, there are likely more forces than these (e.g., stabilizer thrusters)

Knight (2013)

SCIENCE \ SPACE \ SPACEX

Watch SpaceX's Falcon 9 rocket land, tip over, and explode

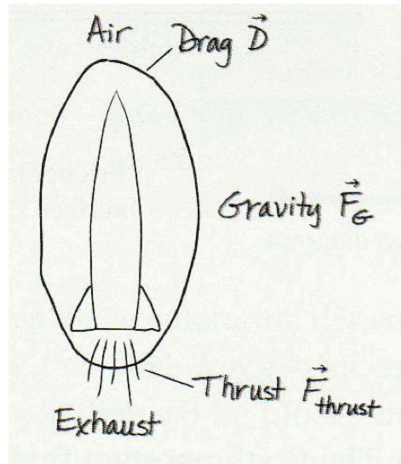
By Sam Byford | @345triangle | Jan 17, 2016, 10:24pm EST

https://www.theverge.com/2016/1/17/10784408/spacex-rocket-landing-explosion-falcon-9

https://www.youtube.com/watch?v=bvim4rsNHkQ

Reminder (re “Mathematical Modeling”)

“We begin with a definition based on the previous discussion: A mathematical model is an abstract, simplified, mathematical construct related to a part of reality and created for a particular purpose. [...] As far as a model is concerned the world can be divided into three parts:



1. Things whose effects are neglected.
2. Things that affect the model but whose behavior the model is not designed to study.
3. Things the model is designed to study the behavior of.

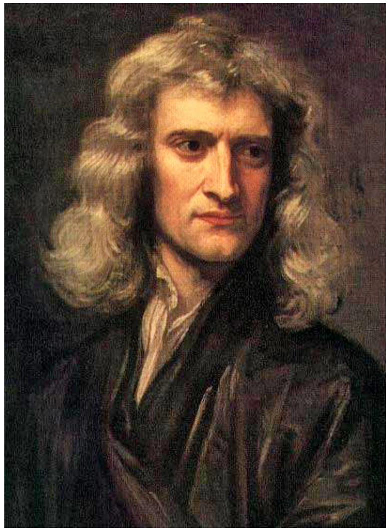
Two key ingredients should be apparent here:

- Figuring out what question you want to try to answer
- What assumptions you are willing to make

Hence:
"It ain't rocket science"

Review: Newton's Laws

I. Newton (1643 -1727)



- Three (seemingly innocuous) rules for motion/forces

Newton's first law of motion: A body in uniform motion remains in uniform motion, and a body at rest remains at rest, unless acted on by a nonzero net force.

Newton's second law of motion: The rate at which a body's momentum changes is equal to the net force acting on the body:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \text{(Newton's 2}^{\text{nd}}\text{ law)} \qquad (4.2)$$

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \text{(Newton's 2}^{\text{nd}}\text{ law, constant mass)}$$

→ We will be using this one (a LOT)

Newton's third law of motion: If object A exerts a force on object B, then object B exerts an oppositely directed force of equal magnitude on A.

- Wrapped up here are other key notions such as *inertia* and *momentum*
  (we'll be revisiting these plenty)

Aside: Fundamental Forces

- Four (er, three?) fundamental forces that govern, well, everything

1. Gravity
2. Electromagnetic
3. Weak nuclear (deals w/ radioactive decay, e.g., β–decay)
4. Strong nuclear (deals w/ what holds sub-atomic particles together)

- Our daily life perceptions are dominated by the first two:

→ PHYS 2010 chiefly focuses only on two of these (which ones!?!)

Aside: Trying to "unite" these = major goal in physics (e.g., "Standard model", string theory)

Newton's 1st law (the law of inertia):

A body remains in its state of rest or in uniform linear motion as long as no external forces act to change that state.

$$\mathbf{p} \equiv m\mathbf{v}\,,$$

where m is the inertial mass of the particle, $\mathbf{v}$ its velocity, and $\mathbf{p}$ the momentum. By "$\equiv$" we mean "equal to, by definition". Mathematically, we can express the law of inertia as

$$\text{no external force} \;\Rightarrow\; \mathbf{p} = \text{constant vector.}$$

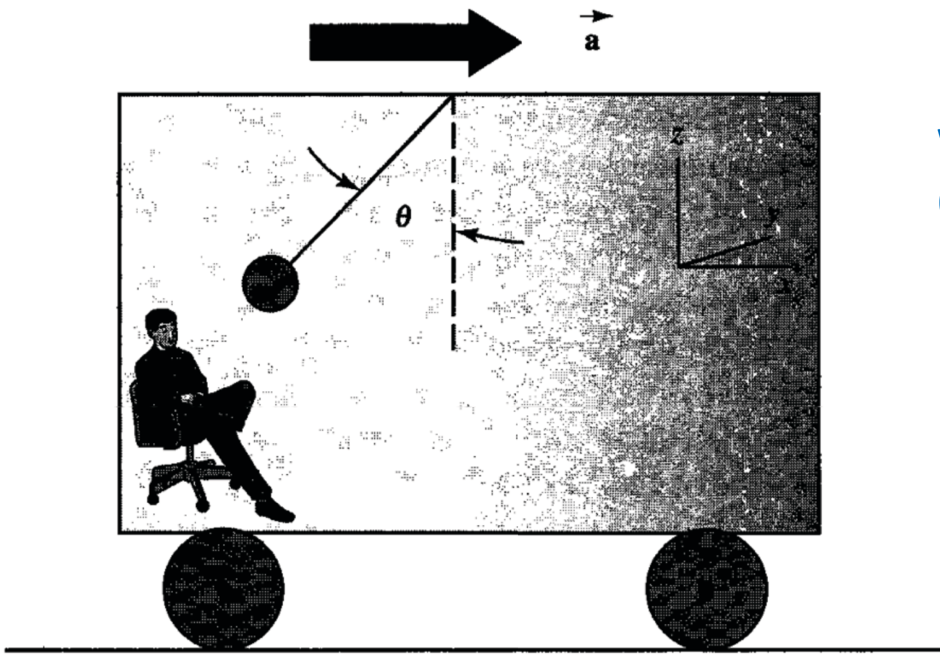
In this way the *law of inertia* coincides with the *law of conservation of momentum for a particle.*

→ Already the notions of *mass, force,* and *momentum* come to bear!

Knudsen & Hjorth

Two further considerations:

The quantitative measure of inertia is called *mass*.



What does the person in the (accelerating) "boxcar" surmise?

→ Frame of reference matters (e.g., inertial vs non inertial frames). We will come back to this a bit later in the semester....

Fowles & Cassidy

Newton's 2nd Law

The change in the momentum of a body is proportional to the force that acts on the body and takes place in the direction of that external force.

$$\frac{d\mathbf{p}}{dt} = \mathbf{F},$$

If mass remains constant:

$$m\mathbf{a} = \mathbf{F},$$

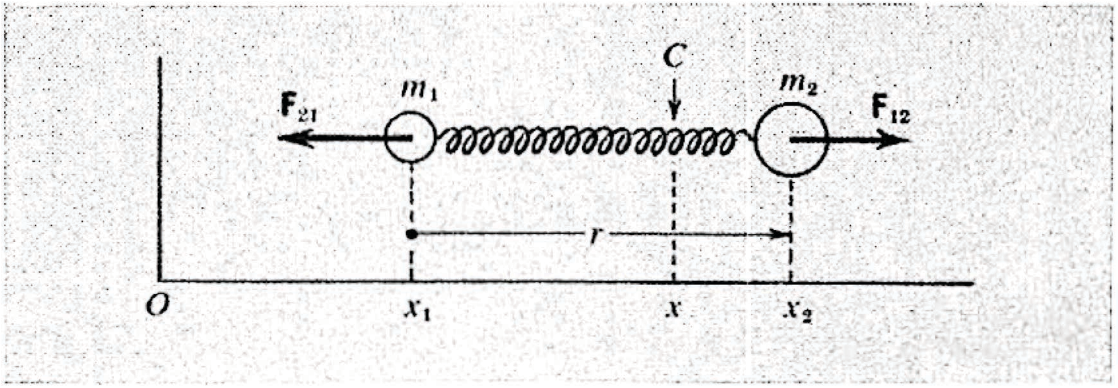
The second-order differential equation that results when some force law is supplied to the second law is called the *equation of motion* for the particle.

An important property of the concept of a force acting on a given object is that the force has its origin in some other material body.

Thus, Newton introduced the concept of *mechanical force* as the cause of the acceleration of an object. This causal description of the motion of an object constitutes what we call *dynamics*.

The quantitative measure of inertia is called *mass*.

Consider two masses connected by a spring, initially at rest in an inertial reference frame



French

Now assume someone pushes them together, compressing the spring, then let's go. The ratio of their velocities is

$$\frac{m_2}{m_1} = \left|\frac{\mathbf{v}_1}{\mathbf{v}_2}\right|$$

Equivalently: $\Delta(m_1\mathbf{v}_1) = -\Delta(m_2\mathbf{v}_2)$ Momentum again!

