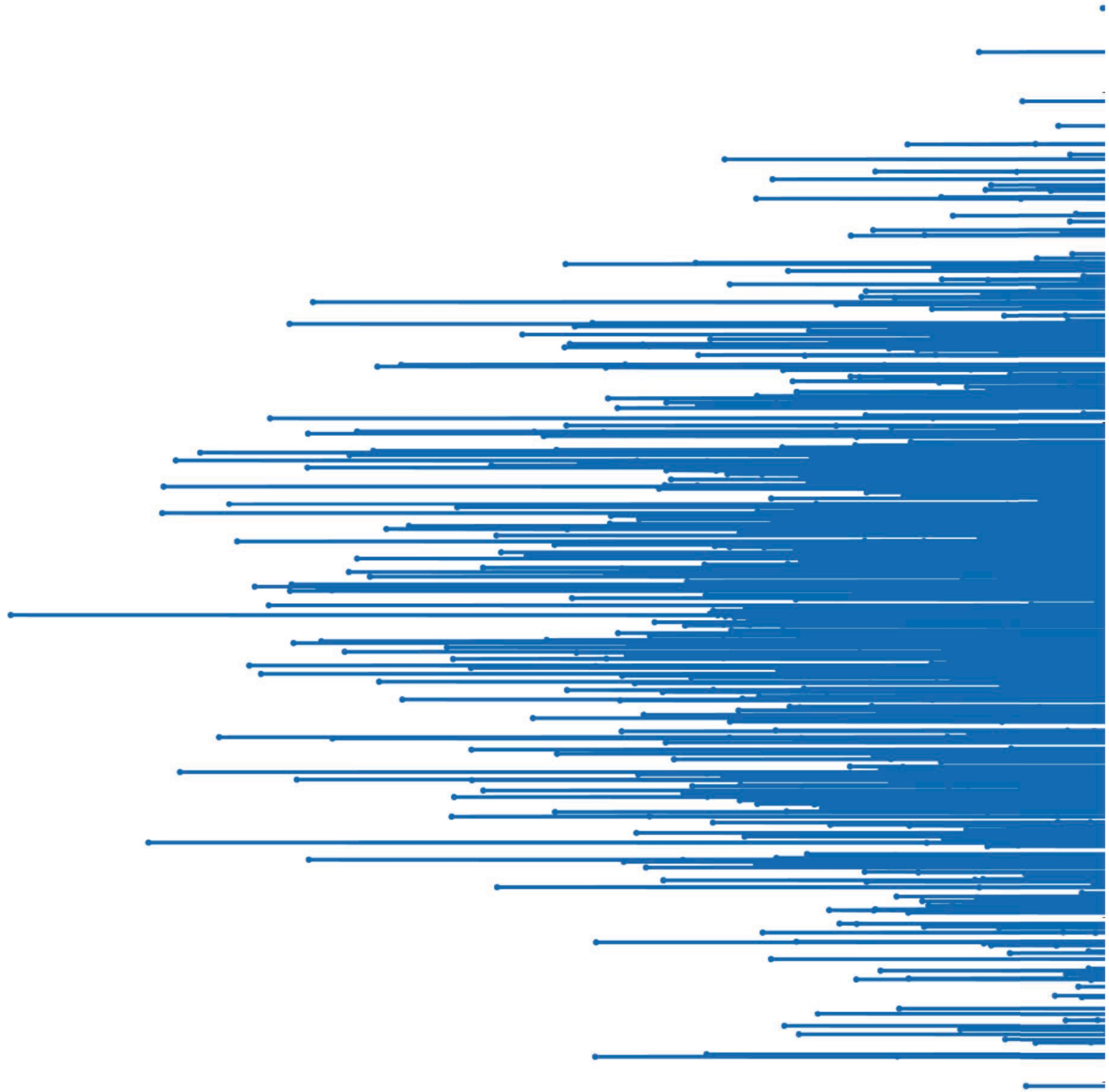
PHYS 2010 (W20)
Classical Mechanics



2020.01.30

Relevant reading:

Knudsen & Hjorth: 15.1-15.3

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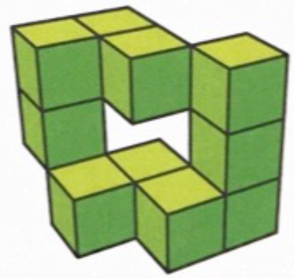
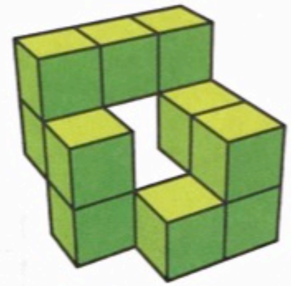
Ref.s:
Knudsen & Hjorth (2000), Fowles & Cassidy (2005)

228. Ten-Cube Ring


A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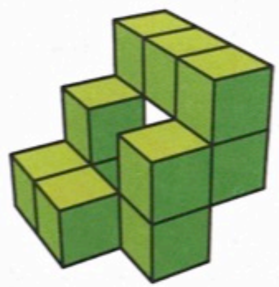
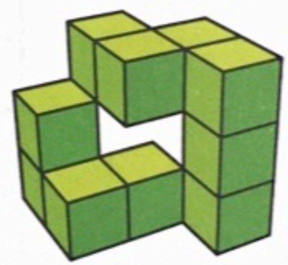

C


D

Which view of 10 cubes in a ring does not match the other three?

A B C D

"Separable" Forces

Definition: $$\mathbf{F} = \mathbf{i}F_x(x) + \mathbf{j}F_y(y) + \mathbf{k}F_z(z)$$

Note this useful property:
(useful exercise to do on the back of an envelope!)

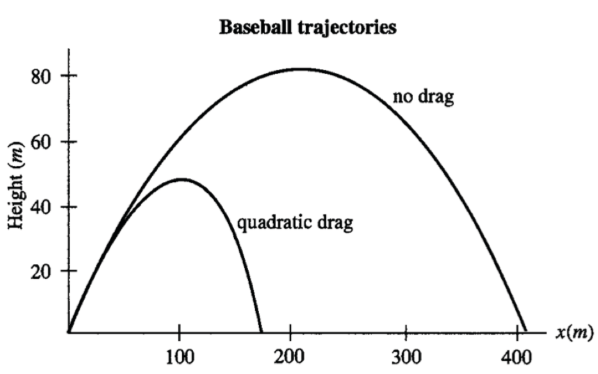
$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x(x) & F_y(y) & F_z(z) \end{vmatrix} = 0$$

$$F_x = m\ddot{x}$$
$$F_y = m\ddot{y}$$
$$F_z = m\ddot{z}$$

Now if $F_x = F_x(x)$, $F_y = F_y(y)$, etc..., then these can be dealt with relatively straightforwardly

But ones of this flavor are typically a bit more common....

$$m\ddot{x} = F_x(x, \dot{x}, t)$$



Fowles & Cassidy

Review: Uniform Gravitational Field

Newton's Law of Gravitation:
(magnitude only)

$$F = G\frac{Mm}{r^2}$$

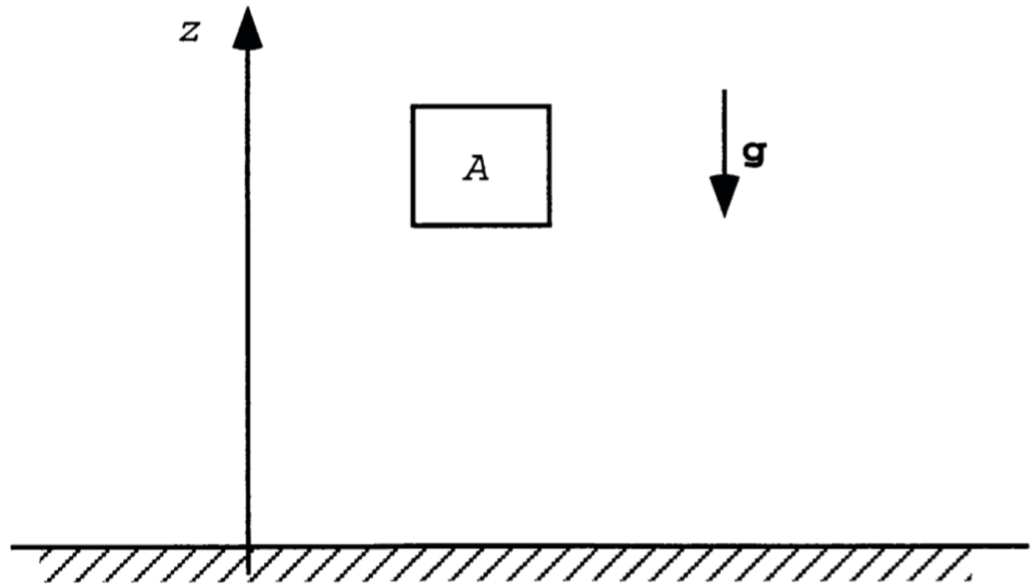


Fig. 3.3.

where: $G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$

Note

Our knowledge of the masses of the objects in the solar system rests entirely on the determination of G. △

Review: Uniform Gravitational Field

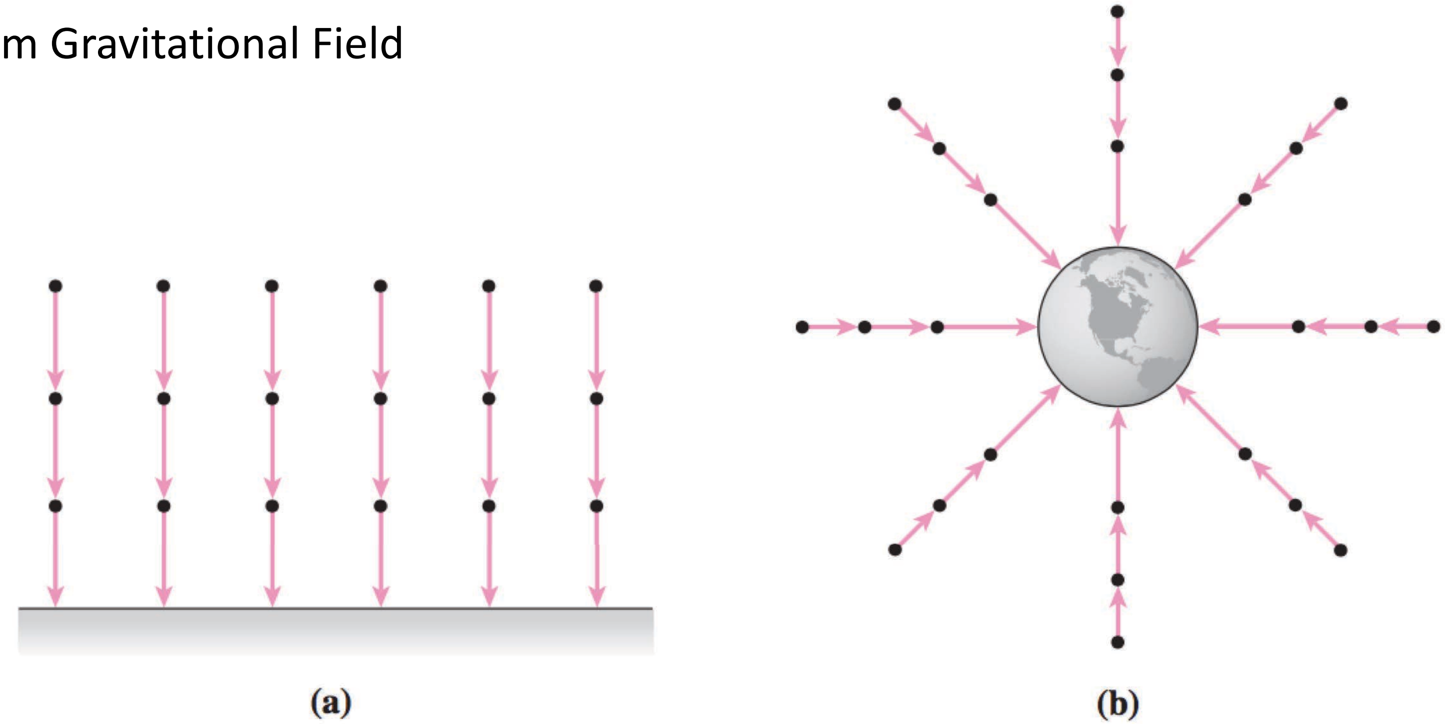


FIGURE 8.14 Gravitational field vectors at points (a) near Earth's surface and (b) on a larger scale.

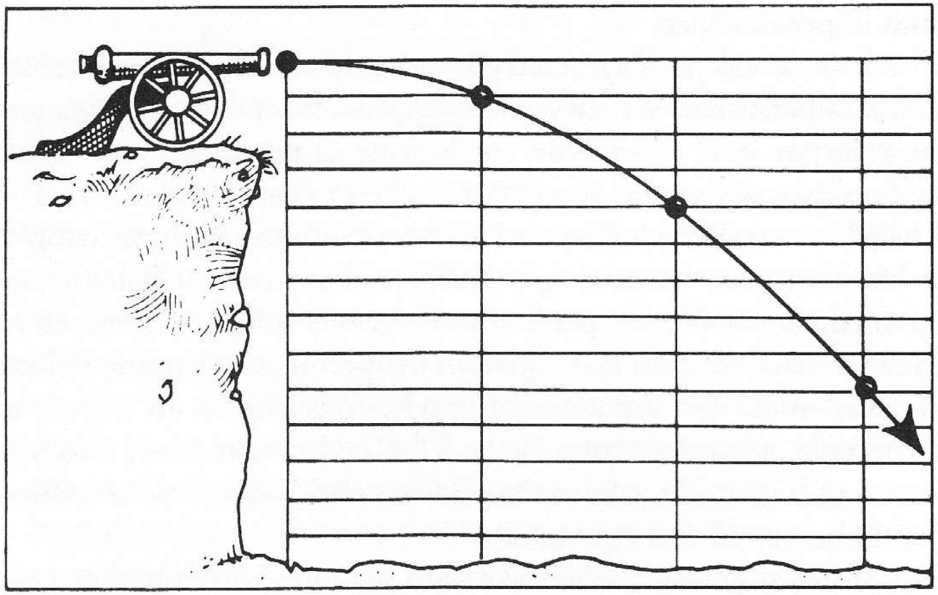
$$\vec{g} = -g\hat{j} \qquad \text{(gravitational field near Earth's surface)}$$

$$\vec{g} = -\frac{GM}{r^2}\hat{r} \qquad \text{(gravitational field of a spherical mass } M\text{)}$$

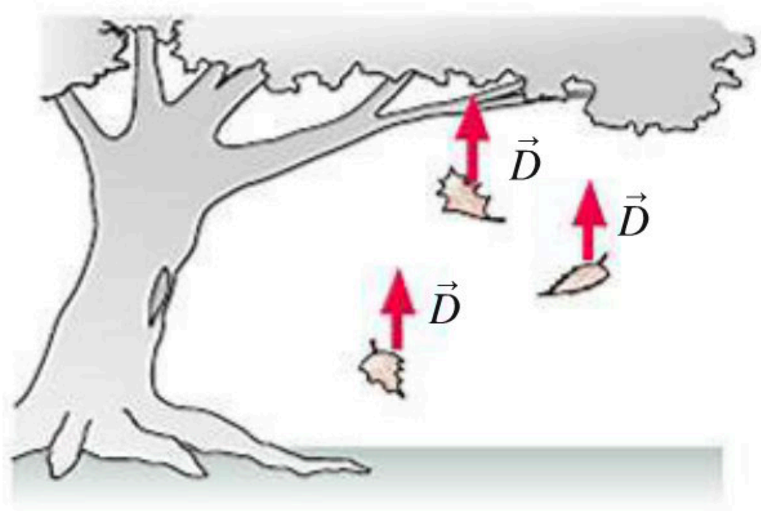
Exercise: Compute the condition re Newton's LoG for "*close to Earth's surface*" such that $\mathbf{F}$ can be considered approximately "constant". By what order of magnitude is that in error for something falling 1 m?

