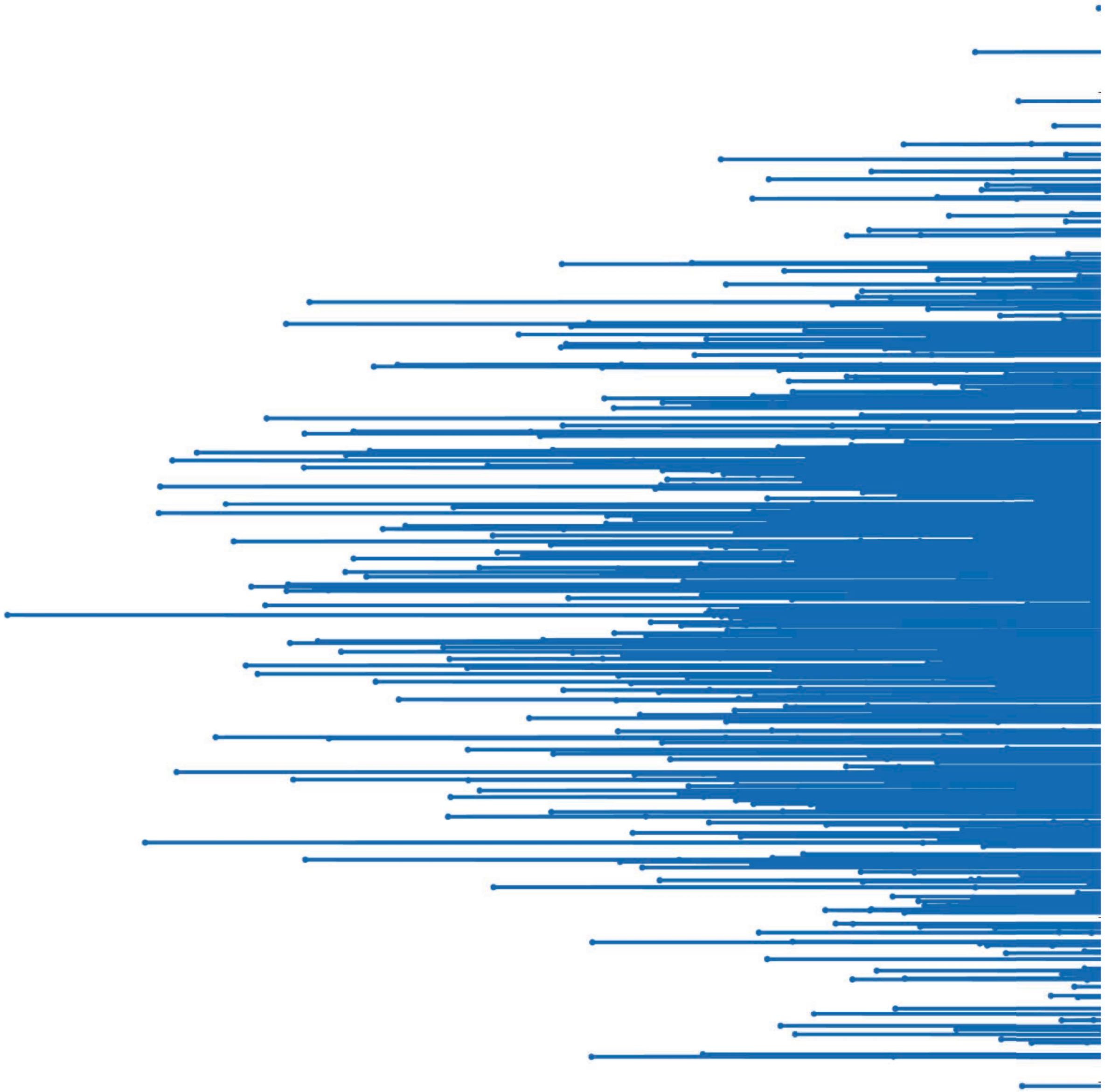
PHYS 2010 (W20)
Classical Mechanics



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Tutorial II

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Ref. (re images):
Knudsen & Hjorth (2000), Kesten & Tauck (2012)

Ex.