Dividing by Δt and taking the limit: $$\frac{d}{dt}(m_1\mathbf{v}_1) = -\frac{d}{dt}(m_2\mathbf{v}_2)$$

Fowles & Cassidy

Revisiting Newton's Laws

Newton's 2nd Law

$$\frac{d}{dt}(m_1\mathbf{v}_1) = -\frac{d}{dt}(m_2\mathbf{v}_2)$$

The change in the momentum of a body is proportional to the force that acts on the body and takes place in the direction of that external force.

$$\mathbf{F} = k\frac{d(m\mathbf{v})}{dt}$$

Assuming mass to be independent of velocity (i.e., not at "relativistic" speeds), then:

$$\mathbf{F} = km\frac{d\mathbf{v}}{dt} = km\mathbf{a}$$

$$\boxed{\mathbf{F} = \frac{d(m\mathbf{v})}{dt} = m\mathbf{a}}$$

Note that:

$$\frac{d}{dt}(m_1\mathbf{v}_1) = -\frac{d}{dt}(m_2\mathbf{v}_2)$$

is equivalent to:

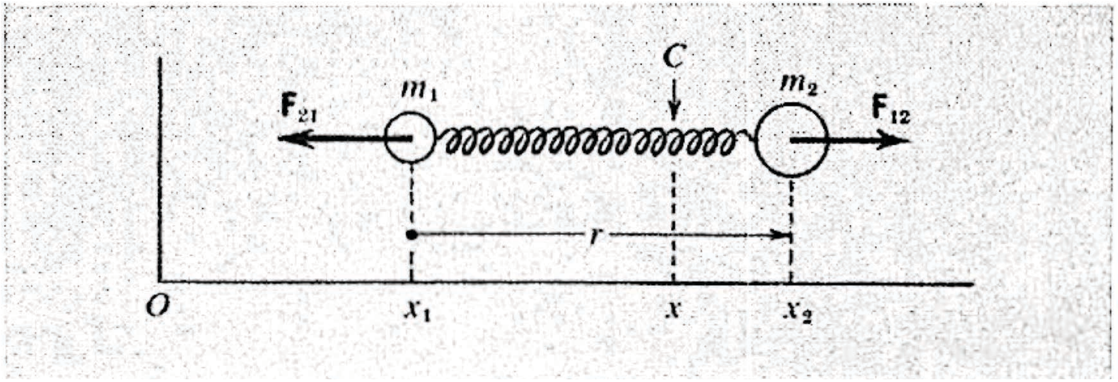
$$\mathbf{F}_1 = -\mathbf{F}_2$$

→ Newton's 3rd Law!

Fowles & Cassidy

Newton's 3rd Law

$$\mathbf{F}_1 = -\mathbf{F}_2$$



If a given body A acts on another body B with a force, then B will also act on A with a force equal in magnitude but opposite in direction.

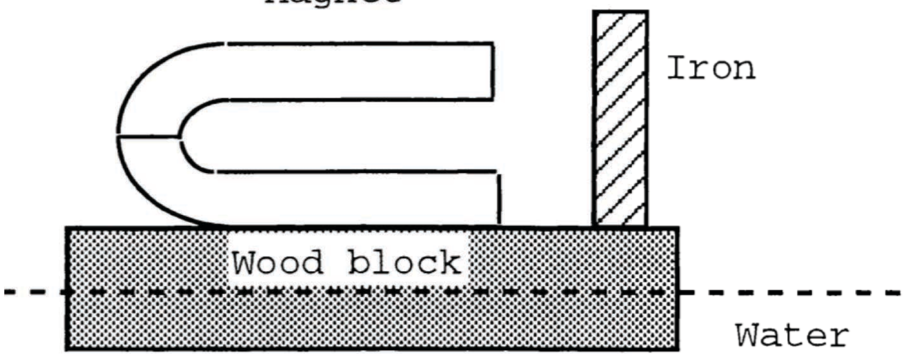


Fig. 2.1. Illustration of Newton's third law

Note:

If the law of action and reaction did not hold, one could assemble systems that perpetually increased their velocity through the action of internal forces.

Knudsen & Hjorth

Newton's 3rd Law

Let's recast Newton's 2nd Law a slightly different way:

$$\mathbf{F} = \frac{d(m\mathbf{v})}{dt} = m\mathbf{a}$$

$$\mathbf{p} = m\mathbf{v} \qquad \mathbf{F} = \frac{d\mathbf{p}}{dt}$$

$$\frac{d}{dt}(m_1\mathbf{v}_1) = -\frac{d}{dt}(m_2\mathbf{v}_2)$$

is equivalent to:

$$\frac{d}{dt}(\mathbf{p}_1 + \mathbf{p}_2) = 0$$

$$\boxed{\mathbf{p}_1 + \mathbf{p}_2 = \text{constant}}$$

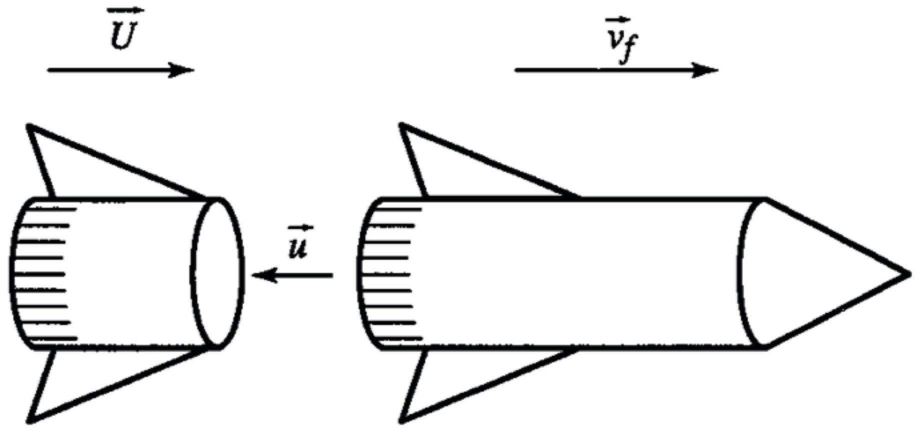
Newton's 3rd Law thus implies that the total momentum of two mutually interacting bodies is a constant

→ Thus shakes out *conservation of momentum*!

Fowles & Cassidy

Newton's 3rd Law

$$\mathbf{p}_1 + \mathbf{p}_2 = \text{constant}$$



→ Rocket propulsion is an excellent/fun/practical example we will return to shortly.....

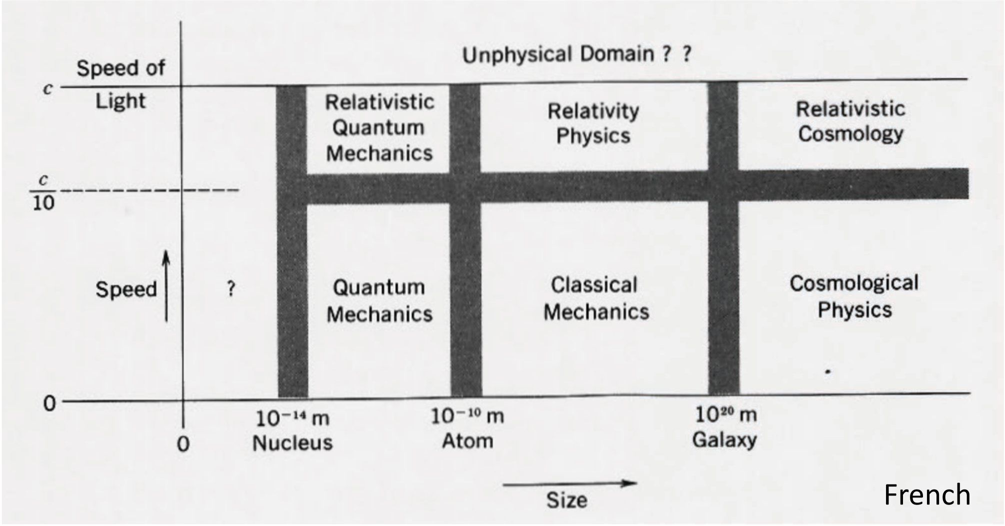
Fowles & Cassidy

Newton's 4th law (the postulate of absolute time):

Absolute, true, and mathematical time, of itself, and from its own nature, flows equally *without relation to anything external*, and by another name is called duration: relative, apparent and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year.

Recall this assumption (i.e., mass is independent of velocity) ?

$$\mathbf{F} = km\frac{d\mathbf{v}}{dt} = km\mathbf{a}$$



Knudsen & Hjorth

Newton's 5th law (the postulate of absolute space):

Absolute space, in its own nature, *without relation to anything external*, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our senses determine by its position to bodies; and which is commonly taken for immovable space; such is the dimension of a subterranous, an arial, or celestial space, determined by its position in respect of the Earth. Absolute and relative space is the same in figure and magnitude; but they do not always remain numerically the same. For if the Earth, for instance, moves, a space of our air, which relatively and in respect of the Earth remains always the same, will at one time be in one part of the absolute space into which the air passes; at another time it will be in another part of the same, and so, absolutely understood, it will be continually changed.