Wolfson (2012)

Projectile Motion: Conservative Gravitational Field + Non-conservative Drag



Air resistance is a significant force on falling leaves. It points opposite the direction of motion.

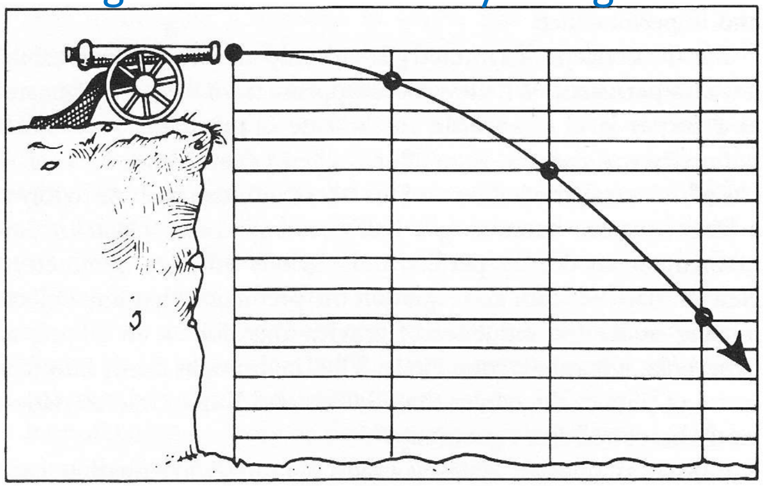


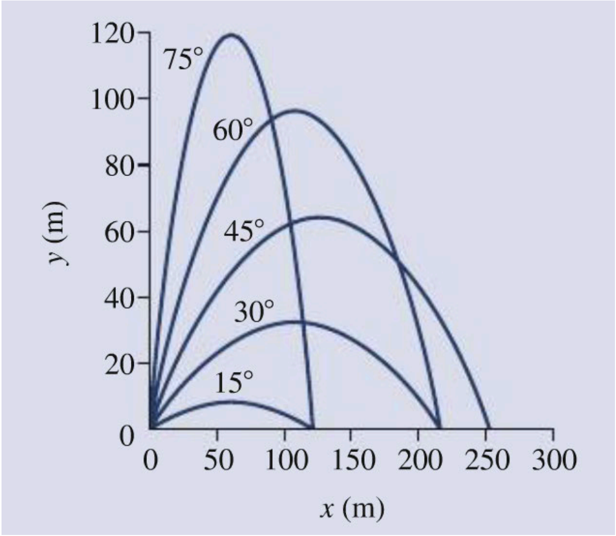
Knight (2013)

<u>Recall</u>: Higher Dimensions & Projectile Motion

Niccolò Tartaglia (1499-1557)

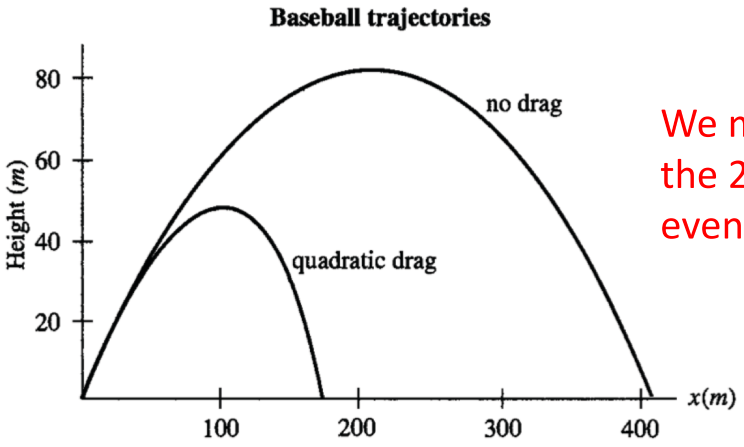
No drag case was relatively straightforward...





1-D nonlinear drag case less so....

$$m\frac{dv}{dt} = mg - c_2 v^2 = mg\left(1 - \frac{c_2}{mg}v^2\right)$$



Fowles & Cassidy

We might expect the 2-D case to be even "messier"!

<u>Recall</u>: Projectile Motion 2) Vector calculation (**No Drag**)

$$\mathbf{r}(t) = \mathbf{i}bt + \mathbf{j}\left(ct - \frac{gt^2}{2}\right) + \mathbf{k}0$$

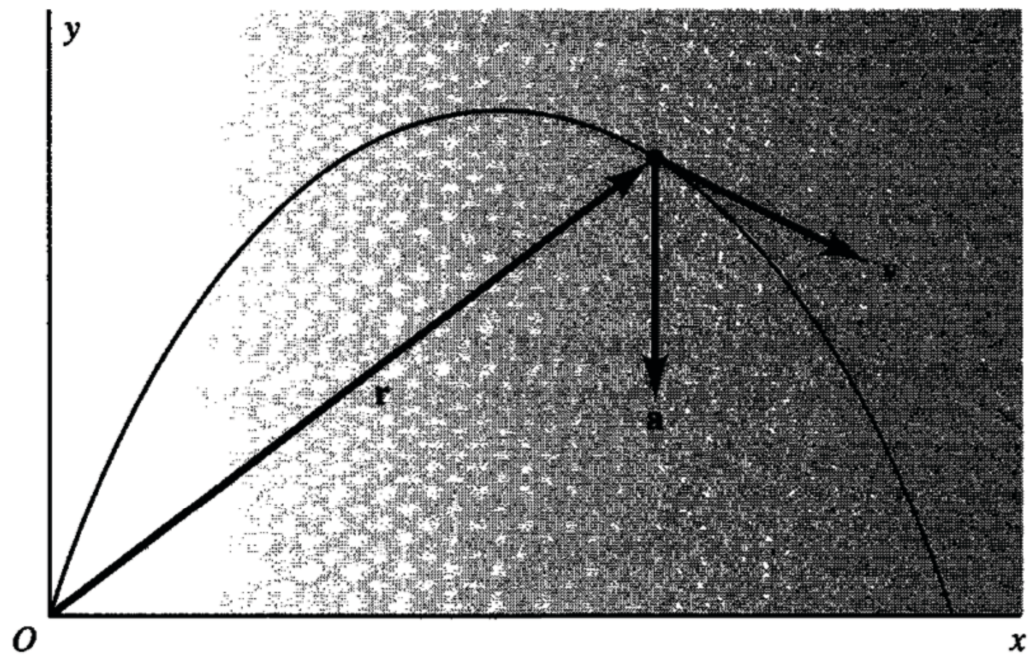
$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \mathbf{i}b + \mathbf{j}(c - gt)$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -\mathbf{j}g$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = -\mathbf{k}gt + \mathbf{v}_0$$

Note the (slight) change in coord system as assumed at start of lecture!

$$v = [b^2 + (c - gt)^2]^{1/2}$$



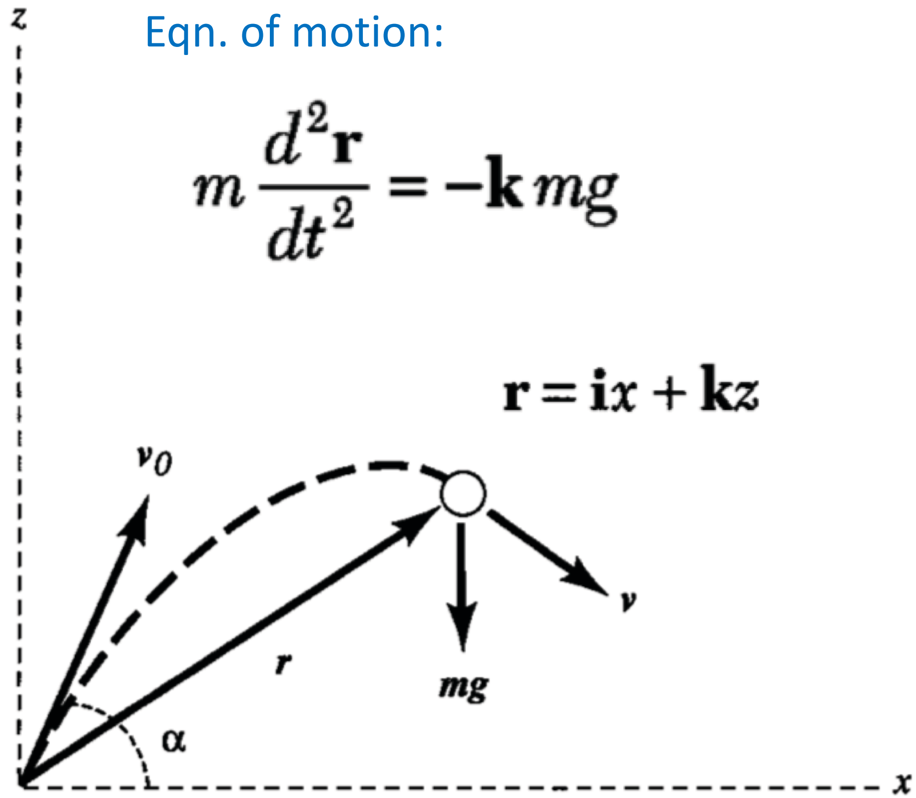
Fowles & Cassidy

Revisiting Projectile Motion: No Air Resistance

Eqn. of motion:

$$m\frac{d^2\mathbf{r}}{dt^2} = -\mathbf{k}\,mg$$

$$\mathbf{r} = \mathbf{i}x + \mathbf{k}z$$



Integrate to yield:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = -\mathbf{k}gt + \mathbf{v}_0$$

$$\mathbf{r} = -\mathbf{k}\tfrac{1}{2}gt^2 + \mathbf{v}_0 t + \mathbf{r}_0$$

Or in component form:

$$\mathbf{v} = \mathbf{i}v_0\cos\alpha + \mathbf{k}(v_0\sin\alpha - gt)$$

$$\mathbf{r} = \mathbf{i}(v_0\cos\alpha)t + \mathbf{k}\left((v_0\sin\alpha)t - \tfrac{1}{2}gt^2\right)$$

And the expected bits fall right out....

$$x = \dot{x}_0 t = (v_0\cos\alpha)t$$

$$y = \dot{y}_0 t \equiv 0$$

$$z = \dot{z}_0 t - \tfrac{1}{2}gt^2 = (v_0\sin\alpha)t - \tfrac{1}{2}gt^2$$

$$t = \frac{x}{v_0\cos\alpha}$$

(e.g., parabolas!)

$$z = (\tan\alpha)x - \left(\frac{g}{2v_0^2\cos^2\alpha}\right)x^2$$

Fowles & Cassidy

Revisiting Projectile Motion: No Air Resistance

And the expected bits fall right out....

Max height:

$$z_{max} = \frac{v_0^2 \sin^2\alpha}{2g}$$

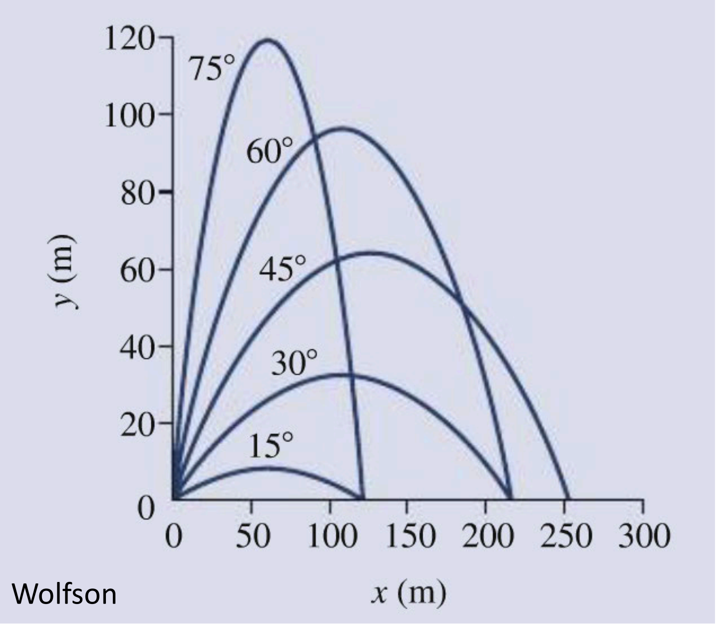
Time to max height:

$$t_{max} = \frac{v_0 \sin\alpha}{g}$$

"Range":

$$R = x = \frac{v_0^2 \sin^2 2\alpha}{g}$$

R has its maximum value $R_{max} = v_0^2/g$ at $\alpha = 45°$.



Fowles & Cassidy

Connecting back to 1st year physics....

So this compact expression....

$$m\frac{d^2\mathbf{r}}{dt^2} = -\mathbf{k}mg$$

... implicitly contains all these parts!

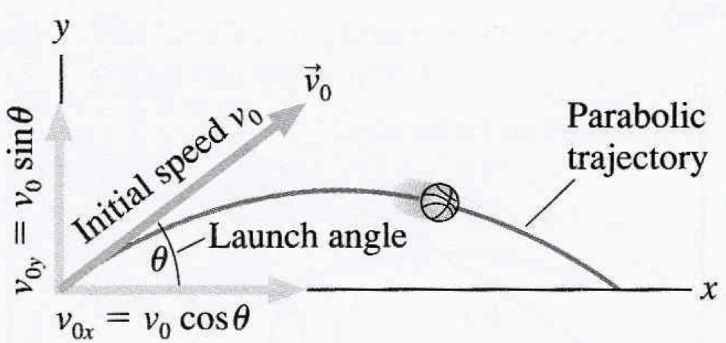
$$\vec{F}_{net} = m\vec{a}$$

Product of object's mass and its acceleration; not a force

Net force: the vector sum of all real, physical forces acting on an object

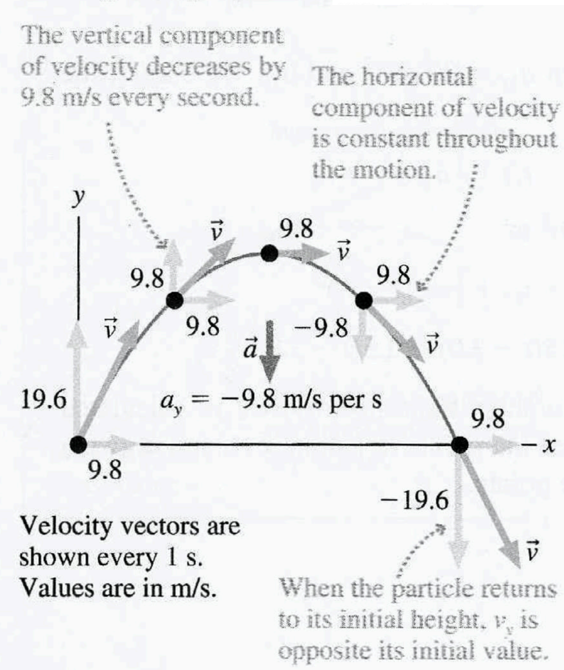
Equal sign indicates that the two sides are mathematically equal — but that doesn't mean they're the same physically. Only $\vec{F}_{net}$ involves physical forces.