Can a man, standing against a wall so that his right shoulder and right leg are in contact with the wall (Fig. 27), raise his left leg and in so doing not lose equilibrium?

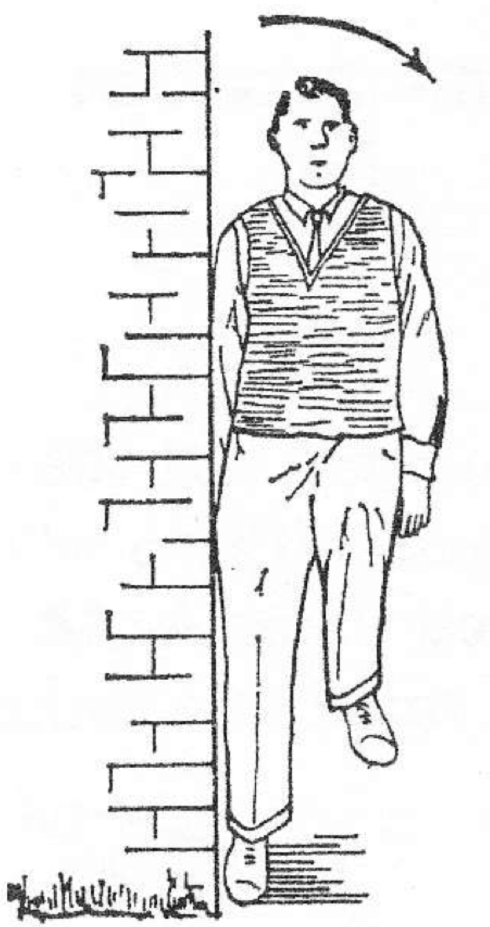


FIG. 27

Shaskol'Skaya & El'Tsin (1963)

Ex. (SOL)

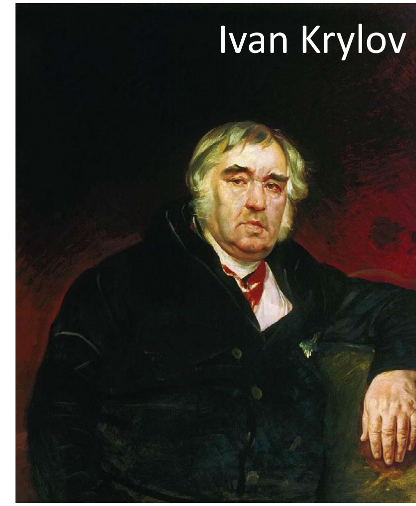
No. A man can raise his left leg and not lose balance only in the case when the vertical line passing through his centre of gravity passes also through the sole of his right foot. In the position described, this cannot be so.

→ Draw a free-body diagram to convince yourself this is the case!

Shaskol'Skaya & El'Tsin (1963)

Ex.

In what cases could the heroes of Krylov's famous fable, the swan, the pike and the crab, not have moved the cart in fact, assuming that they are all of equal strength and that there is no friction between cart and ground?*



* In this fable, a swan, a pike and a crab pull at a cart in three opposing directions. The cart does not move.

Shaskol'Skaya & El'Tsin (1963)

http://www.bartleby.com/380/poem/885.html

Ex. (SOL)

1st case: all three equal forces act in the same plane and make an angle of 120° with each other, their resultant equalling zero (Fig. 173a). 2nd case: the pike and the crab pull in directly opposite directions while the swan pulls vertically upwards (Fig. 173b); then the swan's strength would be less than the weight of the cart (though the last condition is not given in the fable).