Remember this problem?

2. What are the properties of two vectors $\mathbf{a}$ and $\mathbf{b}$ such that

(*a*) $$\mathbf{a} + \mathbf{b} = \mathbf{c} \quad \text{and} \quad a + b = c,$$

(*b*) $$\mathbf{a} + \mathbf{b} = \mathbf{a} - \mathbf{b},$$

(*c*) $$\mathbf{a} + \mathbf{b} = \mathbf{c} \quad \text{and} \quad a^2 + b^2 = c^2.$$

→ It would totally suck without Newton's "5^{th} Law"!

Putting Newton's Laws to work....

Let's consider 1-D for the moment:

$$\mathbf{F}_{\mathbf{net}} = \sum \mathbf{F}_i = m \frac{d^2\mathbf{r}}{dt^2} = m\mathbf{a}$$

$$F_x(x, \dot{x}, t) = m\ddot{x}$$

And consider constant force (i.e., acceleration):

$$\ddot{x} = \frac{dv}{dt} = \frac{F}{m} = \text{constant} = a$$

And voila!

Simple integration leads to:

$$\dot{x} = v = at + v_0$$

$$x = \frac{1}{2}at^2 + v_0 t + x_0$$

And upon eliminating t:

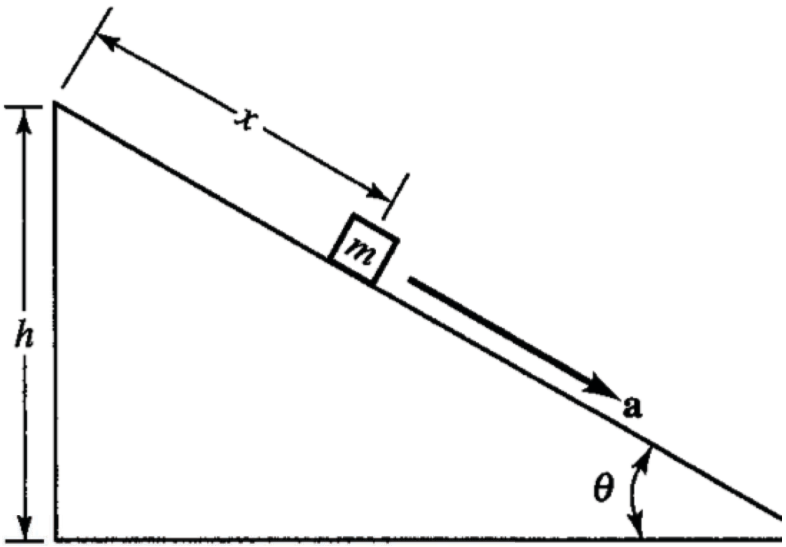
$$2a(x - x_0) = v^2 - v_0^2$$

Table 2.1 Equations of Motion for Constant Acceleration

Equation	**Contains**
$v = v_0 + at$	v, a, t; no x
$x = x_0 + \frac{1}{2}(v_0 + v)t$	x, v, t; no a
$x = x_0 + v_0 t + \frac{1}{2}at^2$	x, a, t; no v
$v^2 = v_0^2 + 2a(x - x_0)$	x, v, a; no t

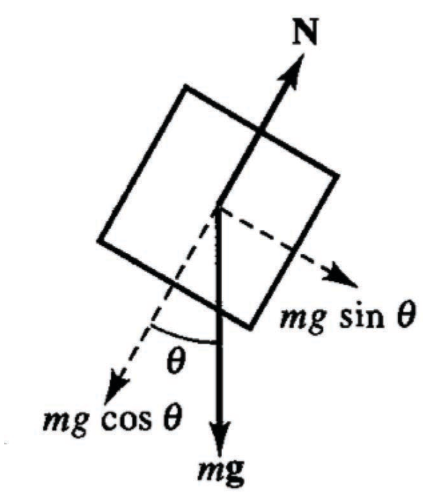
Wolfson

Ex. Block sliding down an incline w/ friction



- Set up a coordinate system (e.g., x along the incline)
- Draw free-body diagram
- Consider "simpler" case first (i.e., no friction)

No friction



$$\ddot{x} = a = \frac{F_x}{m} = g\sin\theta$$

$$v^2 = 2(g\sin\theta)\left(\frac{h}{\sin\theta}\right) = 2gh$$

Distance the block has slid:

$$x - x_0 = \frac{h}{\sin\theta}$$

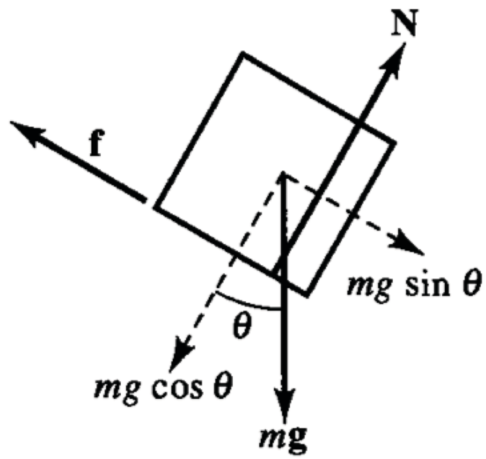
→ Note that an energy-based argument would end up at the same place

Fowles & Cassidy

Ex. Block sliding down an incline w/ friction

w/ friction

Note: We will ignore static friction at the moment....



Kinetic friction: $f = \mu_\kappa N$

Note: "Normal" force!

$$f = \mu_\kappa mg\cos\theta$$

Total force action on block along x: $mg\sin\theta - \mu_\kappa mg\cos\theta$

$$\ddot{x} = \frac{F_x}{m} = g(\sin\theta - \mu_k\cos\theta)$$

The speed of the particle increases if the expression in parentheses is positive—that is, if $\theta > \tan^{-1}\mu_\kappa$. The angle, $\tan^{-1}\mu_\kappa$, usually denoted by ϵ, is called the *angle of kinetic friction*. If $\theta = \epsilon$, then $a = 0$, and the particle slides down the plane with constant speed. If $\theta < \epsilon$, a is negative, and so the particle eventually comes to rest. For motion *up* the plane, the direction of the frictional force is reversed; that is, it is in the positive x direction. The acceleration (actually deceleration) is then $\ddot{x} = g(\sin\theta + \mu_\kappa\cos\theta)$.

Fowles & Cassidy

Tangent/Review: Friction

"We will ignore static friction at the moment...."