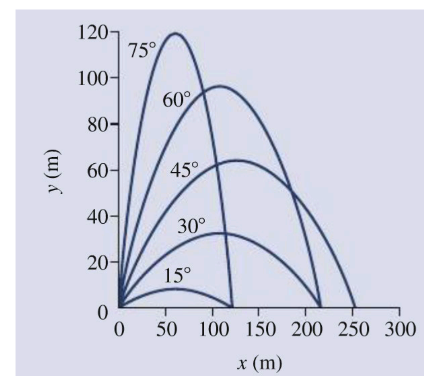
FIGURE 4.14 A projectile launched with initial velocity $\vec{v}_0$.



Knight

FIGURE 4.15 The velocity acceleration vectors of a moving along a parabol





Wolfson

Revisiting Projectile Motion: Linear Air Resistance

Eqn. of motion: $$m\frac{d^2\mathbf{r}}{dt^2} = -m\gamma\mathbf{v} - \mathbf{k}mg$$

Or in component form:

$$\ddot{x} = -\gamma\dot{x}$$
$$\ddot{y} = -\gamma\dot{y}$$
$$\ddot{z} = -\gamma\dot{z} - g$$

And we have already seen how to deal with (i.e., integrate):

$$\dot{x} = \dot{x}_0 e^{-\gamma t}$$
$$\dot{y} = \dot{y}_0 e^{-\gamma t}$$
$$\dot{z} = \dot{z}_0 e^{-\gamma t} - \frac{g}{\gamma}(1 - e^{-\gamma t})$$

$$x = \frac{\dot{x}_0}{\gamma}(1 - e^{-\gamma t})$$
$$z = \left(\frac{\dot{z}_0}{\gamma} + \frac{g}{\gamma^2}\right)(1 - e^{-\gamma t}) - \frac{g}{\gamma}t$$

Solution in vector form: $$\mathbf{r} = \left(\frac{\mathbf{v}_0}{\gamma} + \frac{\mathbf{k}g}{\gamma^2}\right)(1 - e^{-\gamma t}) - \mathbf{k}\frac{gt}{\gamma}$$

And don't forget the odd things that can happen with even linear drag... (e.g., x asymptotically approaches a limiting value, but never quite gets there)

Fowles & Cassidy

Revisiting Projectile Motion: Linear Air Resistance

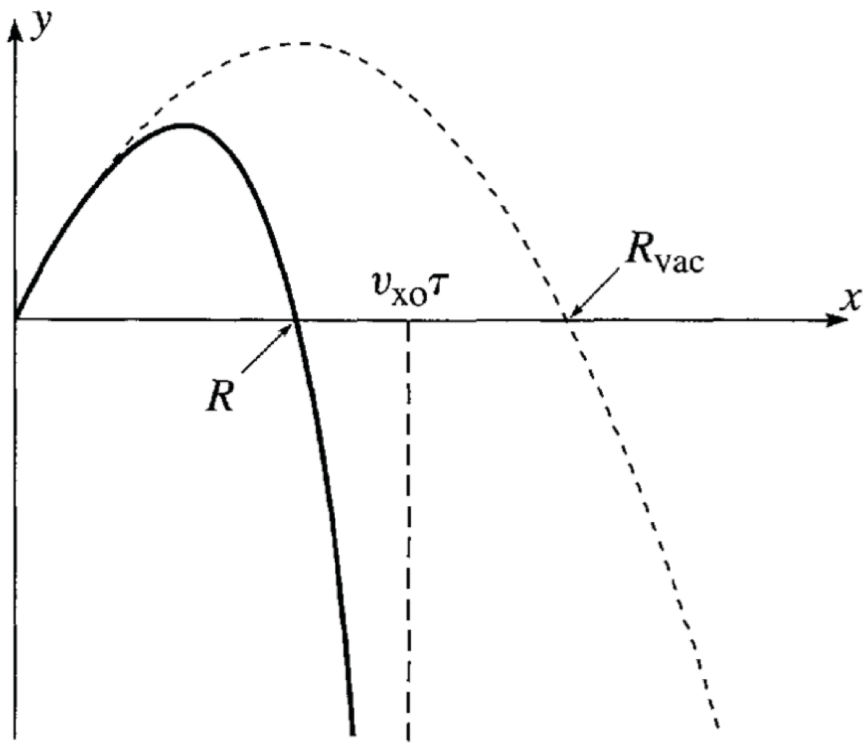


Figure 2.7 The trajectory of a projectile subject to a linear drag force (solid curve) and the corresponding trajectory in a vacuum (dashed curve). At first the two curves are very similar, but as t increases, air resistance slows the projectile and pulls its trajectory down, with a vertical asymptote at $x = v_{xo}\tau$. The horizontal range of the projectile is labeled R, and the corresponding range in vacuum R_{vac}.

Taylor (2005)

Revisiting Projectile Motion: Linear Air Resistance

$$x = \frac{\dot{x}_0}{\gamma}(1 - e^{-\gamma t})$$

$$z = \left(\frac{\dot{z}_0}{\gamma} + \frac{g}{\gamma^2}\right)(1 - e^{-\gamma t}) - \frac{g}{\gamma}t$$

Rearranging the equation for x to get t :

$$t = -\gamma^{-1}\ln(1 - \gamma x/\dot{x}_0)$$

To get the horizontal range (i.e., $x_{\max}$), set $z = 0$ and plug in that value for t:

$$\left(\frac{\dot{z}_0}{\gamma} + \frac{g}{\gamma^2}\right)\frac{\gamma x_{max}}{\dot{x}_0} + \frac{g}{\gamma^2}\ln\left(1 - \frac{\gamma x_{max}}{\dot{x}_0}\right) = 0$$

→ Transcendental equation for $x_{\max}$!

(i.e., the equation contains the variable being solved for; sometimes they are solvable, sometimes they are not...)

Aside:

$$x = e^{-x}$$
$$x = \cos x$$
$$2^x = x^2$$

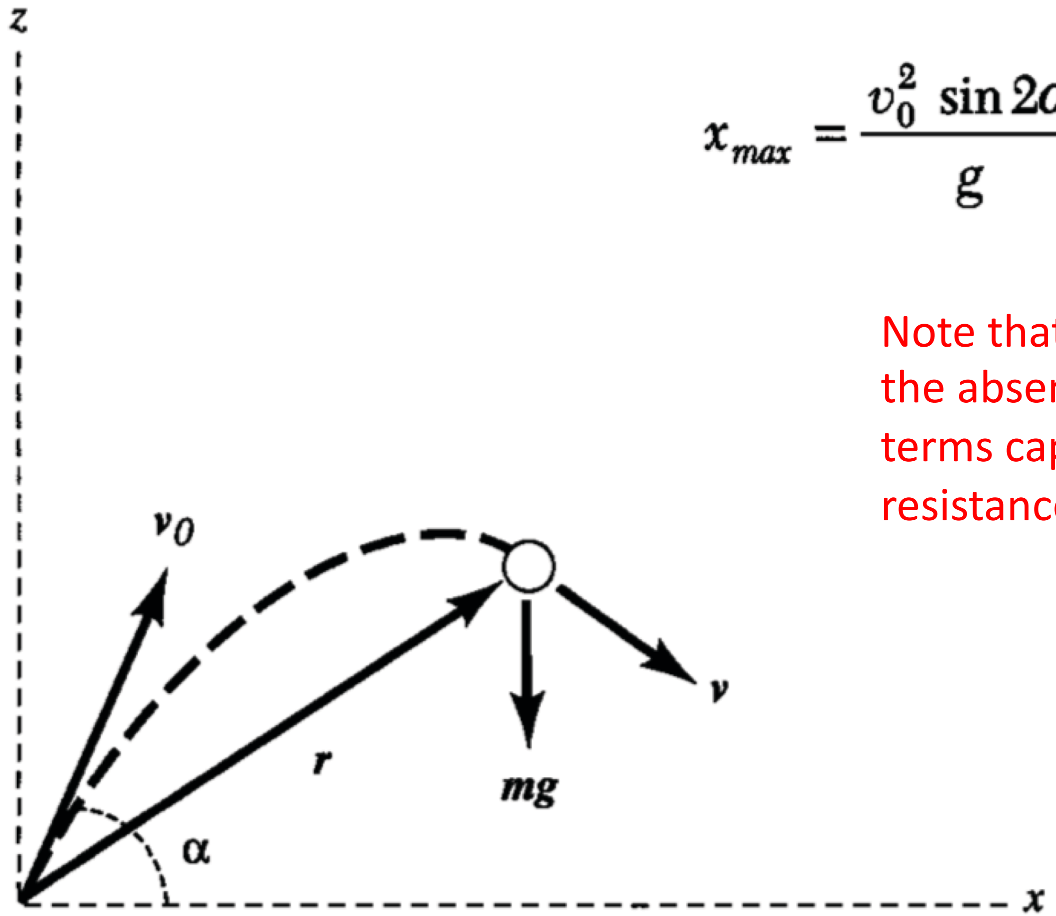
Using the following series expansion, this one is solvable:

$$\ln(1-u) = -u - \frac{u^2}{2} - \frac{u^3}{3} - \cdots$$

$$x_{max} = \frac{2\dot{x}_0\dot{z}_0}{g} - \frac{8\dot{x}_0\dot{z}_0^2}{3g^2}\gamma + \cdots$$

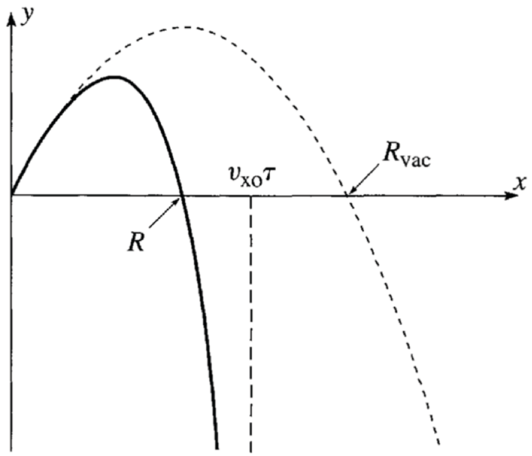
Fowles & Cassidy

Revisiting Projectile Motion: Linear Air Resistance



$$x_{max} = \frac{v_0^2 \sin 2\alpha}{g} - \frac{4v_0^3 \sin 2\alpha \sin \alpha}{3g^2}\gamma + \cdots$$

Note that the first term is the range in the absence of drag, the higher order terms capturing the decrease due to air resistance



Fowles & Cassidy

Revisiting Projectile Motion: Quadratic Drag

Note: There is a ***duality*** here. On one hand, this is a gross oversimplification (for what drag forces an object will experience). But on the other, the nonlinear nature of things greatly complicates analysis....

Assume drag force goes as square of velocity:

$$\mathbf{F}_D(v) = -c_2 |v| \mathbf{v}$$

Eqn. of motion:

$$m\ddot{\mathbf{r}} = -c_2 |v| \mathbf{v} - mg\mathbf{k}$$

In component form:

$$m\ddot{x} = -c_2 |v| \dot{x}$$

$$m\ddot{z} = -c_2 |v| \dot{z} - mg$$

Let: $\gamma = c_2/m$

Leaving us to deal with:

$$\ddot{x} = -\gamma(\dot{x}^2 + \dot{z}^2)^{1/2}\dot{x}$$

$$\ddot{z} = -\gamma(\dot{x}^2 + \dot{z}^2)^{1/2}\dot{z} - g$$

→ Set of coupled nonlinear ODEs. Not possible(?) to solve analytically in closed form.....

... so one possible strategy might be to examine from a *numerical* approach

Fowles & Cassidy

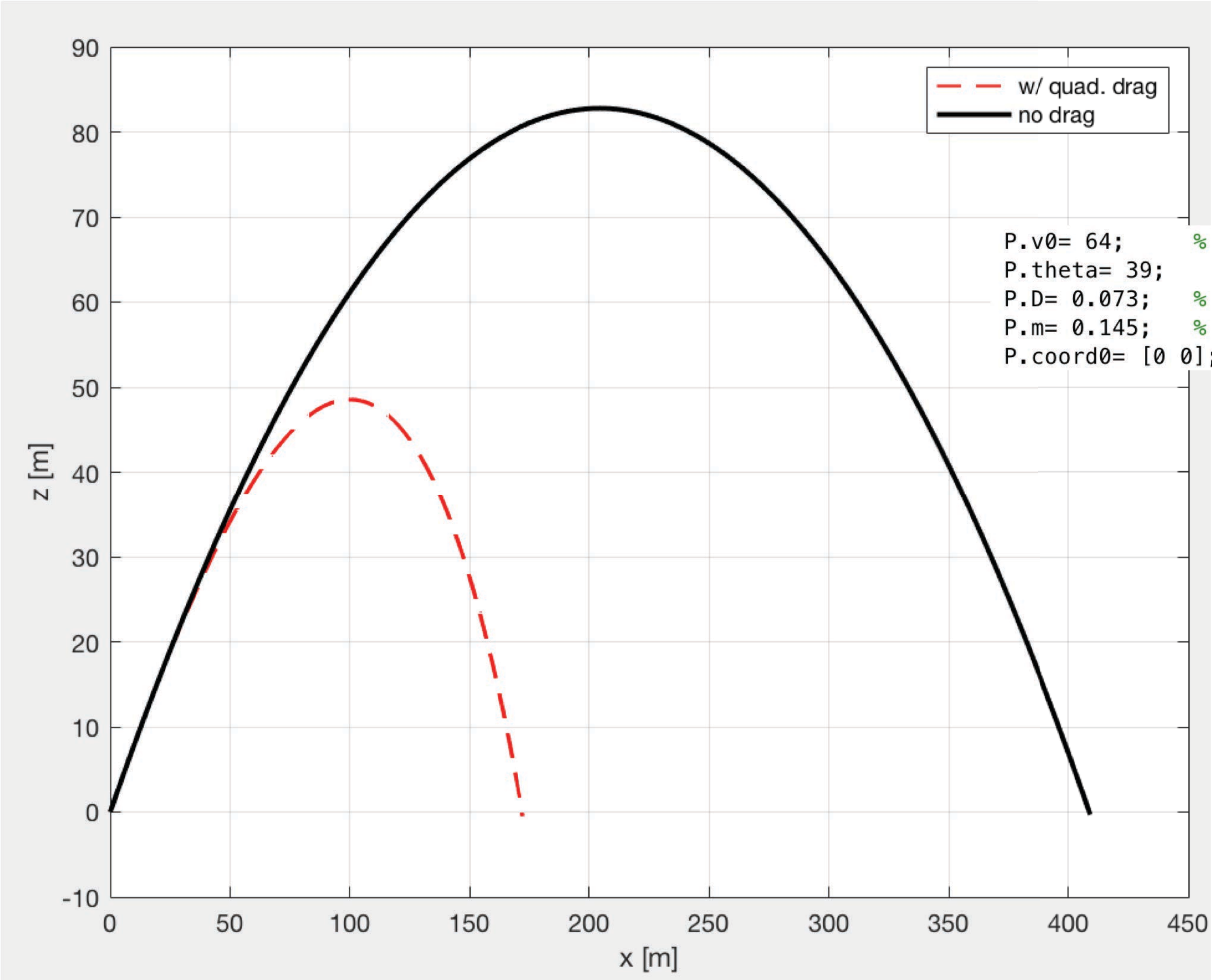
```
% ### EXprojectile.m ###      2020.01.28  C. Bergevin
% [REF: ex.4.3.2 from Fowles & Cassidy 2005]
% Purpose: Solve/plot 2-D projectile motion for spherical object w/ (optional)
% quadratic drag (x is horiz. position, z vert. pos.)
% ---- Notes
% o v0= 143.2 mph ~ 64 m/s
clear
% ------------------------------------------
P.g= 9.8;      % grav. const. [m^2/s] {9.8}
P.drag= 0;      % boolean to incl. drag: 0=no drag, 1=drag {1}
P.v0= 64;      % launch velocity [m/s] {64}
P.theta= 39;      % launch angle [degrees] {45}
P.D= 0.073;    % diameter of object [m] {0.073}
P.m= 0.145;    % mass of object [kg] {0.145}
P.coord0= [0 0];  % initial [x z] coords [m] {[0 0]}
P.tLim= [0 10];  % time limits of integration [s]
P.tRez= 300;      % # of (interp.) time points for integration interval {300?}
% ------------------------------------------
% --- derived params.
if (P.drag==0), P.gamma= 0;   % determine assoc. const. from input params.
else P.gamma= 0.15*P.D^2/P.m;  end
P.theta= pi*P.theta/180;   % convert launch angle to rads
P.y0(1)= P.coord0(1); P.y0(3)= P.coord0(2);  % initial horiz. and vert. positions
P.y0(2)= P.v0*cos(P.theta);  % initial horiz. velocity
P.y0(4)= P.v0*sin(P.theta);  % initial vert. velocity
% --- use built-in solver ode45 to numerically integrate
[t vals] = ode45('PROJECTILEfunction',linspace(P.tLim(1),P.tLim(2),P.tRez),P.y0,[],P);
% --- kludge: find when object hits the ground (and indicate if it hasn't)
indxG= find(vals(:,3)<0,1); flag= 0;
if (isempty(indxG)), disp('Longer int. time needed (to hit ground)'); indxG=size(vals,1); flag=1; end
indxH= find(vals(:,4)<0,1);  % index where velocity flips sign
% --- rename vars. (excluding those in the ground!)
x= vals(1:indxG,1); xdot= vals(1:indxG,2);
z= vals(1:indxG,3); zdot= vals(1:indxG,4);

% --- spit back a few vals. to screen
if (flag==0), disp(['total flight time= ',num2str(t(indxG)),' s']);
    disp(['horizontal dist. covered= ',num2str(x(indxG)),' m']);
    disp(['max. vert. height= ',num2str(z(indxH)),' m']);   end
% ---- visualize
figure(1); clf; h1= plot(x,z,'k-','LineWidth',1); hold on; grid on;
xlabel('x [m]');  ylabel('z [m]');
```

```
function [out1] = PROJECTILEfunction(t,y,flag,P)
% ----------------------------------------------------------------------
% o 2-D equations for projectile motion (see ex.4.3.2 from Fowles & Cassidy 2005)
% o x is the horizontal position, z the vertical position
% [see XX.m for further notes]
%   y(1) ... horiz. position x
%   y(2) ... horiz. velocity dx/dt
%   y(3) ... vert. position z
%   y(4) ... vert. velocity dz/dt

out1(1)= y(2);  % --> integrates to x(t)
out1(2)= -P.gamma* sqrt(y(2)^2 + y(4)^2)*y(2);  % --> integrates to dx/dt(t)
out1(3)= y(4);  % --> integrates to z(t)
out1(4)= -P.gamma* sqrt(y(2)^2 + y(4)^2)*y(4)- P.g;  % --> integrates to dz/dt(t)
out1= out1';    % wants output as a column vector
```