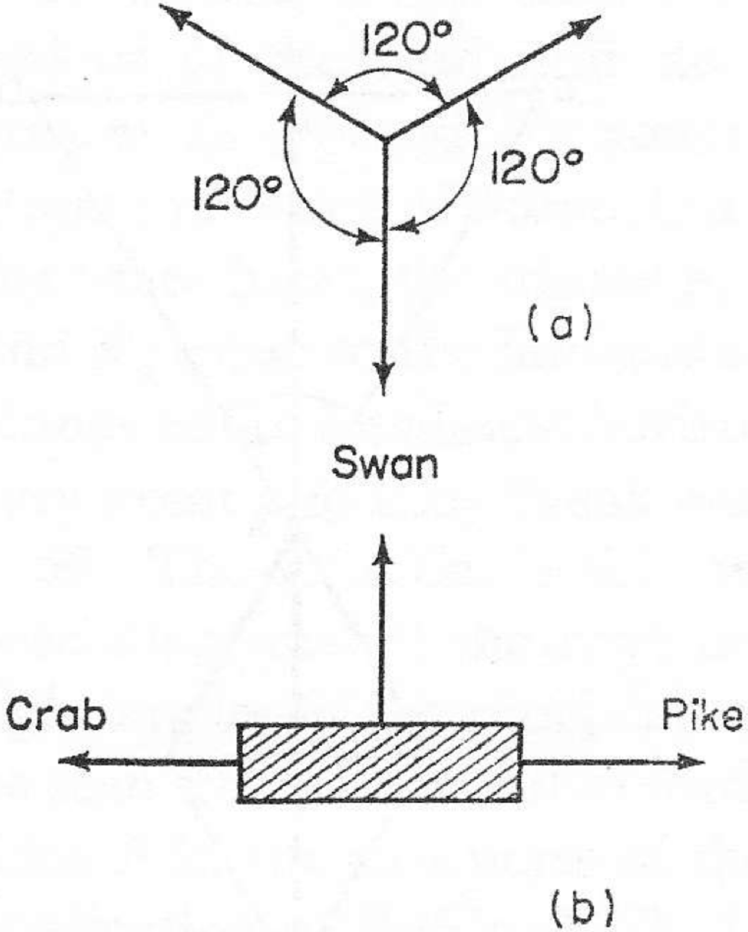


FIG. 173

Shaskol'Skaya & El'Tsin (1963)

Ex.

A lamp hangs from a bracket whose three arms each have one end fixed in the wall, the other ends meeting at a point. The two upper arms form an isosceles triangle with an angle of 60° between the arms. The plane of this triangle is at right angles to the third arm, which makes an angle of 30° with the wall. The bulb and shade weigh 1 kg. Find the stresses in the arms (Fig. 28).