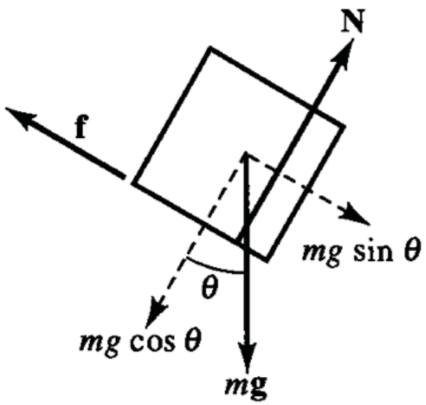
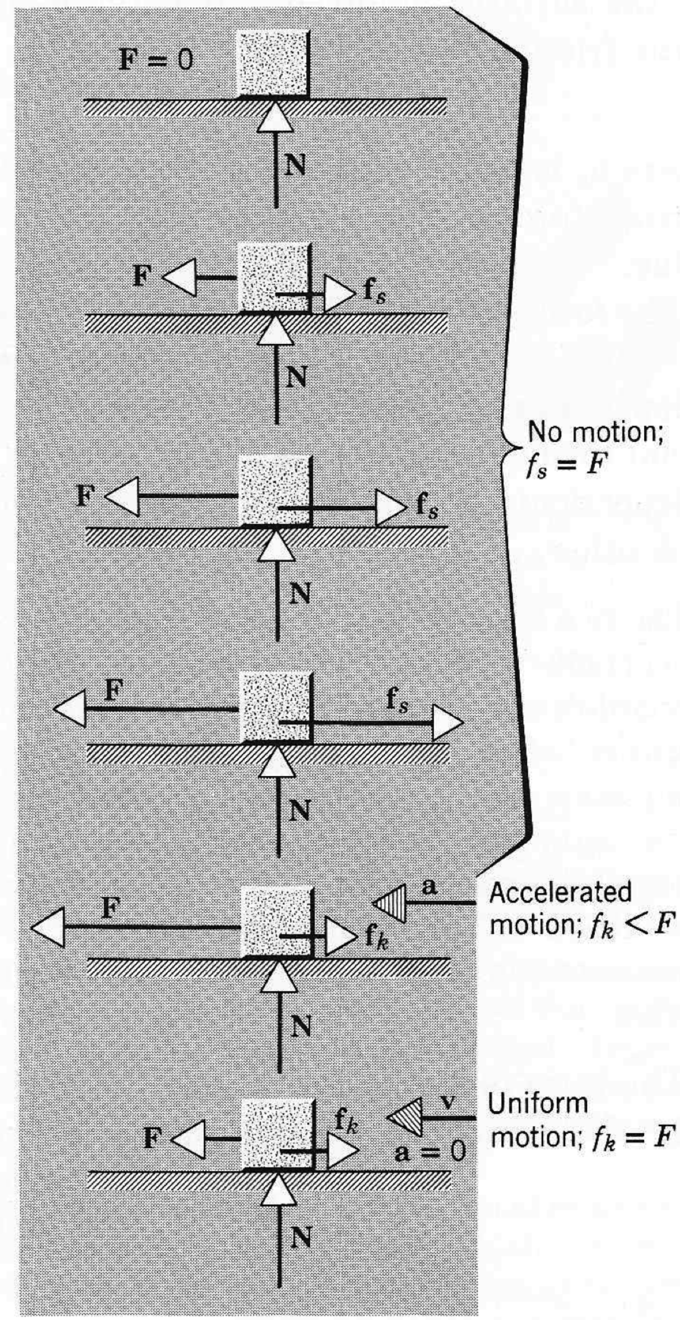
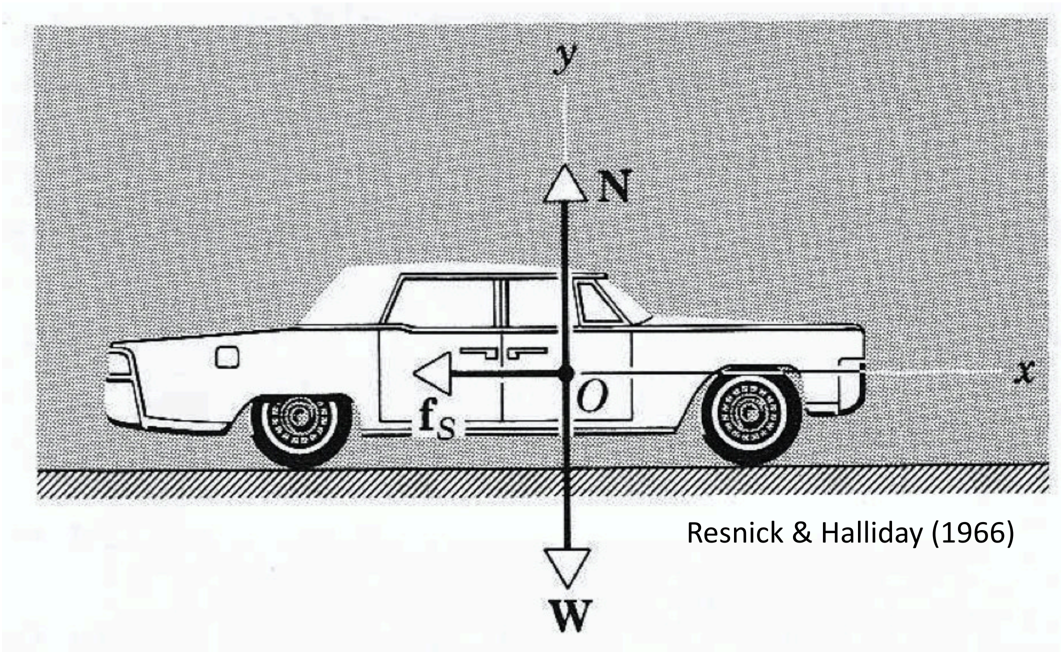


Fig. 6–1 A block being put into motion as applied force **F** overcomes frictional forces. In the first four drawings the applied force is gradually increased from zero to magnitude $\mu_s N$. No motion occurs until this point because the frictional force always just balances the applied force. The instant F becomes greater than $\mu_s N$, the block goes into motion, as is shown in the fifth drawing. In general, $\mu_k N < \mu_s N$; this leaves an unbalanced force to the left and the block accelerates. In the last drawing F has been reduced to equal $\mu_k N$. The net force is zero, and the block continues with constant velocity.



Resnick & Halliday (1966)

Review: Friction



Resnick & Halliday (1966)

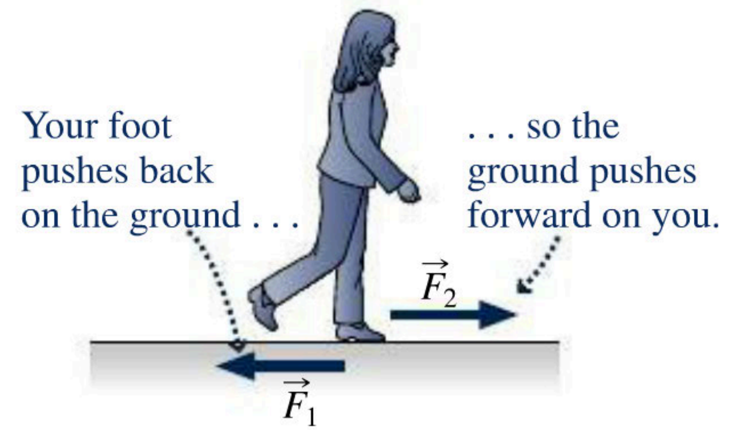


FIGURE 5.21 Walking.

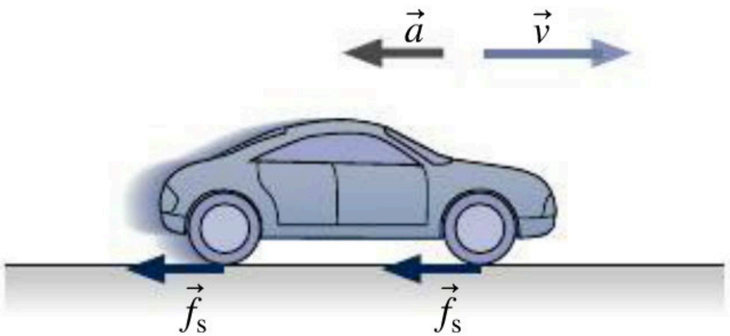


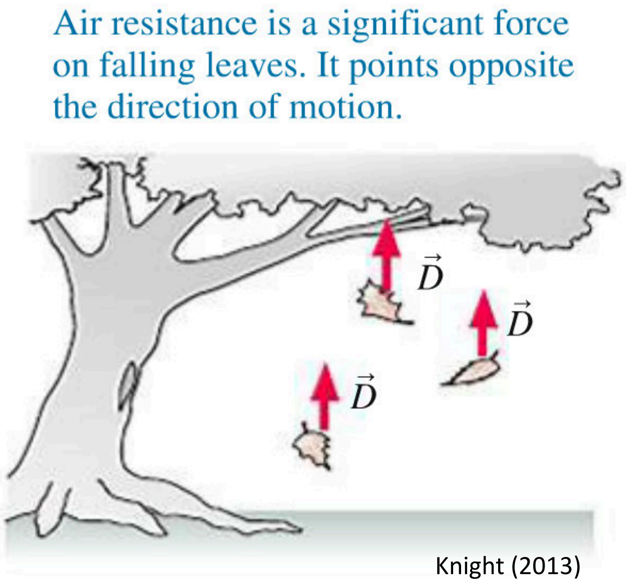
FIGURE 5.22 Friction stops the car.

Wolfson

→ Without friction, it'd be hard to make it anywhere in the first place!

Review: Friction

- "Friction" comes in many forms....

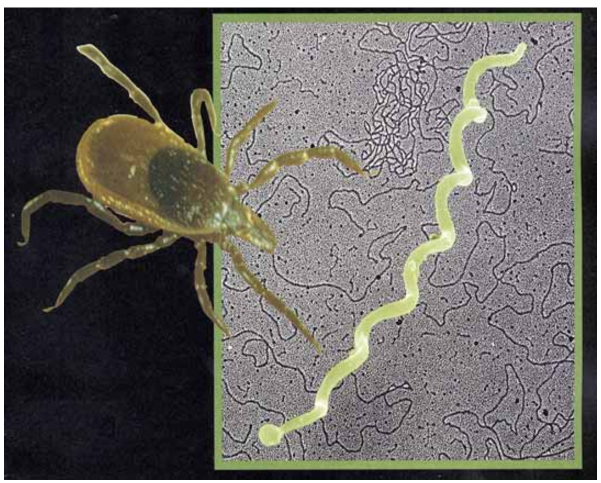


Knight (2013)