EXprojectile.m




```
P.v0= 64;          % launch velocity [m/s] {64}
P.theta= 39;          % launch angle [degrees] {45}
P.D= 0.073;        % diameter of object [m] {0.073}
P.m= 0.145;        % mass of object [kg] {0.145}
P.coord0= [0 0];   % initial [x z] coords [m] {[0 0]}
```

w/ drag

```
total flight time= 6.2542 s
horizontal dist. covered= 172.1954 m
max. vert. height= 48.5492 m
```

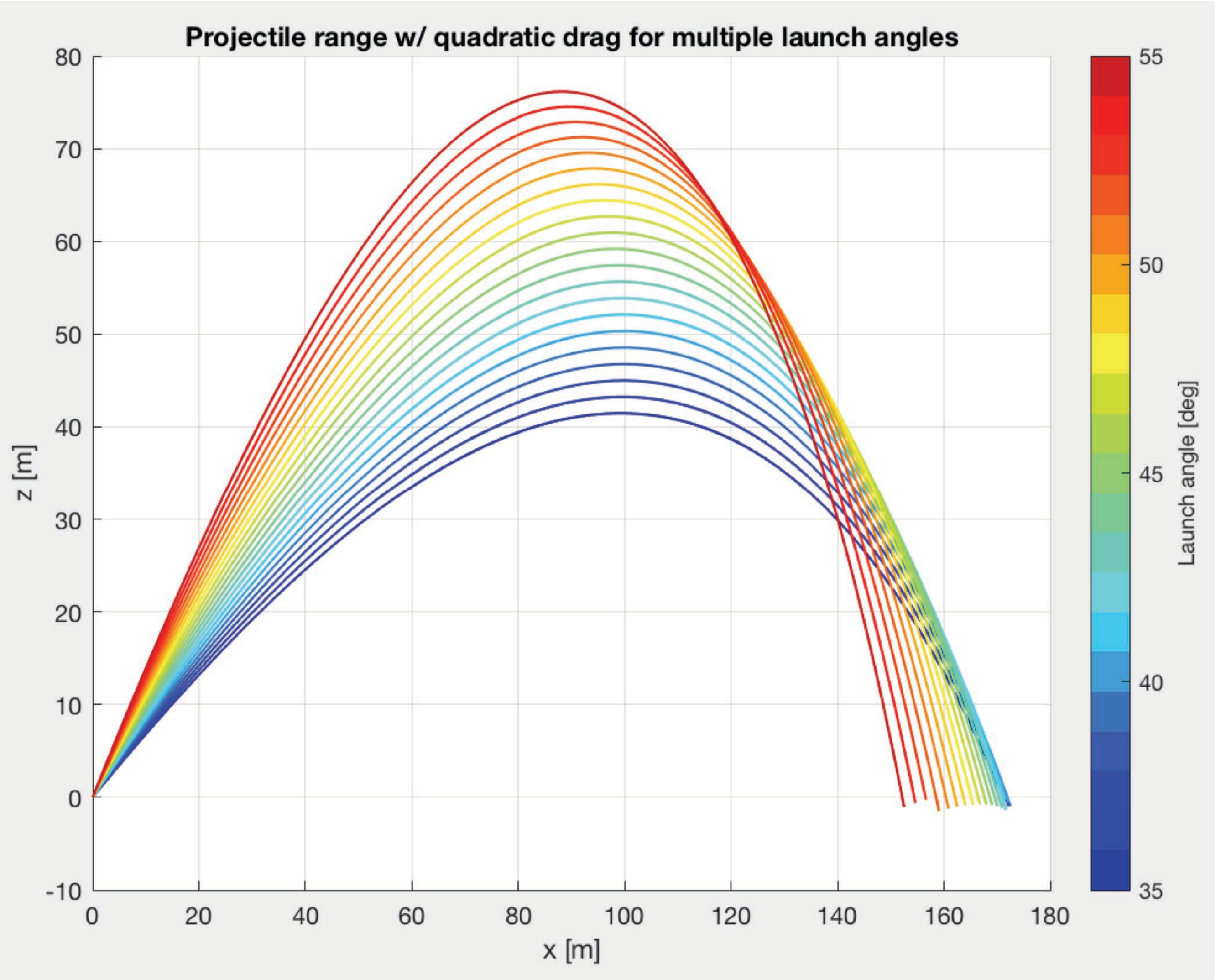
No drag

```
total flight time= 8.2274 s
horizontal dist. covered= 409.2102 m
max. vert. height= 82.7651 m
```

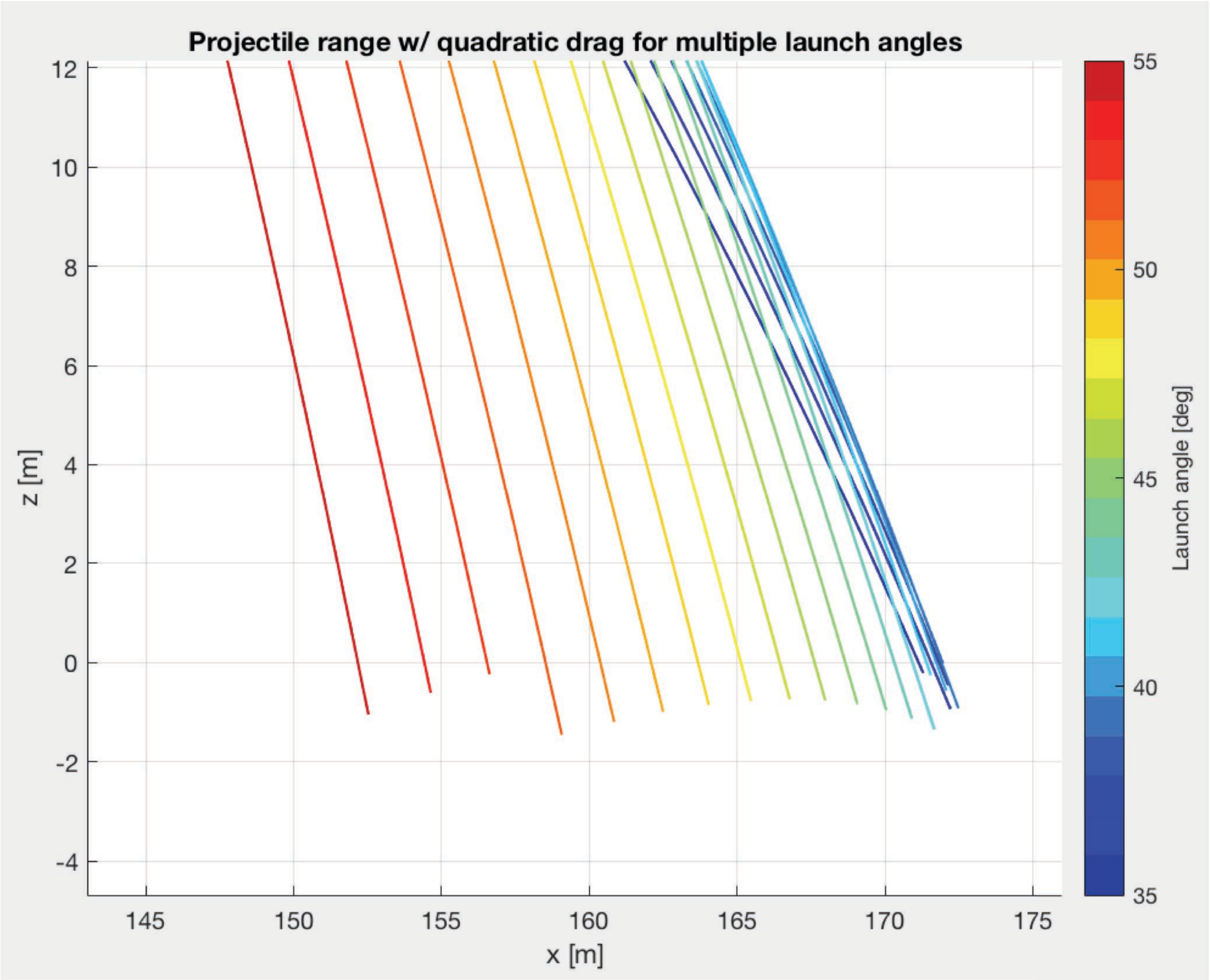
```
% ### EXprojectileMOD.m ###      2020.01.28  C. Bergevin

clear
% ------------------------------------------
P.thetaA= linspace(35,55,21);     % launch angle [degrees] {45}
P.g= 9.8;      % grav. const. [m^2/s] {9.8}
P.drag= 1;      % boolean to incl. drag: 0=no drag, 1=drag {1}
P.v0= 64;      % launch velocity [m/s] {64}
%P.theta= 39;      % launch angle [degrees] {45}
P.D= 0.073;    % diameter of object [m] {0.073}
P.m= 0.145;    % mass of object [kg] {0.145}
P.coord0= [0 0];  % initial [x z] coords [m] {[0 0]}
P.tLim= [0 15];  % time limits of integration [s]
P.tRez= 300;     % # of (interp.) time points for integration interval {300?}
% ------------------------------------------
% --- derived params.
if (P.drag==0), P.gamma= 0;   % determine assoc. const. from input params.
else P.gamma= 0.15*P.D^2/P.m;  end
P.y0(1)= P.coord0(1); P.y0(3)= P.coord0(2);  % initial horiz. and vert. positions
% --- set up fig. plus color-coding scheme
colormap(jet(numel(P.thetaA)))
jetcustom = jet(numel(P.thetaA));
figure(1); clf; hold on; grid on;
% ---
for nn=1:numel(P.thetaA)
    P.theta= P.thetaA(nn);
    P.theta= pi*P.theta/180;   % convert launch angle to rads
    P.y0(2)= P.v0*cos(P.theta);  % initial horiz. velocity
    P.y0(4)= P.v0*sin(P.theta);  % initial vert. velocity
    % --- use built-in solver ode45 to numerically integrate
    [t vals] = ode45('PROJECTILEfunction',linspace(P.tLim(1),P.tLim(2),P.tRez),P.y0,[],P);
    indxG= find(vals(:,3)<0,1);  % find when object hits the ground
    % --- rename vars. (excluding those in the ground!) & plot
    x= vals(1:indxG,1); xdot= vals(1:indxG,2);
    z= vals(1:indxG,3); zdot= vals(1:indxG,4);
    plot(x,z,'-','LineWidth',1,'Color',jetcustom(nn,:));
end
xlabel('x [m]');  ylabel('z [m]');
hC=colorbar; caxis([min(P.thetaA) max(P.thetaA)]);
ylabel(hC, 'Launch angle [deg]')
title('Projectile range w/ quadratic drag for multiple launch angles')
```