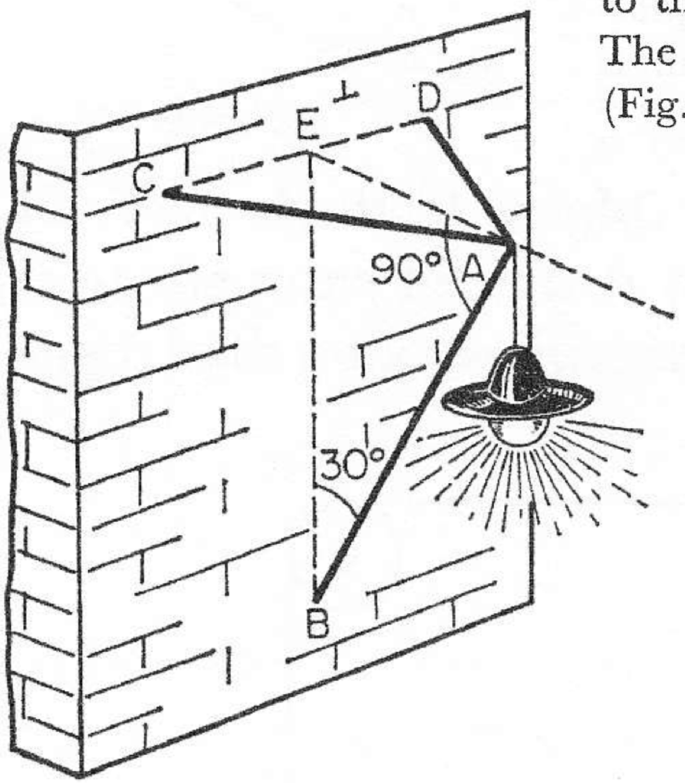


FIG. 28

Shaskol'Skaya & El'Tsin (1963)

57. Let us resolve the weight of lamp and shade in two directions at right angles to one another: along the arm *AB* and along the line of *AE*, the height of the isosceles triangle *ACD*, produced (Fig. 175a). Since arm *AB* makes an angle of 30° with the line of hang of the lamp, we can easily find that the force which acts in the line of *AE*, the height of the triangle *ADC*, and which exerts a stress on the upper arm *AC* and *AD*, equals $\frac{1}{2}$ kg, while the force which exerts a pressure on arm *AB* equals $\sqrt{1-(\frac{1}{2})^2} = \frac{\sqrt{3}}{2}$ kg. Now let us resolve the $\frac{1}{2}$ kg force along arms *AC* and *AD* (Fig. 175b). In the case given, the parallelogram of forces will be a

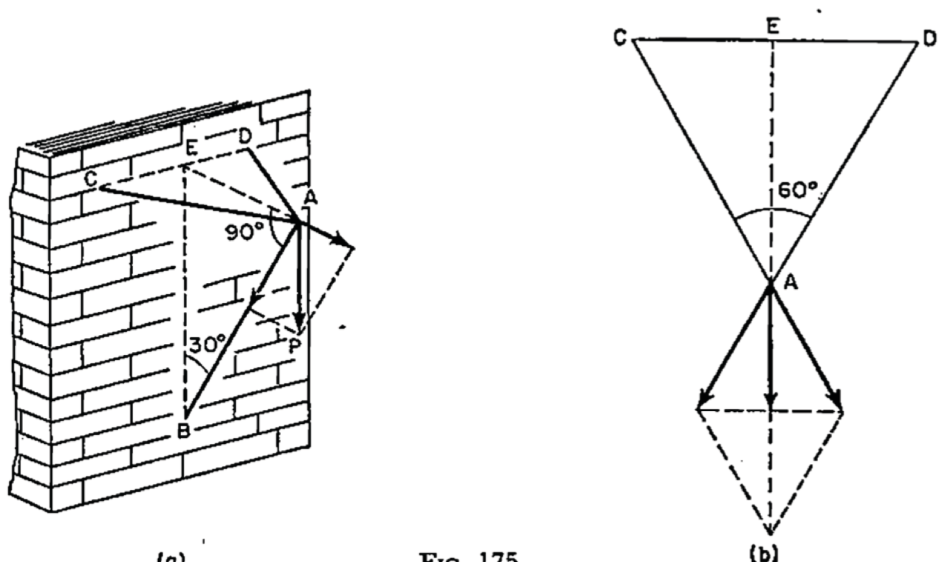


FIG. 175

rhombus, in which, of course, the diagonals are at right angles to each other, and since the angle at vertex *A* of the rhombus equals 60°, we can easily find that the unknown force *F*, which exerts a stress on each of the upper arms satisfies the equation

$$F^2 - \left(\frac{F}{2}\right)^2 = (\tfrac{1}{4})^2$$

hence

$$F = \frac{1}{2\sqrt{3}}$$

Ex.

A load is attached to two strings *AB* and *AC* of equal length and suspended from them (Fig. 29). In what case will the strings break most easily, when they hang down almost vertically, or when they are stretched almost horizontally? Neglect the weight of the strings.