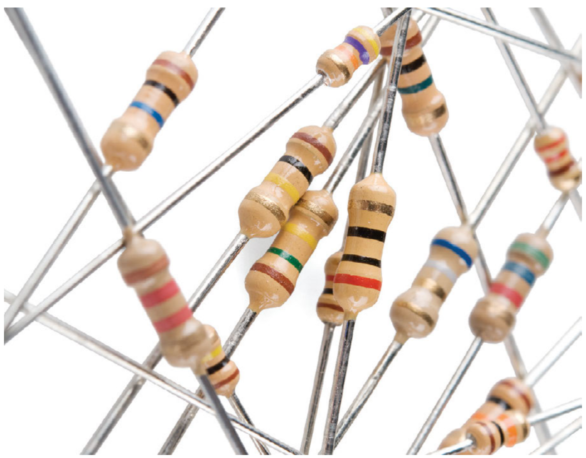
Drag → Bacterial motion

Electrical resistance

Viscosity

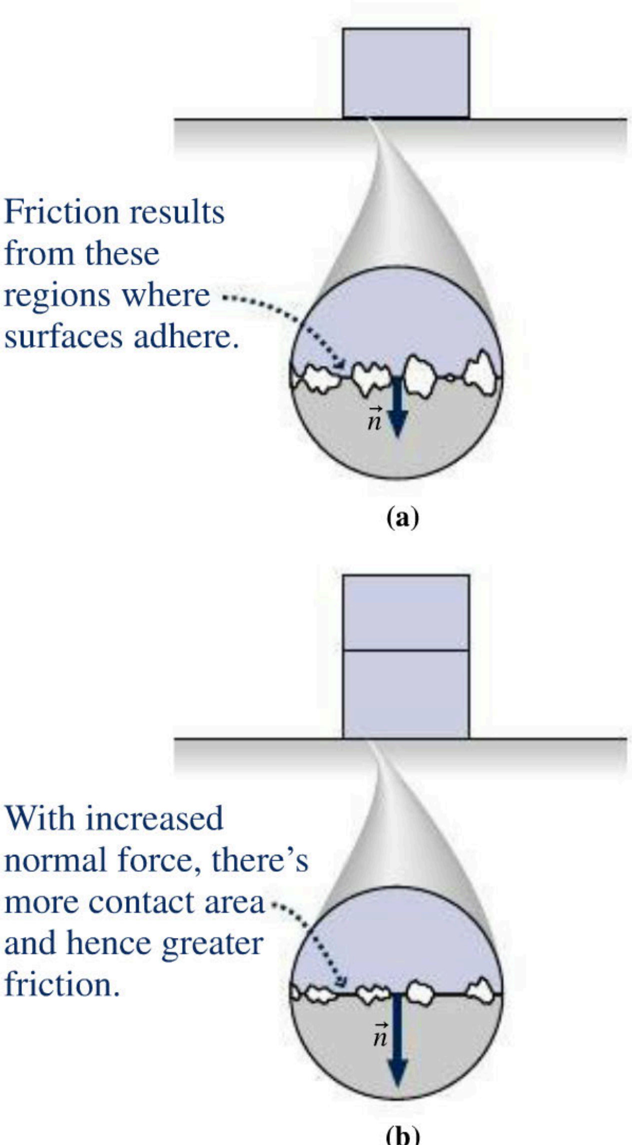


Ticks → Spirochetes → Lyme Disease



→ Focus initially on contact friction

Review: Friction



Wolfson

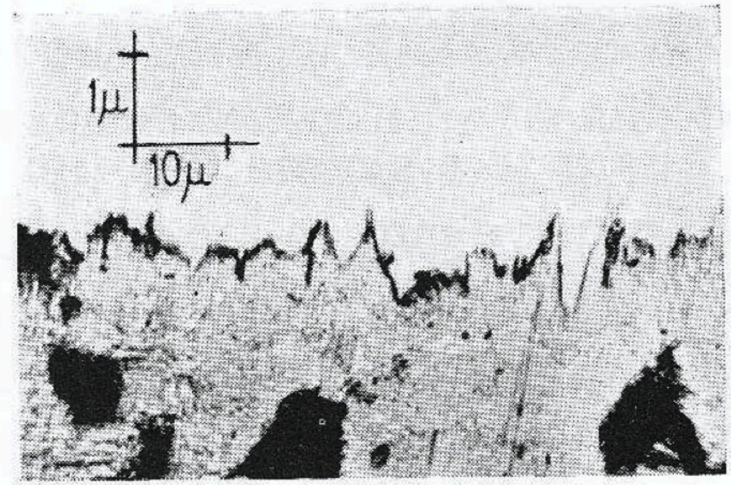


Fig. 6–2 A highly magnified view of a section of a finely polished steel surface. The section was cut at an angle so that vertical distances are exaggerated by a factor of ten with respect to horizontal distances. The surface irregularities are several thousand atomic diameters high. From *Friction and Lubrication of Solids*, by F. P. Bowden and D. Tabor, Clarendon Press, 1950.

Resnick & Halliday (1966)

<u>Review</u>: Friction

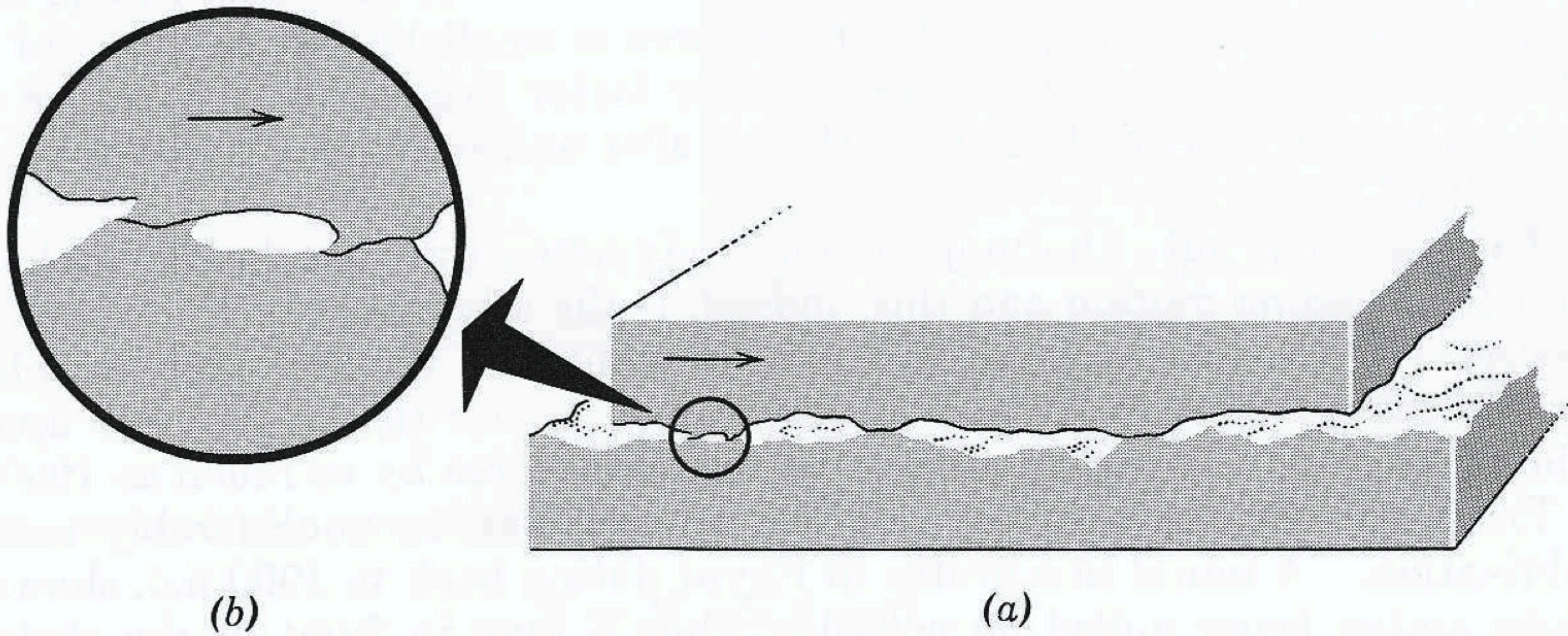


Fig. 6–3 Sliding friction. (*a*) The upper body is sliding to the right over the lower body in this enlarged diagram. (*b*) A further enlarged view showing two spots where surface adhesion has occurred. Force is required to break these welds apart and maintain the motion.

→ "break these welds apart" is a bit vague, but alludes to molecular considerations still not entirely all that well understood....