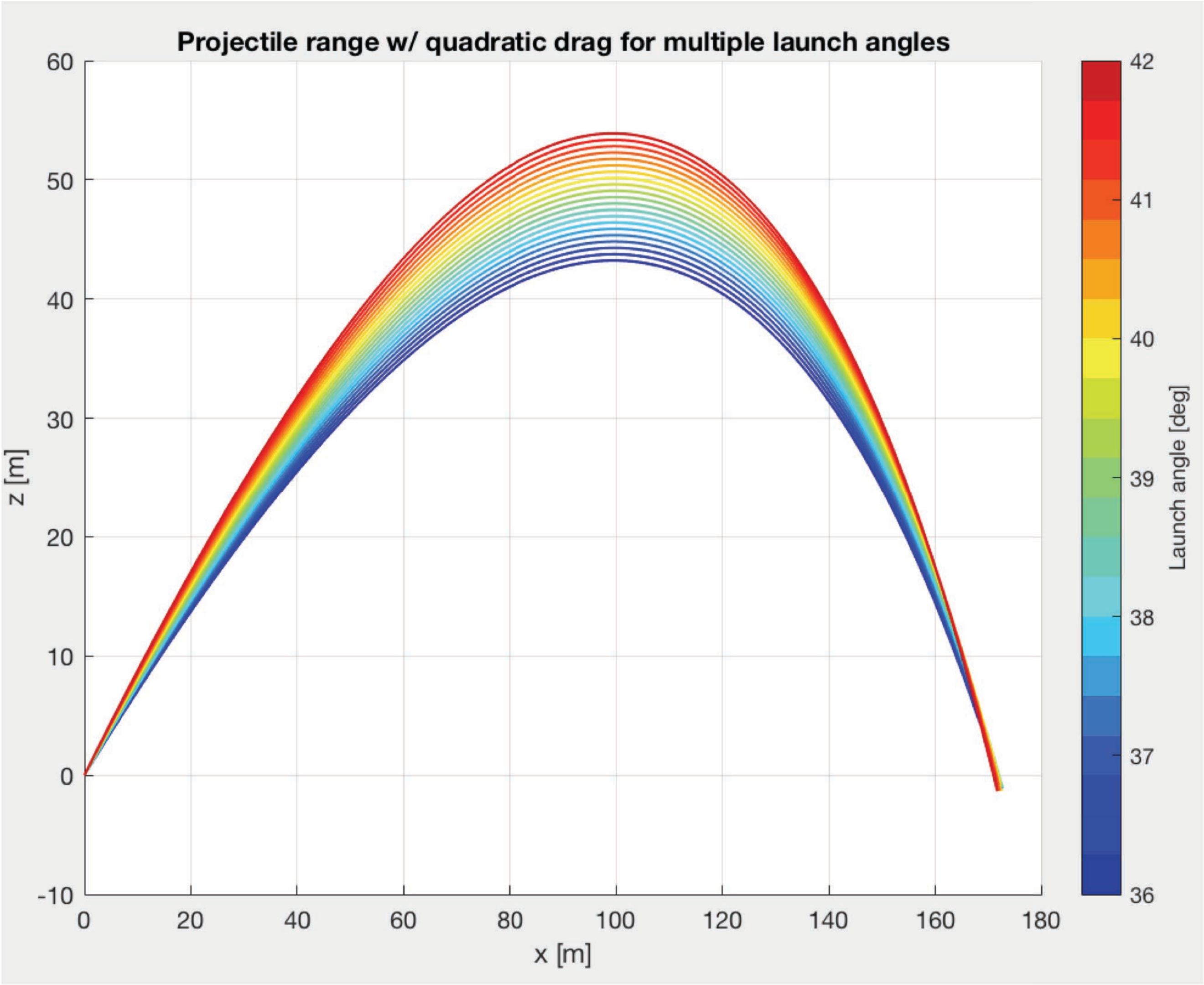
```
P.thetaA= linspace(55,35,21);  % launch angle [degrees]
```



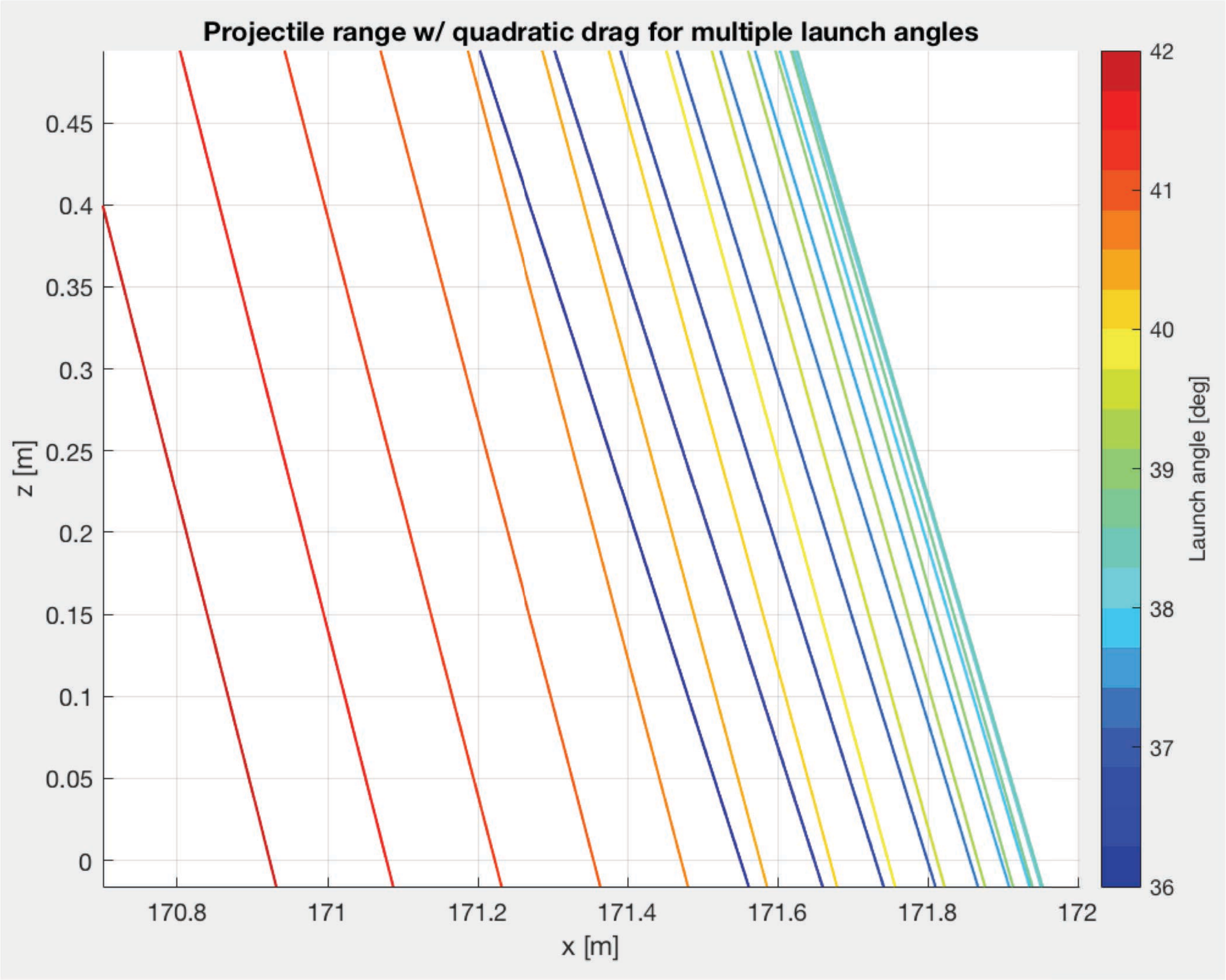
```
P.thetaA= linspace(55,35,21);  % launch angle [degrees]
```



```
P.thetaA= linspace(36,42,21);   % launch angle [degrees]
```

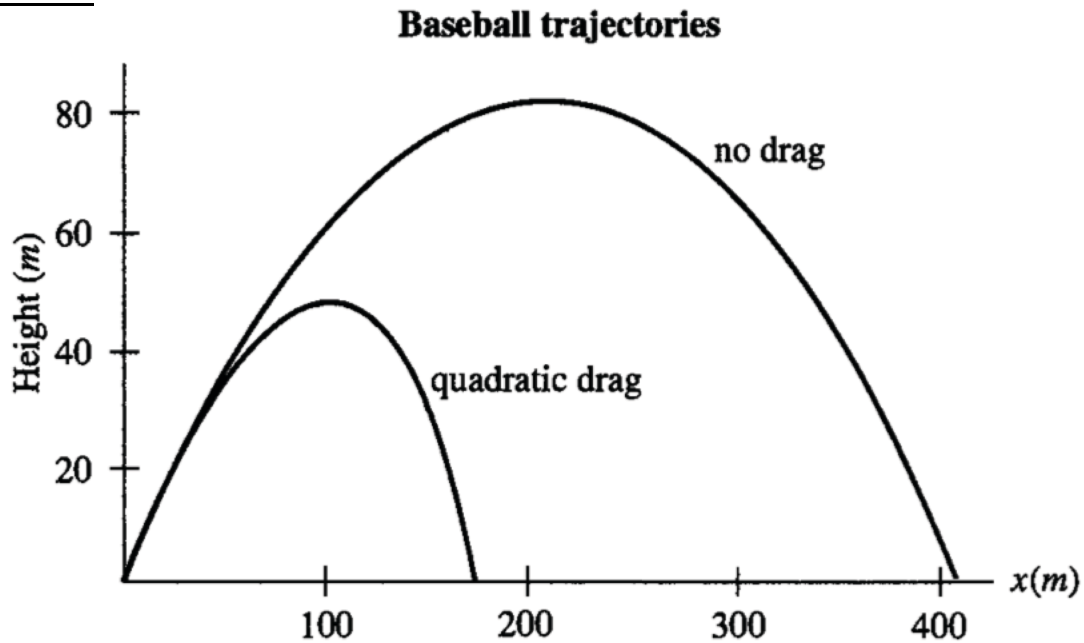


```
P.thetaA= linspace(36,42,21);  % launch angle [degrees]
```



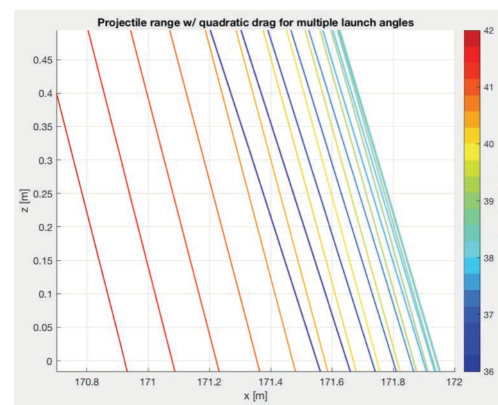
Ballpark 38-39° seems the optimal angle in this particular case....

Recall



Fowles & Cassidy

Short version: In the "real world", Tartaglia was wrong (you want a launch angle a bit less than 45°)



“Tartaglia’s correct theoretical answer of 45° surprised the experts; they thought it would be smaller [...] but he refrained from publication. The reason for his diffidence is highly creditable: He felt it would be immoral to use science to help [soliders] slaughter [soliders] more efficiently”

→ So the "experts" were right!

Recall: Nonconservative Force Fields

Assume $\mathbf{F}'$ is NOT conservative → There is no potential function V →

$\mathbf{F}' \cdot d\mathbf{r}$ is not an exact differential

But what is conservative ($\mathbf{F}$) and non-conservative forces ($\mathbf{F}'$) are both at play?

Work done over an increment is: $(\mathbf{F} + \mathbf{F}') \cdot d\mathbf{r} = -dV + \mathbf{F}' \cdot d\mathbf{r} = dT$

Work-energy theorem becomes: $\int_A^B \mathbf{F}' \cdot d\mathbf{r} = \Delta(T + V) = \Delta E$

→ So total energy is not constant, but increases or decreases depending upon $\mathbf{F}'$

Note: If the force is dissipative (e.g., drag, air resistance), then: $\mathbf{F}' \cdot d\mathbf{r} < 0$

Largely in part due to the fact that $\mathbf{F}'$ would have to be opposite the direction of motion!

Fowles & Cassidy

Energy-based Arguements re Nonconservative Force Fields....

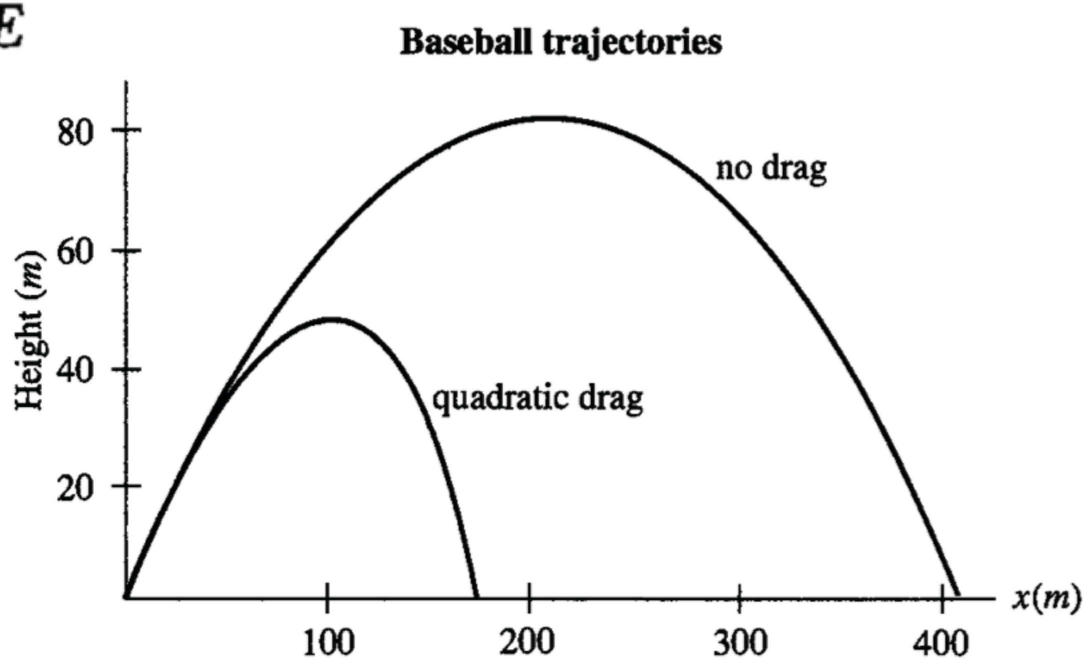
$$\mathbf{F} = -\nabla V$$

$$m\ddot{\mathbf{r}} = -c_2 |v| \mathbf{v} - mg\mathbf{k}$$

$\mathbf{F}' \cdot d\mathbf{r}$ is not an exact differential

$$(\mathbf{F} + \mathbf{F}') \cdot d\mathbf{r} = -dV + \mathbf{F}' \cdot d\mathbf{r} = dT$$

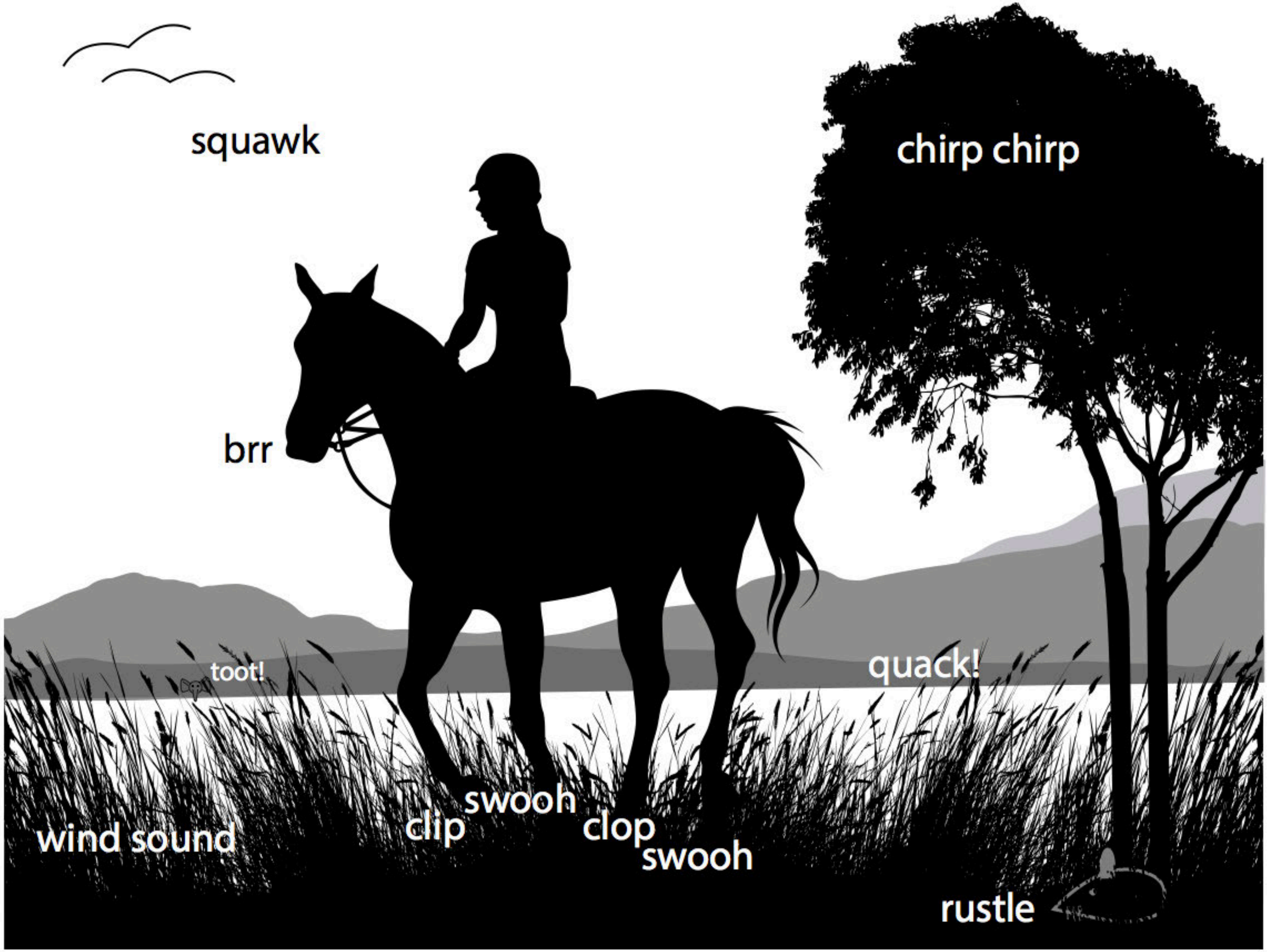
$$\int_A^B \mathbf{F}' \cdot d\mathbf{r} = \Delta(T + V) = \Delta E$$



→ Leads to a reasonable question: Is an energy-based approach any better/worse here?