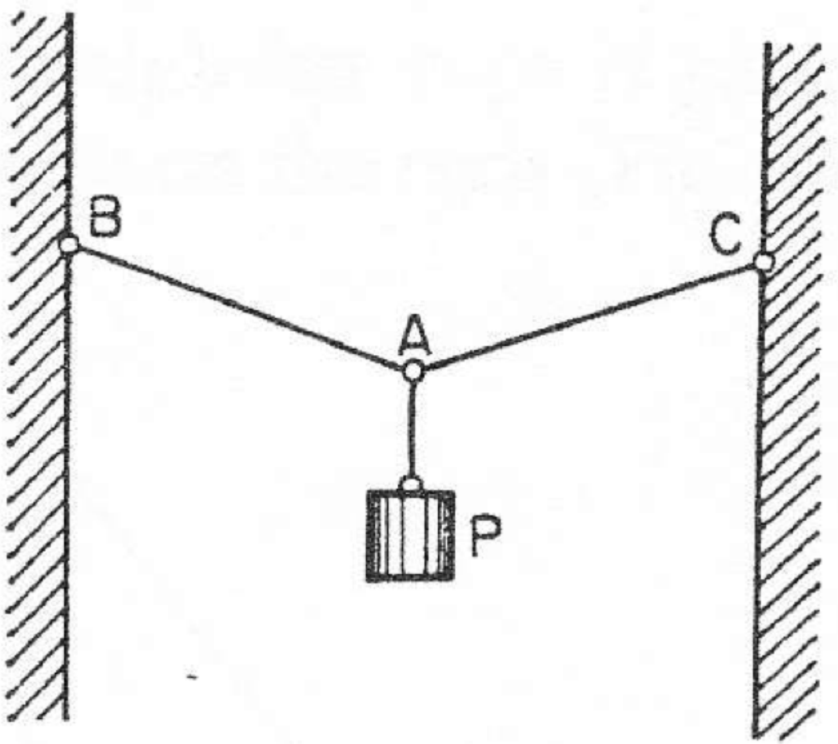


FIG. 29

Shaskol'Skaya & El'Tsin (1963)

58. The strings will break more easily, the nearer they are to a horizontal position. To explain this, let us consider the tension in each string. Tensions of F_1 and F_2 act along the strings. These two forces and the weight P of the body are in equilibrium, i.e. the resultant F of forces F_1 and F_2 is equal and opposite to force P (Fig. 176). If the angle between F_1 and F_2 is 120°, the triangle will be equilateral, and the tension in each string equals the weight of the load. If the angle is less than 120°, each tension is less than the weight of the load and decreases as F_1 and F_2 come closer to P's line of action. On the other hand, the nearer F_1 and F_2 move to the horizontal, the greater their value. When the strings adopt an almost horizontal position, the tension in them is very great and they break easily.

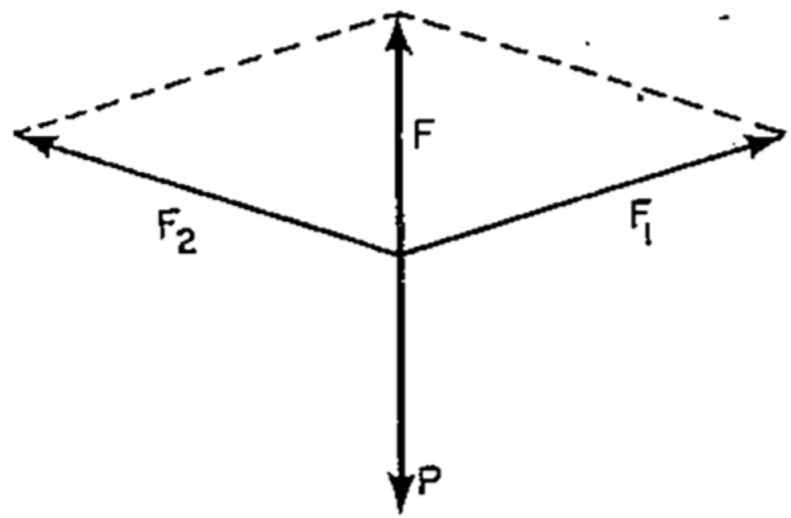


Fig. 176

Ex.