Resnick & Halliday (1966)

Review: Friction

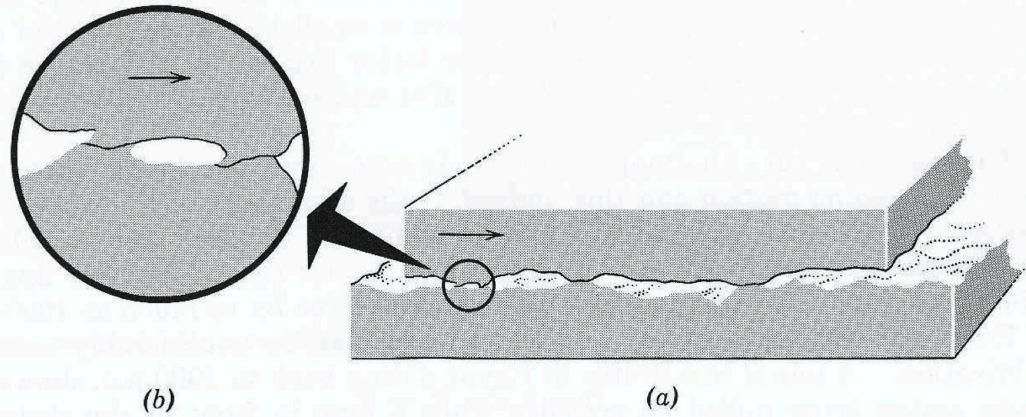


Fig. 6–3 Sliding friction. (*a*) The upper body is sliding to the right over the lower body in this enlarged diagram. (*b*) A further enlarged view showing two spots where surface adhesion has occurred. Force is required to break these welds apart and maintain the motion.

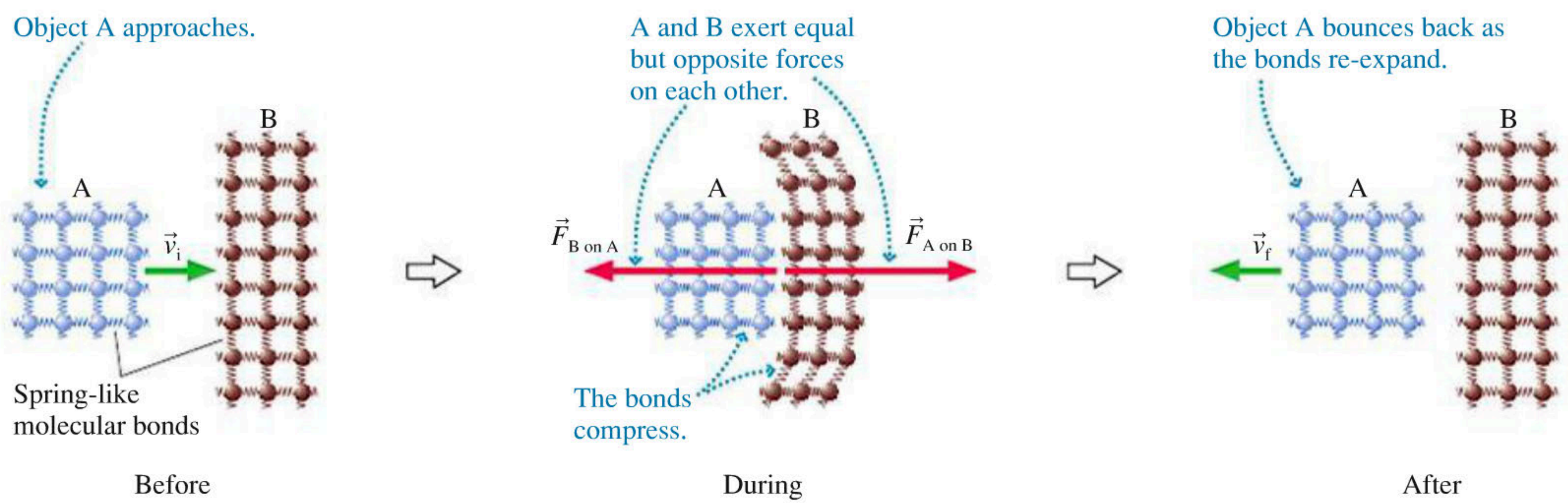
Consider for example how friction relates back to the four fundamental forces:

1. Gravity
2. Electromagnetic
3. Weak nuclear (deals w/ radioactive decay, e.g., β–decay)
4. Strong nuclear (deals w/ what holds sub-atomic particles together)

Resnick & Halliday (1966)

Aside: Molecular Underpinnings of Momentum & Collisions

Atomic model of a collision.



Knight (2013)

June 30, 2016 issue of Nature

676 | NATURE | VOL 534 | 30 JUNE 2016

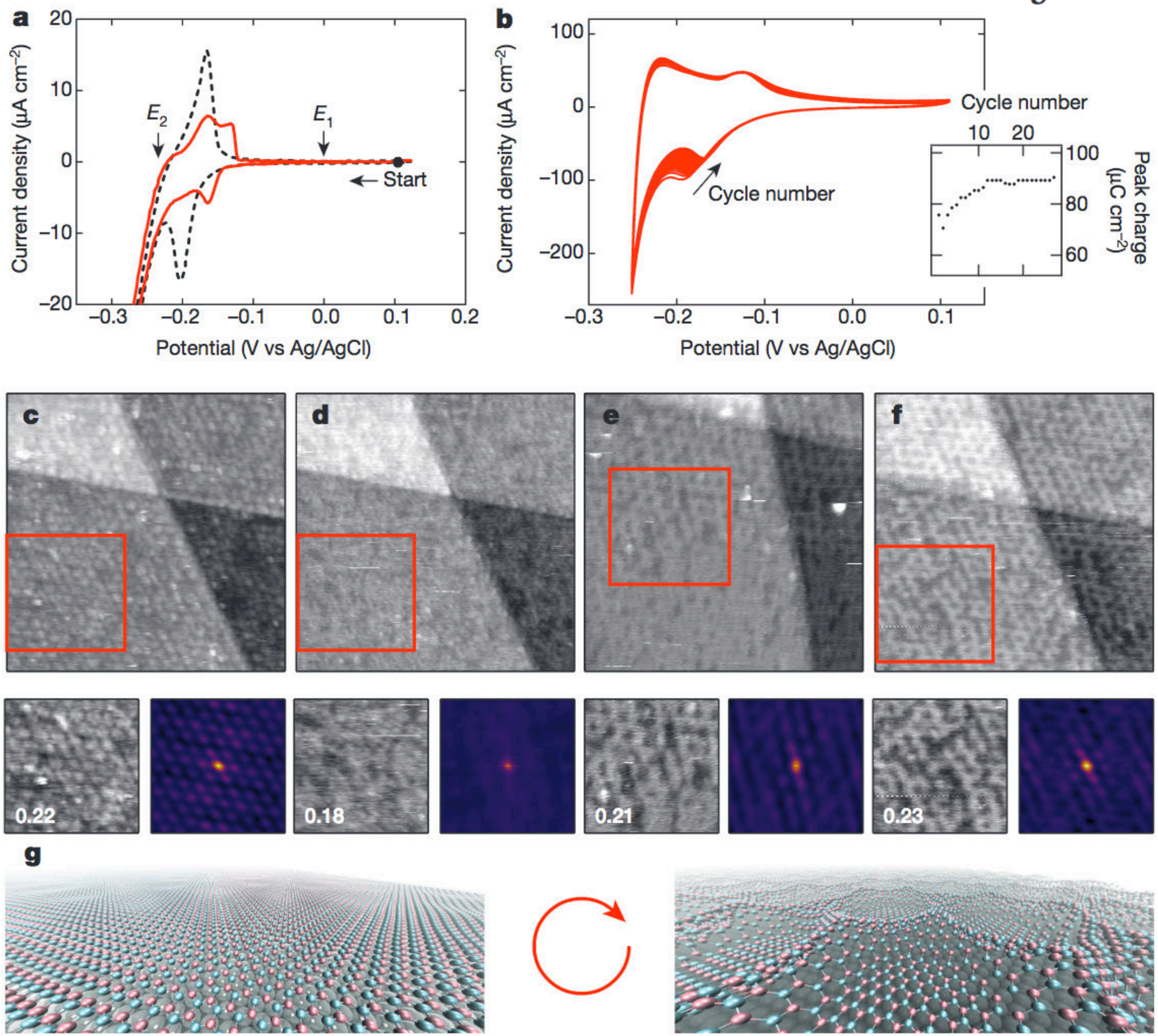
doi:10.1038/nature18275

Switching stiction and adhesion of a liquid on a solid

Stijn F. L. Mertens[1,2]*, Adrian Hemmi[3]*, Stefan Muff[3], Oliver Gröning[4], Steven De Feyter[1], Jürg Osterwalder[3] & Thomas Greber[3]

When a gecko moves on a ceiling it makes use of adhesion and stiction. Stiction—static friction—is experienced on microscopic and macroscopic scales and is related to adhesion and sliding friction[1]. Although important for most locomotive processes, the concepts of adhesion, stiction and sliding friction are often only empirically correlated. A more detailed understanding of these concepts will, for example, help to improve the design of increasingly smaller devices such as micro- and nanoelectromechanical switches[2]. Here we show how stiction and adhesion are related for a liquid drop on a hexagonal boron nitride monolayer on rhodium[3], by measuring dynamic contact angles in two distinct states of the solid–liquid interface: a corrugated state in the absence of hydrogen intercalation and an intercalation-induced flat state. Stiction and adhesion can be reversibly switched by applying different electrochemical potentials to the sample, causing atomic hydrogen to be intercalated or not. We ascribe the change in adhesion to a change in lateral electric field of in-plane two-nanometre dipole rings[4], because it cannot be explained by the change in surface roughness known from the Wenzel model[5]. Although the change in adhesion can be calculated for the system we study[6], it is not yet possible to determine the stiction at such a solid–liquid interface using *ab initio* methods. The inorganic hybrid of hexagonal boron nitride and rhodium is very stable and represents a new class of switchable surfaces with the potential for application in the study of adhesion, friction and lubrication.