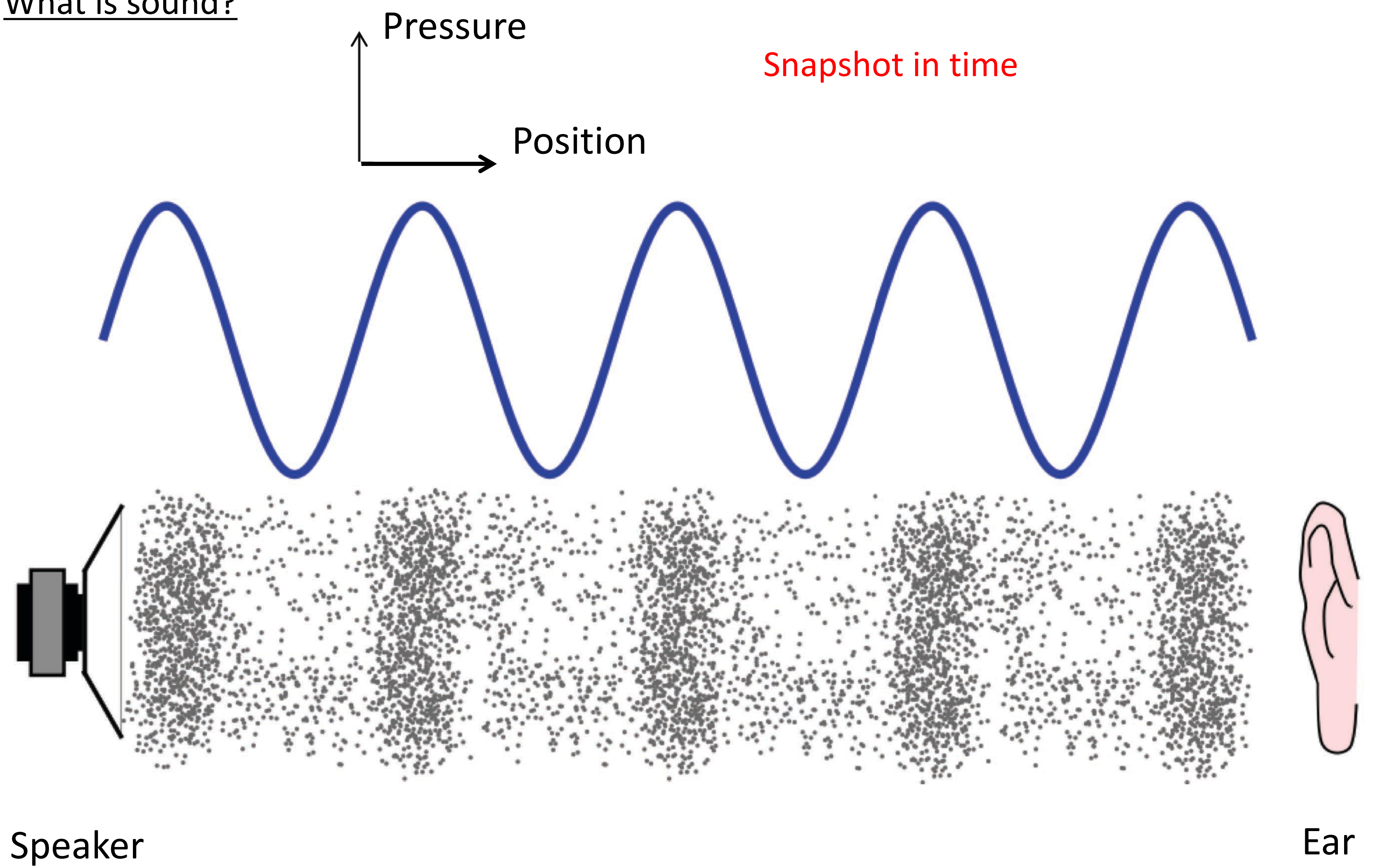
Fowles & Cassidy

Moving along....



Pulkki & Karjalainen (2015)

What is sound?



→ Note the periodic nature present....

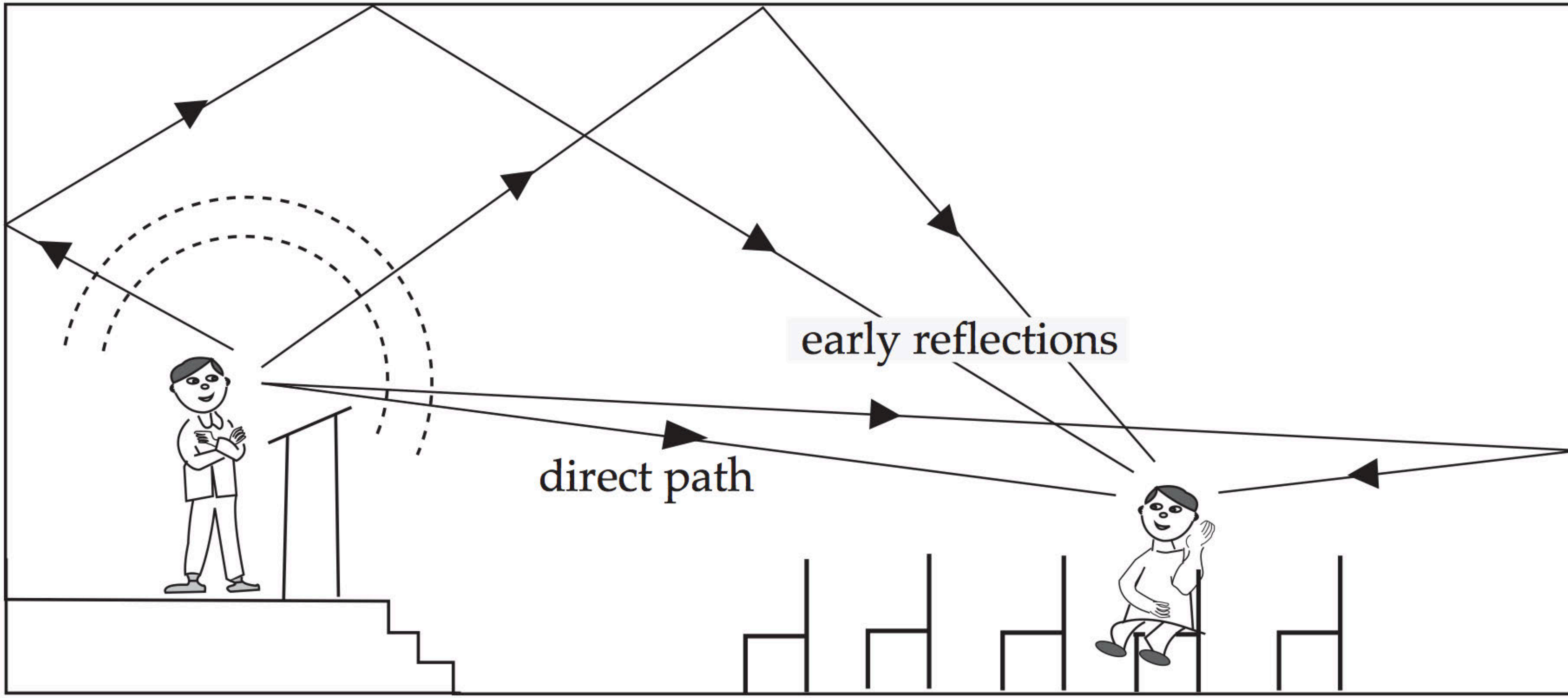
Pulkki & Karjalainen (2015)

Ex.

Why does the sound in a hall filled with people sound deader than in the same hall empty?

Shaskol'Skaya & El'Tsin (1963)

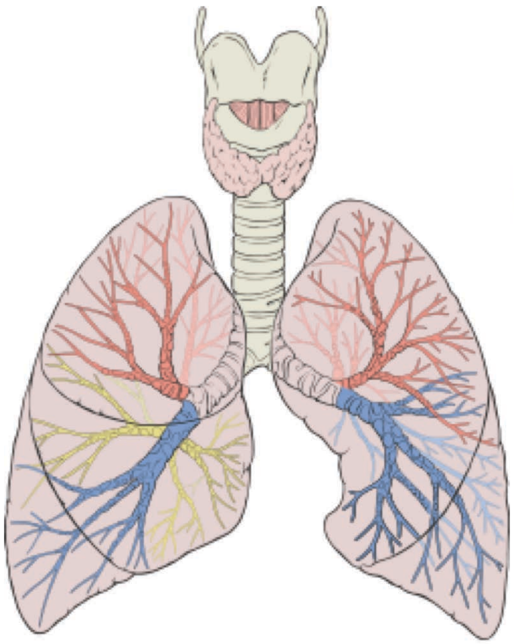
<u>What is sound?</u> (REVISITED)

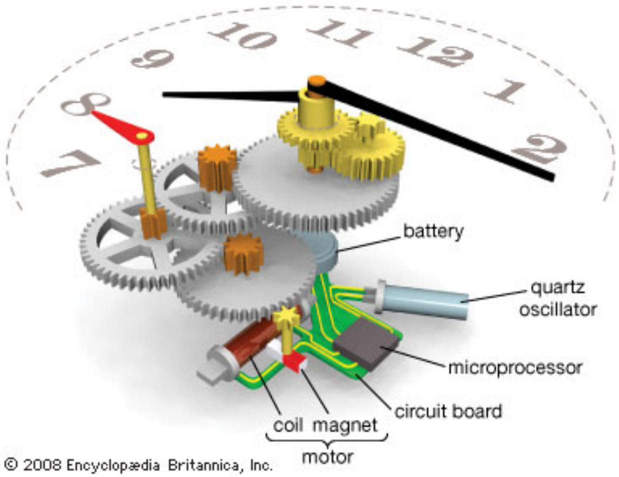


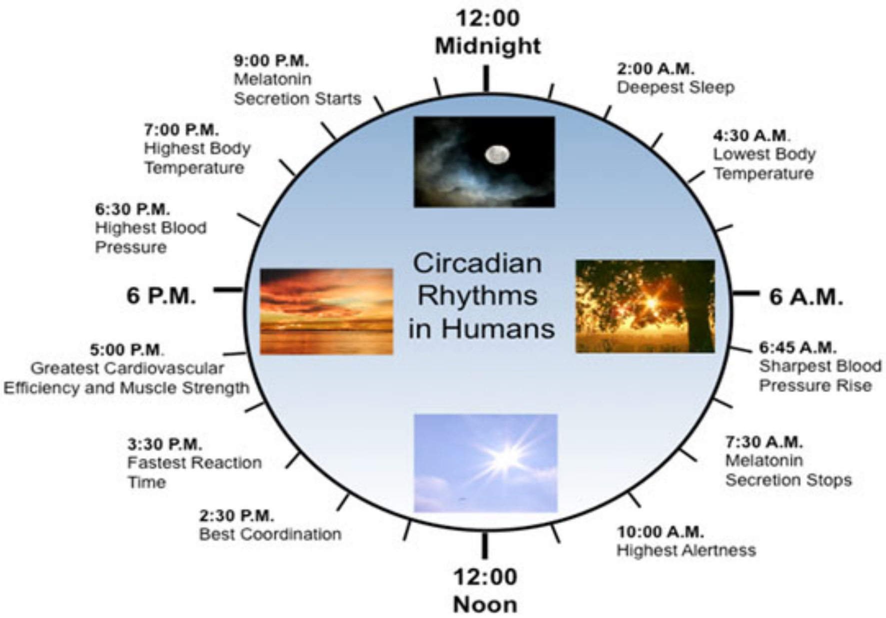
→ The notion of acoustics deals not just with oscillations, but *waves* as well....

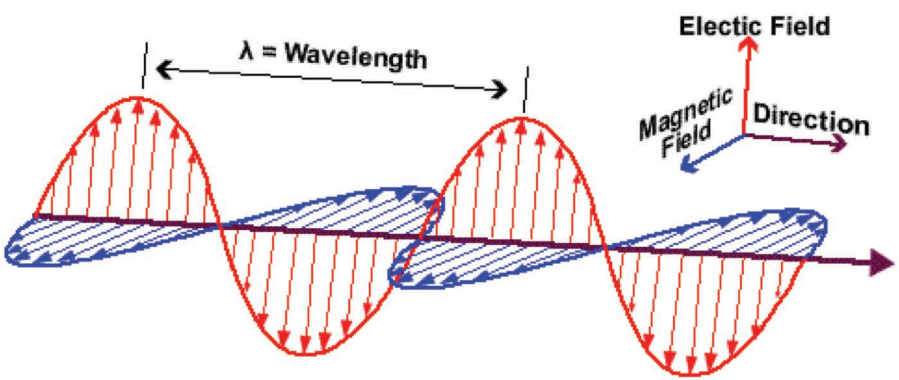
Pulkki & Karjalainen (2015)

Things that oscillate....

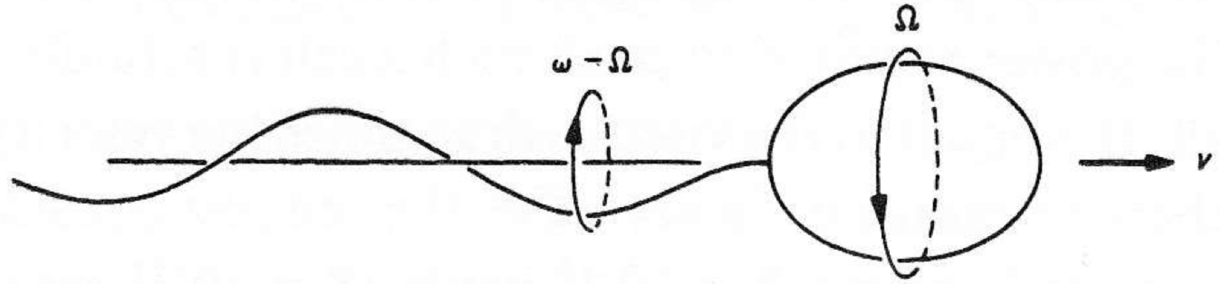








Things that oscillate....