Calculate the work done in pumping oil from the cone-shaped tank in Figure 8.62 to the rim. The oil has density 800 kg/m^3 and its vertical depth is 10 m.

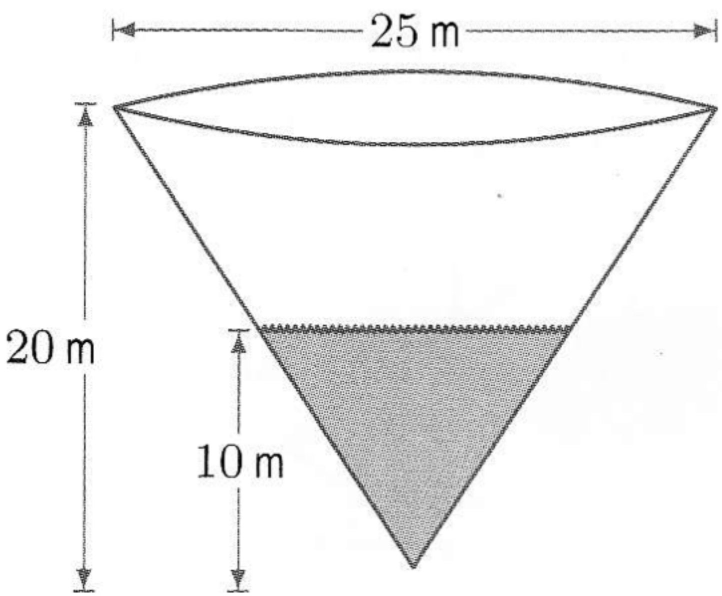


Figure 8.62: Cone-shaped tank containing oil

Hughes-Hallet et al (2005)

Ex. (SOL)

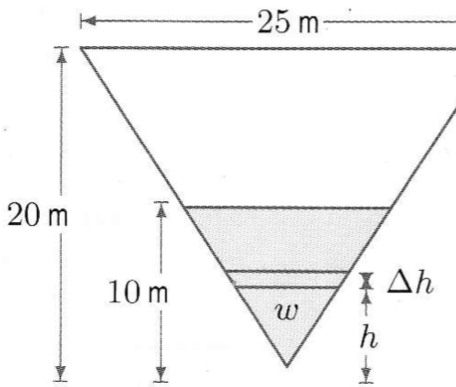


Figure 8.63: Slicing the oil horizontally to compute work

We slice the oil horizontally because each part of such a slice moves the same vertical distance. Each slice is approximately a circular disk with radius $w/2$ m, so

$$\text{Volume of slice} \approx \pi\left(\frac{w}{2}\right)^2 \Delta h = \frac{\pi}{4} w^2 \Delta h \text{ m}^3.$$

$$\text{Force of gravity on slice} = \text{Density} \cdot g \cdot \text{Volume} = 800g\frac{\pi}{4}w^2\Delta h = 200\pi g w^2 \Delta h \text{ nt.}$$

Since each part of the slice has to move a vertical distance of $(20-h)$ m, we have

$$\begin{aligned}\text{Work done on slice} \approx \text{Force} \cdot \text{Distance} &= 200\pi g w^2 \Delta h \text{ nt} \cdot (20-h) \text{ m}\\ &= 200\pi g w^2 (20-h)\Delta h \text{ joules.}\end{aligned}$$