Figure 1 | Voltammetry and electrochemical scanning tunnelling microscopy.

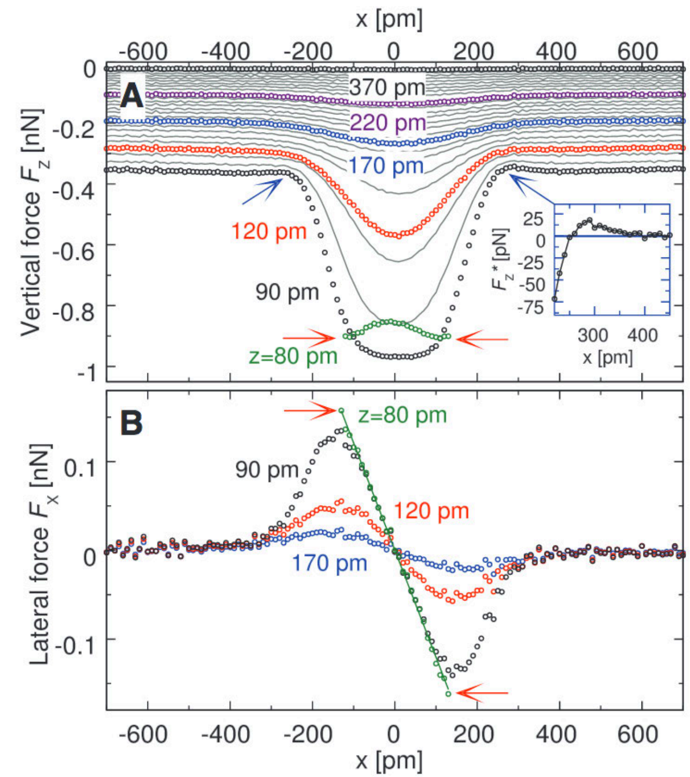


22 FEBRUARY 2008 VOL 319 SCIENCE

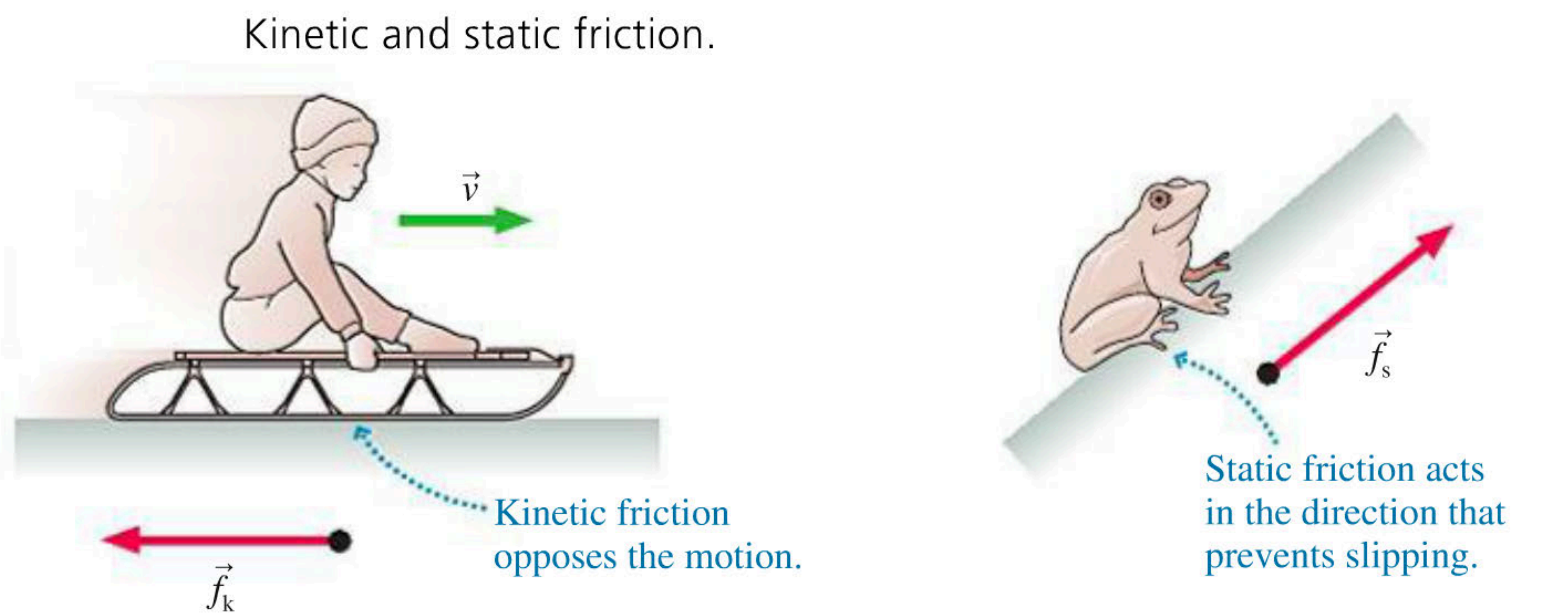
The Force Needed to Move an Atom on a Surface

Markus Ternes,[1]† Christopher P. Lutz,[1] Cyrus F. Hirjibehedin,[1]*
Franz J. Giessibl,[2] Andreas J. Heinrich[1]

Manipulation of individual atoms and molecules by scanning probe microscopy offers the ability of controlled assembly at the single-atom scale. However, the driving forces behind atomic manipulation have not yet been measured. We used an atomic force microscope to measure the vertical and lateral forces exerted on individual adsorbed atoms or molecules by the probe tip. We found that the force that it takes to move an atom depends strongly on the adsorbate and the surface. Our results indicate that for moving metal atoms on metal surfaces, the lateral force component plays the dominant role. Furthermore, measuring spatial maps of the forces during manipulation yielded the full potential energy landscape of the tip-sample interaction.



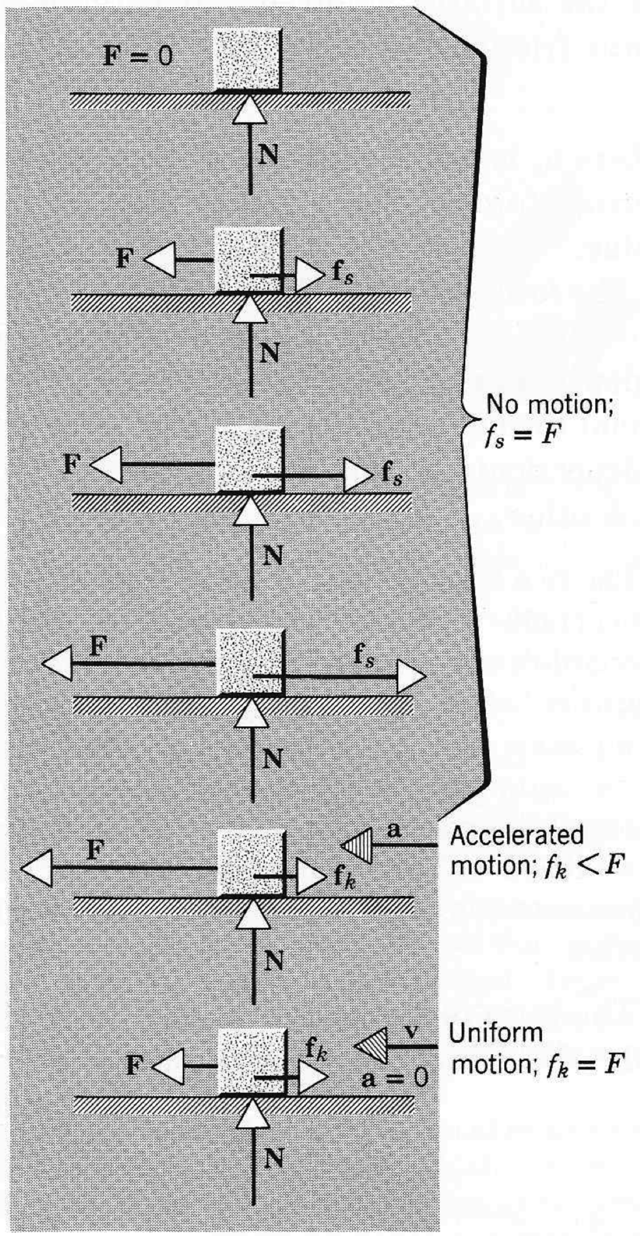
<u>Review</u>: Static vs Kinetic Friction