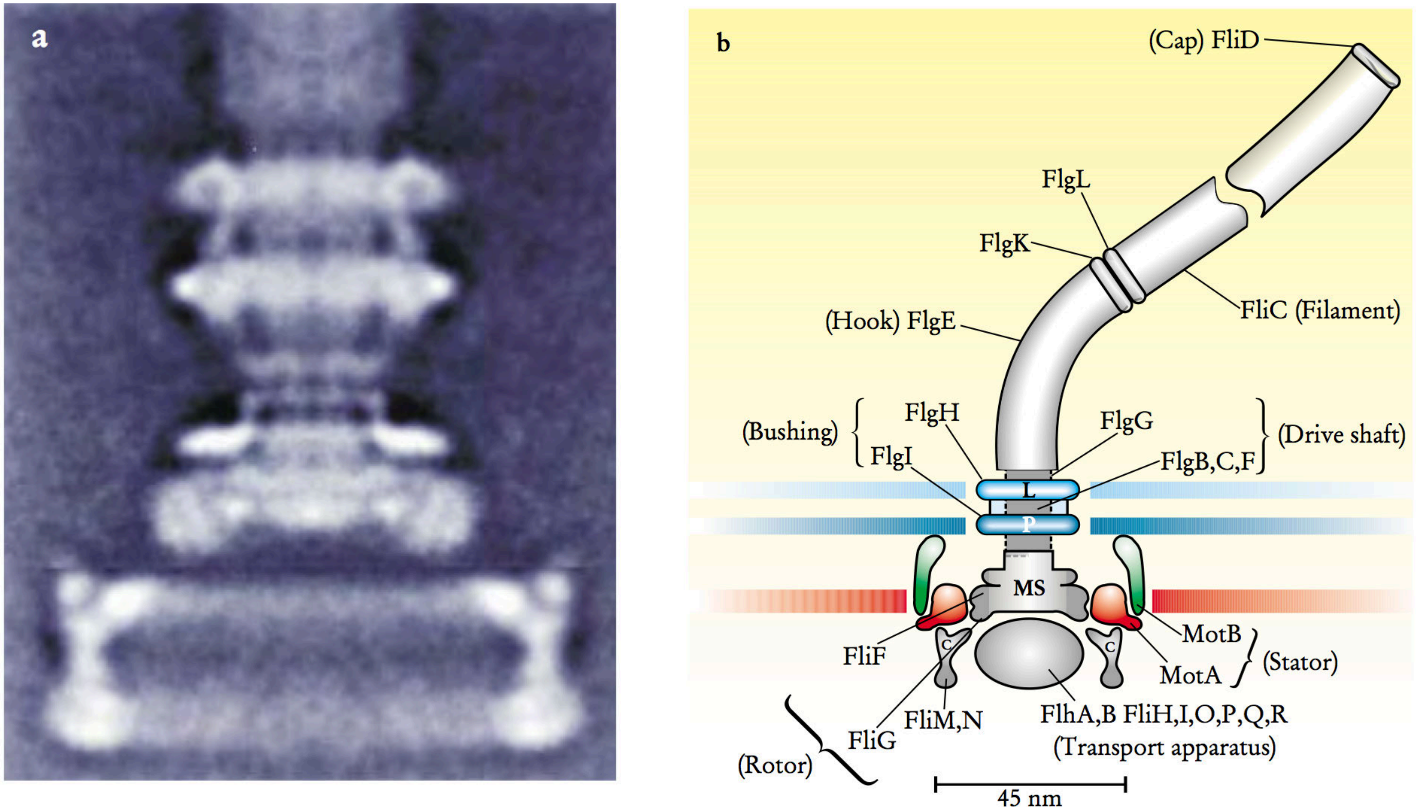


FIGURE 2. BACTERIAL MOTOR AND DRIVE TRAIN. (**a**) Rotationally averaged reconstruction of electron micrographs of purified hook-basal bodies. The rings seen in the image and labeled in the schematic diagram (**b**) are the L ring, P ring, MS ring, and C ring. (Digital print courtesy of David DeRosier, Brandeis University.)

Berg (2000)

<u>Aside</u>: Bacterial motility

Note: This is a 3-D plot!
(try crossing your eyes)

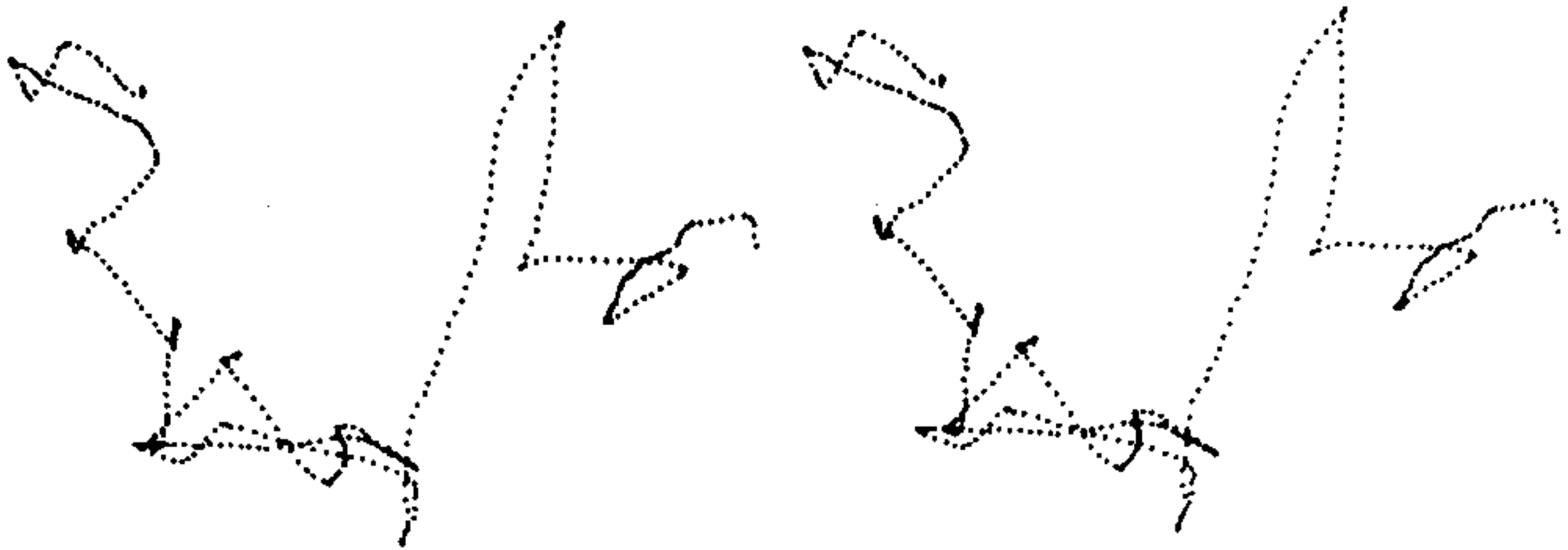
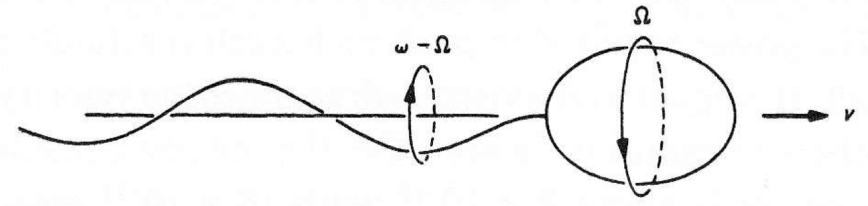


FIGURE 3. MOVEMENT. This stereo plot shows about 30 s in the life of one *Escherichia coli* K-12 bacterium swimming in an isotropic homogenous medium.[18] The track spans about 0.1 mm, left to right. The plot shows 26 runs and tumbles, the longest run (nearly vertical) lasting 3.6 s. The mean speed is about 21 μm/s. To see this plot in three dimensions, look at the left image with your left eye and the right image with your right eye, and relax your eye muscles so that the two images overlap. A stereoscope (pair of lenses) helps.

Berg (2000)

Tangent...

Question:
What differences are there for micro- vs. macro-scopic motors?



Life at low Reynolds number

E. M. Purcell
Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138

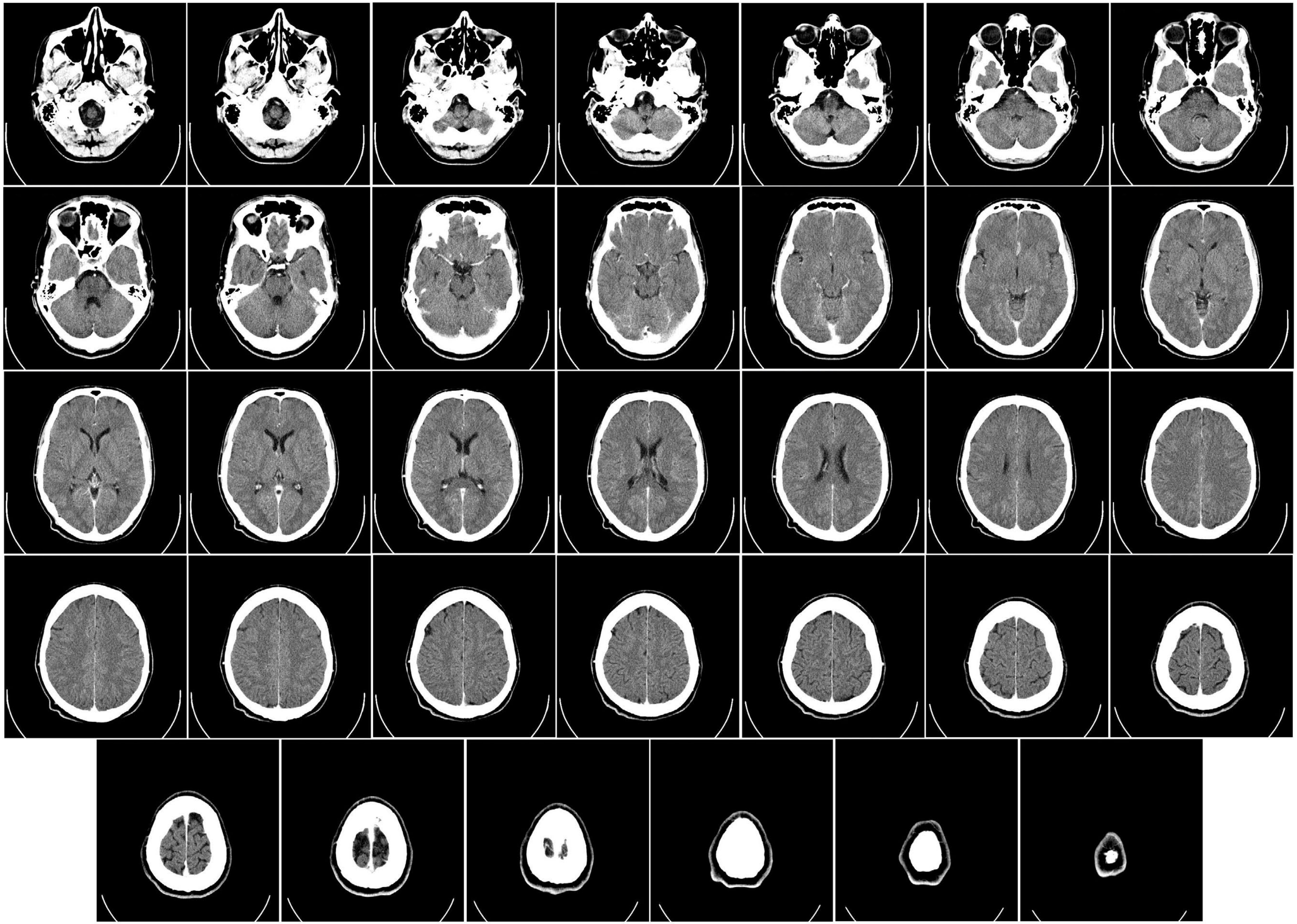
(Received 12 June 1976)

American Journal of Physics, Vol. 45, No. 1, January 1977

But I want to take you into the world of very low Reynolds number—a world which is inhabited by the overwhelming majority of the organisms in this room. This world is quite different from the one that we have developed our intuitions in.

Note: Purcell (1912-1997) won the 1952 Nobel Prize for his work on NMR

Tangent...



Wikipedia

Things that oscillate....

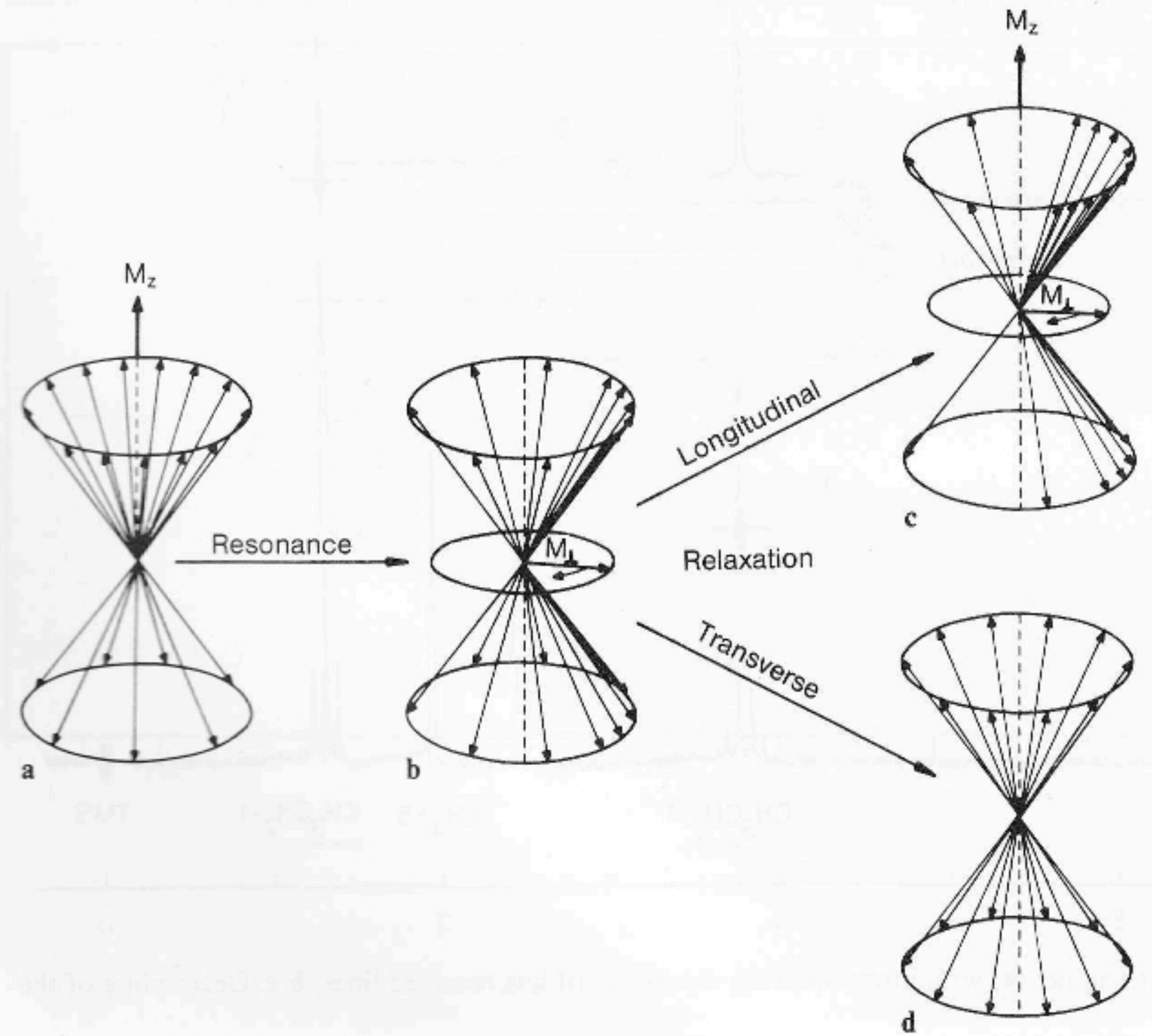
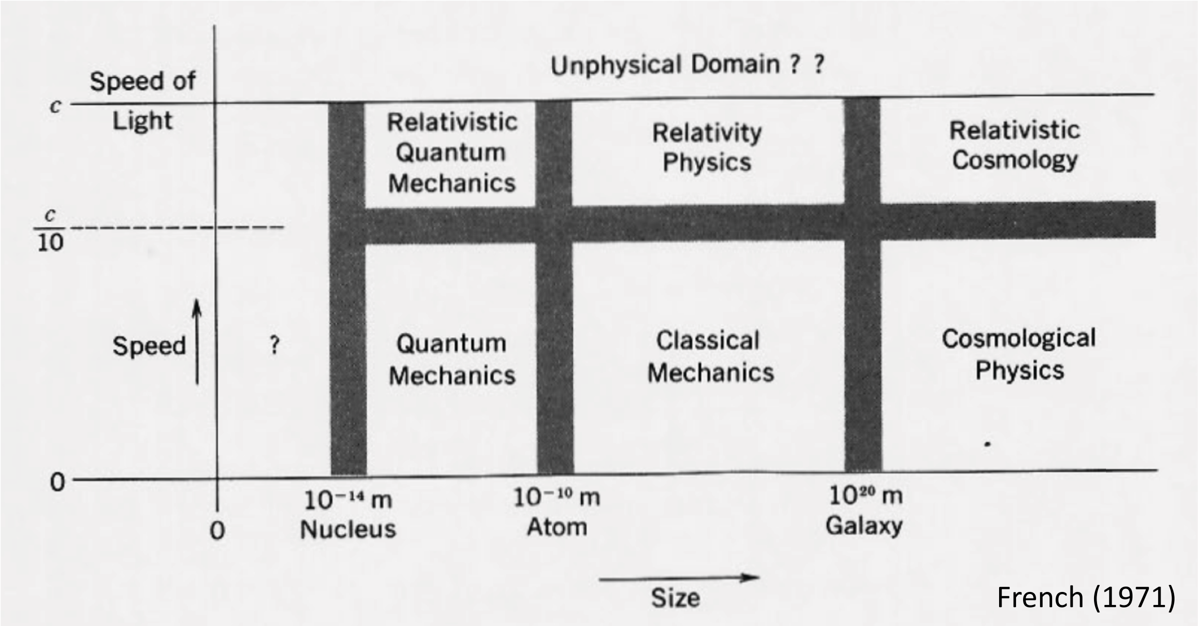


Fig. 3.147a–d. Classical representation of the NMR experiment. **a** In equilibrium the nuclear spins are distributed in the states α and β according to the Boltzmann distribution. **b** At resonance and with a sufficiently strong RF field, the populations of α and β are equalized and the spins precess in phase at the Larmor frequency ω_L. **c** Longitudinal relaxation restores the equilibrium distribution of the spins. **d** The phase coherence of the spins is lost by transverse relaxation. In reality the processes **c** and **d** proceed simultaneously

Hoppe

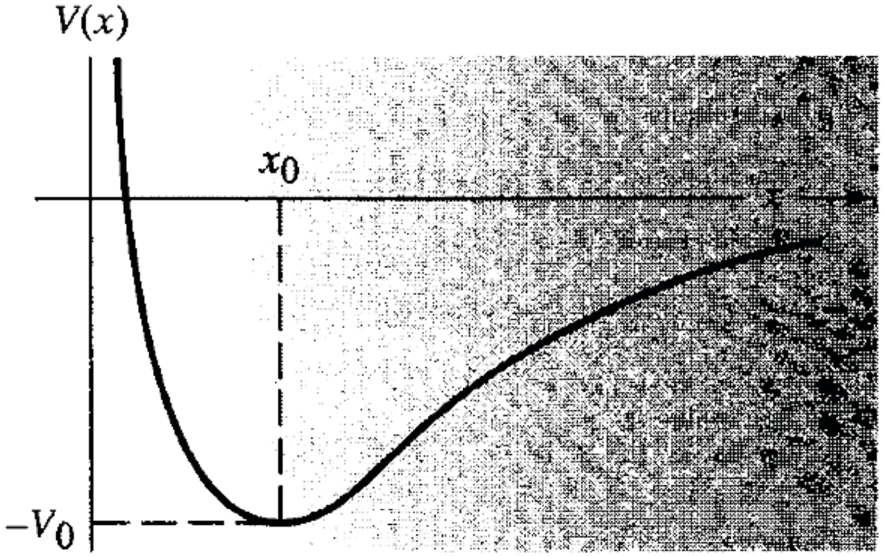
Recall (re the "Vibrating diatomic molecule")

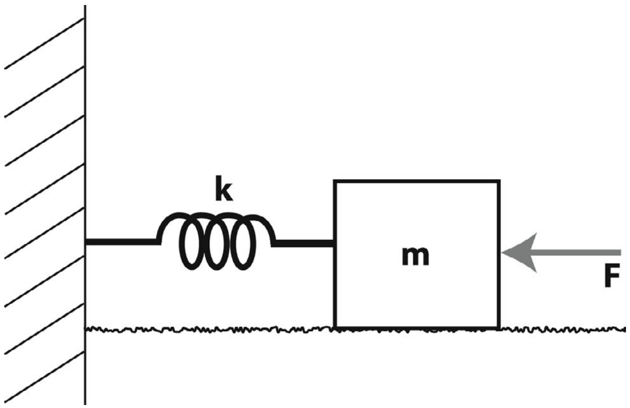


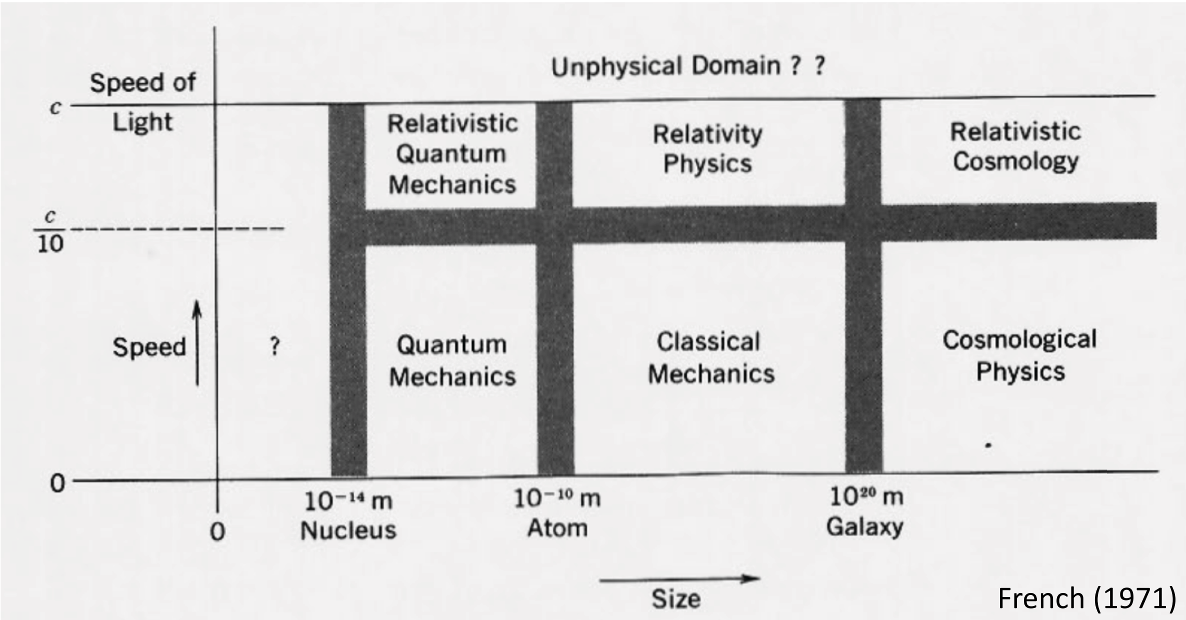
French (1971)

→ An example of an application of classical mechanics applied to systems that aren't really "classical" per se...

(see also Lord Rayleigh and elastic scattering re "Why is the sky blue?")





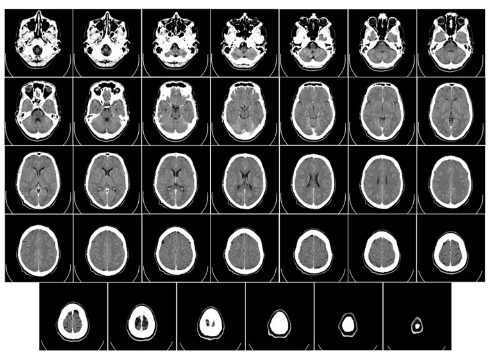


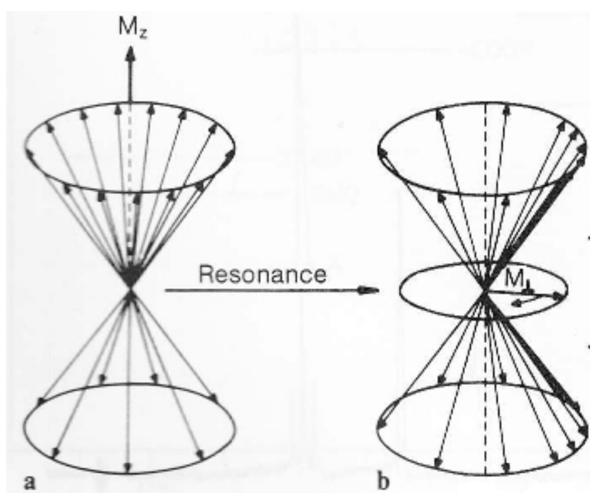
NMR and MRI is another example along these lines....

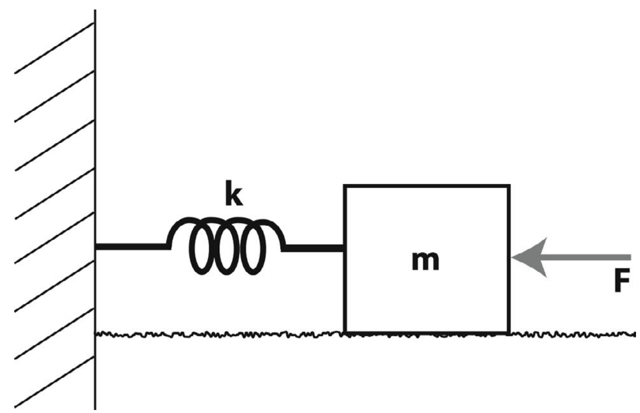
(that also happens to tie right back to the harmonic oscillator...)

Bloch equations

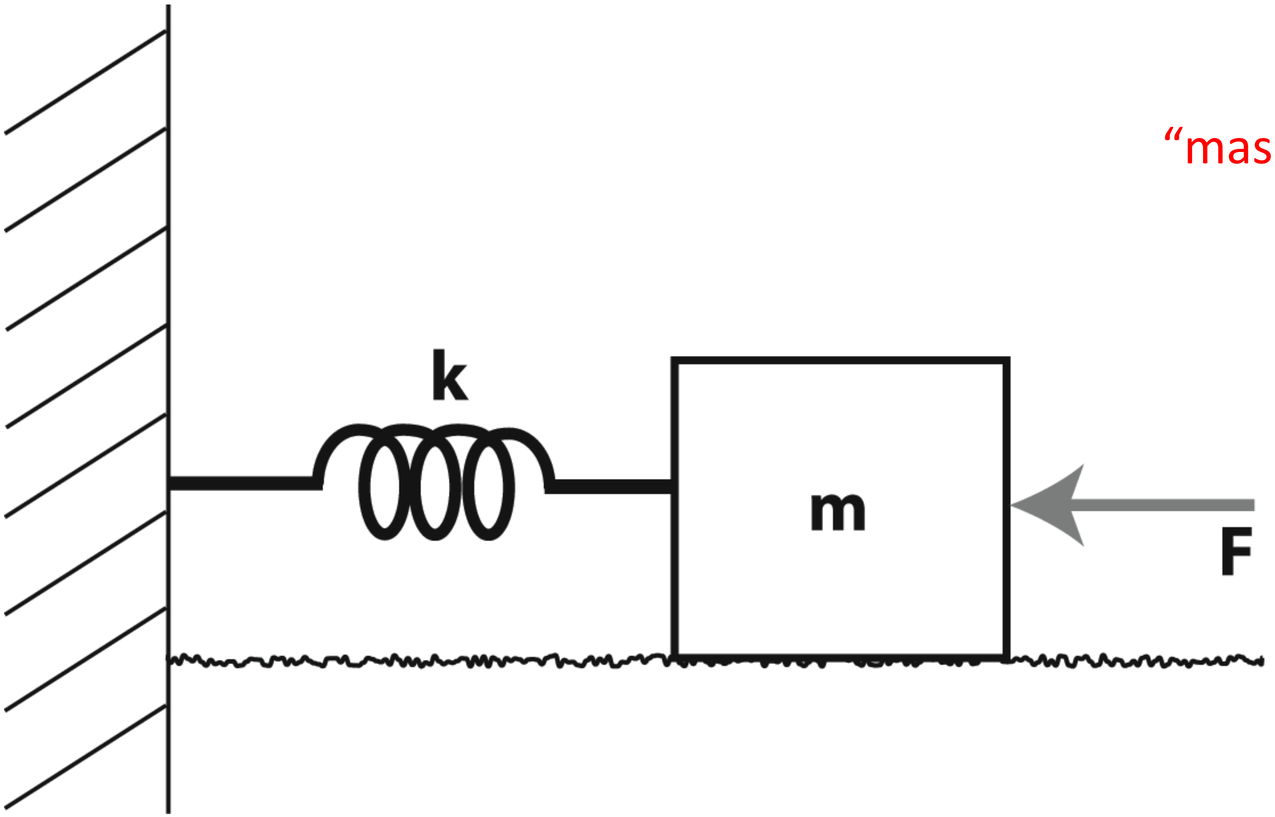
$$\frac{\mathrm{d}M_x}{\mathrm{d}t} = -\frac{M_x}{T_2} + \gamma(\vec{M}\times\vec{B})_x,$$
$$\frac{\mathrm{d}M_y}{\mathrm{d}t} = -\frac{M_y}{T_2} + \gamma(\vec{M}\times\vec{B})_y,$$
$$\frac{\mathrm{d}M_z}{\mathrm{d}t} = \frac{1}{T_1}(M_0 - M_z) + \gamma(\vec{M}\times\vec{B})_z$$







Harmonic oscillator



“mass-on-a-spring”

- One of the more fundamental/canonical problems in all areas of physics...