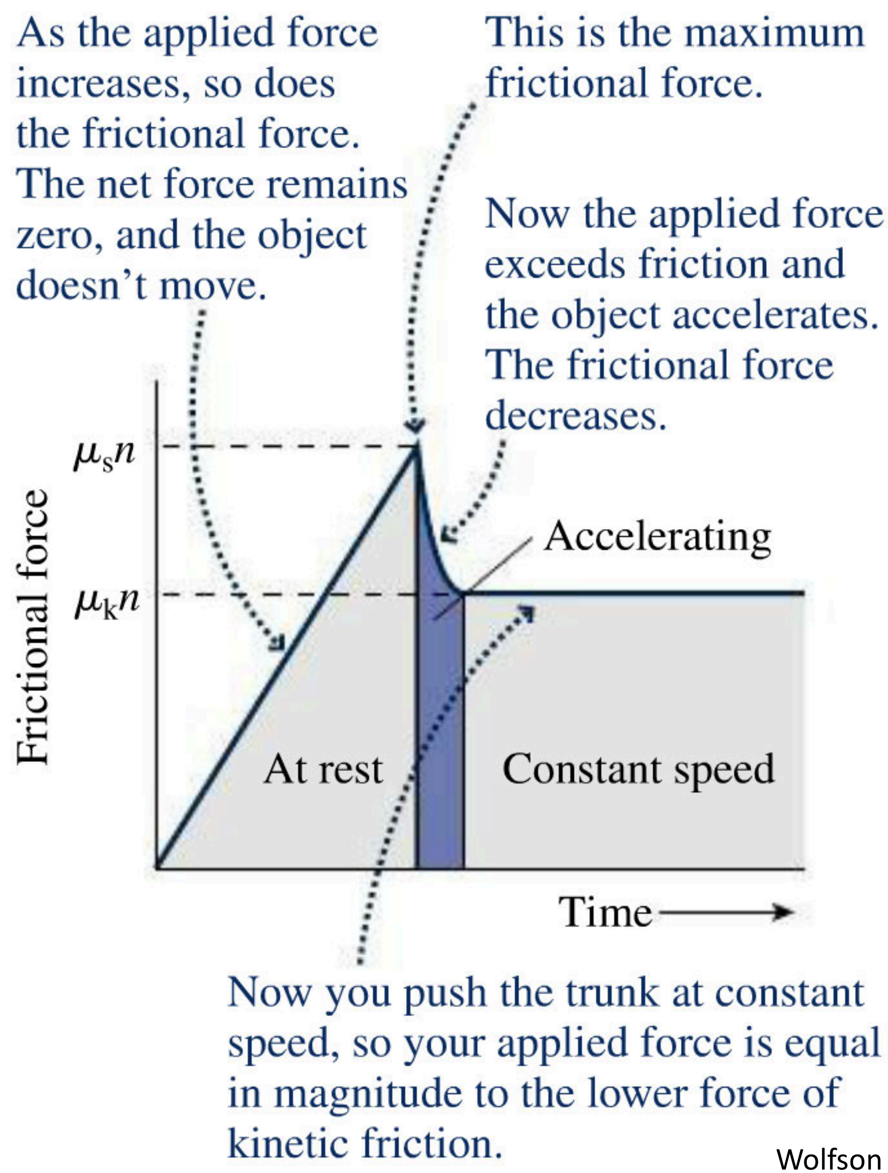


→ Key idea/distinction here is "static" versus "kinetic" friction

Knight (2013)

Review: Static vs Kinetic Friction





Wolfson

→ Two sides of the same coin here

Review: Static vs Kinetic Friction

If f_s represents the magnitude of the force of static friction, we can write

$$f_s \leq \mu_s N, \tag{6–1}$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force. The equality sign holds only when f_s has its maximum value.

The force of kinetic friction f_k between dry, unlubricated surfaces follows the same two laws as those of static friction. (1) It is approximately independent of the area of contact over wide limits and (2) it is proportional to the normal force. The force of kinetic friction is also reasonably independent of the relative speed with which the surfaces move over each other.

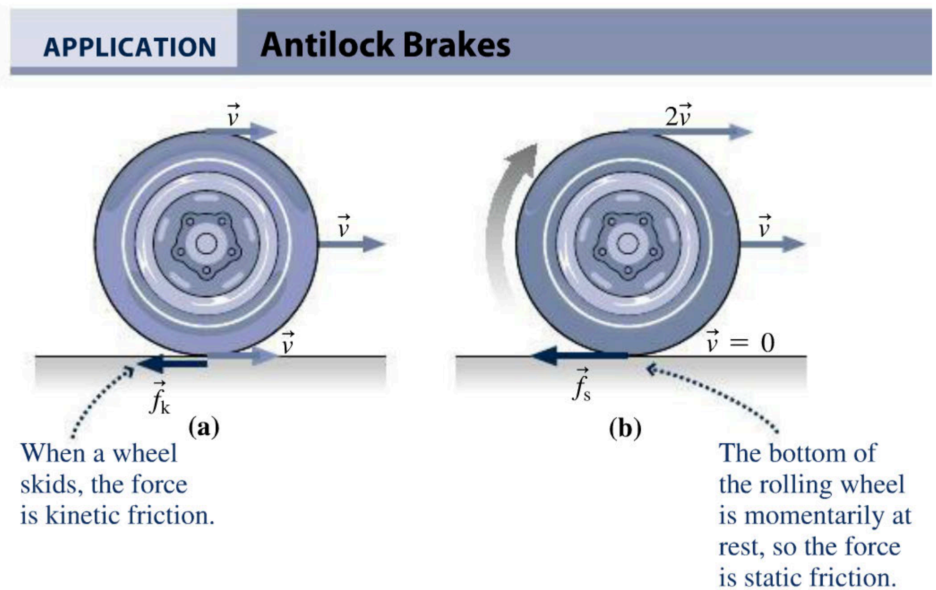
$$f_k = \mu_k N$$

Resnick & Halliday (1966)

Review: Static vs Kinetic vs "Rolling" Friction

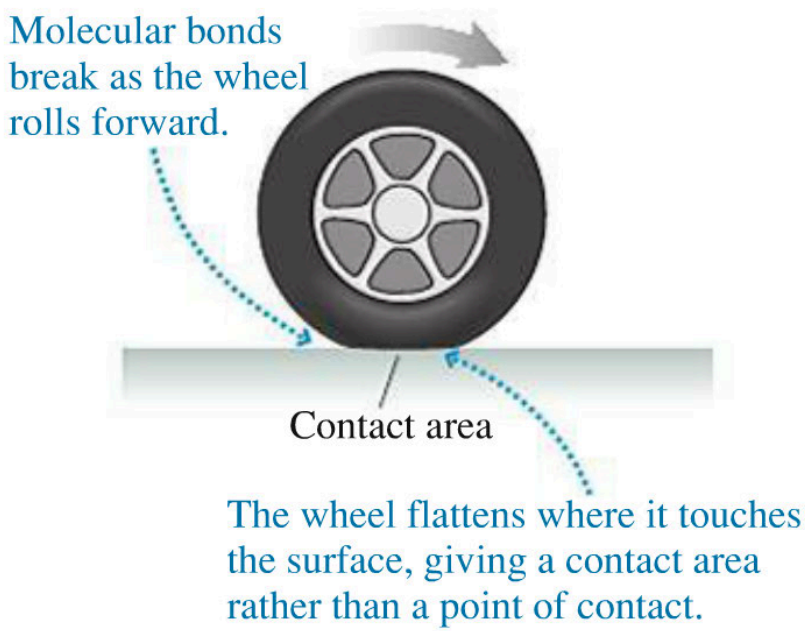
Note: For wheels, the notion of “rolling friction” here (as opposed to static friction) is a bit different re Kesten & Tauck (which is in ch.8!)

$$\text{Static: } \vec{f}_s \leq (\mu_s n, \text{ direction as necessary to prevent motion})$$
$$\text{Kinetic: } \vec{f}_k = (\mu_k n, \text{ direction opposite the motion})$$
$$\text{Rolling: } \vec{f}_r = (\mu_r n, \text{ direction opposite the motion})$$



Wolfson

Rolling friction is due to the contact area between a wheel and the surface.



Knight (2013)

“Laws of Friction”

The two laws of friction above were discovered experimentally by Leonardo da Vinci (1452–1519) and rediscovered, in 1699, by the French engineer G. Amontons. Leonardo’s statement of the two laws was remarkable, coming as it did about two centuries before the concept of force was fully developed by Newton. Leonardo’s formulation was: (1) “Friction made by the same weight will be of equal resistance at the beginning of the movement though the contact may be of different breadths or lengths” and (2) “Friction produces double the amount of effort if the weight be doubled.” The French scientist, Charles A. Coulomb, (1736–1806) did many experiments on friction and pointed out the difference between static and kinetic friction.

Charles-Augustin de Coulomb (1736-1806)

(you’ll see this guy again downstream re another very important inverse-square law in physics re “action-at-a-distance”)

$$F = k_e \frac{q_1 q_2}{r^2}$$

Resnick & Halliday (1966)