To find w in terms of h, we use the similar triangles in Figure 8.63:

$$\frac{w}{h} = \frac{25}{20} \quad \text{so} \quad w = \frac{5}{4}h = 1.25h.$$

Thus,

$$\text{Work done on strip} \approx 200\pi g(1.25h)^2(20-h)\Delta h = 312.5\pi g h^2(20-h)\Delta h \text{ joules.}$$

Summing and taking the limit as $\Delta h \to 0$ gives an integral with upper limit $h = 10$, the depth of the oil.

$$\text{Total work} = \lim_{\Delta h \to 0} \sum 312.5\pi g h^2(20-h)\Delta h = \int_0^{10} 312.5\pi g h^2(20-h)\, dh \text{ joules.}$$

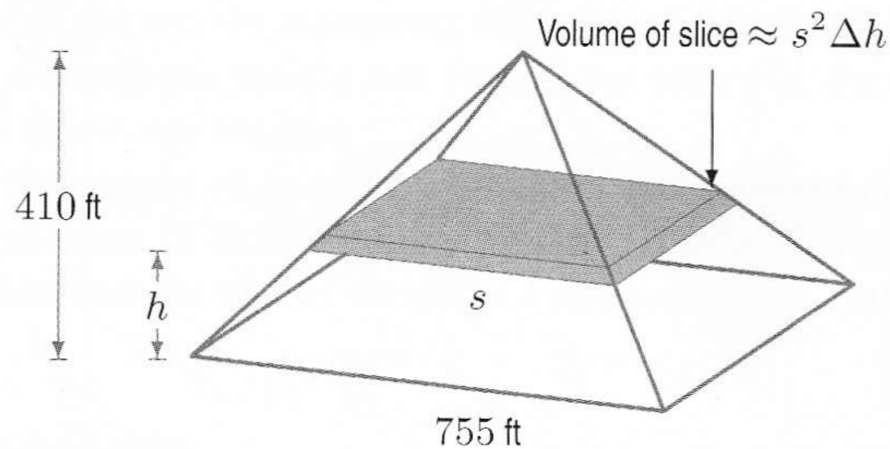
Evaluating the integral using $g = 9.8$ m/sec^2 gives

$$\text{Total work} = 312.5\pi g\left(20\frac{h^3}{3} - \frac{h^4}{4}\right)\Bigg|_0^{10} = 1{,}302{,}083\pi g \approx 4.0 \cdot 10^7 \text{ joules.}$$

Hughes-Hallet et al (2005)

Ex.

It is reported that the Great Pyramid of Egypt was built in 20 years. If the stone making up the pyramid has density 200 pounds per cubic foot, find the total amount of work done in building the pyramid. The pyramid is 410 feet high and has a square base 755 feet by 755 feet. Estimate how many workers were needed to build the pyramid.



Hughes-Hallet et al (2005)

Ex. (SOL)

$$\text{Total work } = \int_0^{410} 200\left(\frac{755}{410}\right)^2 (410-h)^2 h\, dh \approx 1.6\cdot 10^{12} \text{ foot-pounds.}$$

We have calculated the total work done in building the pyramid; now we want to estimate the total number of workers needed. Let's assume every laborer worked 10 hours a day, 300 days a year, for 20 years. Assume that a typical worker lifted ten 50 pound blocks a distance of 4 feet every hour, thus performing 2000 foot-pounds of work per hour (this is a very rough estimate). Then each laborer performed $(10)(300)(20)(2000) = 1.2\cdot 10^8$ foot-pounds of work over a twenty year period. Thus, the number of workers needed was about $(1.6\cdot 10^{12})/(1.2\cdot 10^8)$, or about 13,000.

Hughes-Hallet et al (2005)

Ex.

- How does one measure the speed of a bullet?

A 10 g bullet is fired into a 1200 g wood block hanging from a 150-cm-long string. The bullet embeds itself into the block, and the block then swings out to an angle of 40°. What was the speed of the bullet? (This is called a *ballistic pendulum.*)

Knight (2013)

Ex. (SOL)

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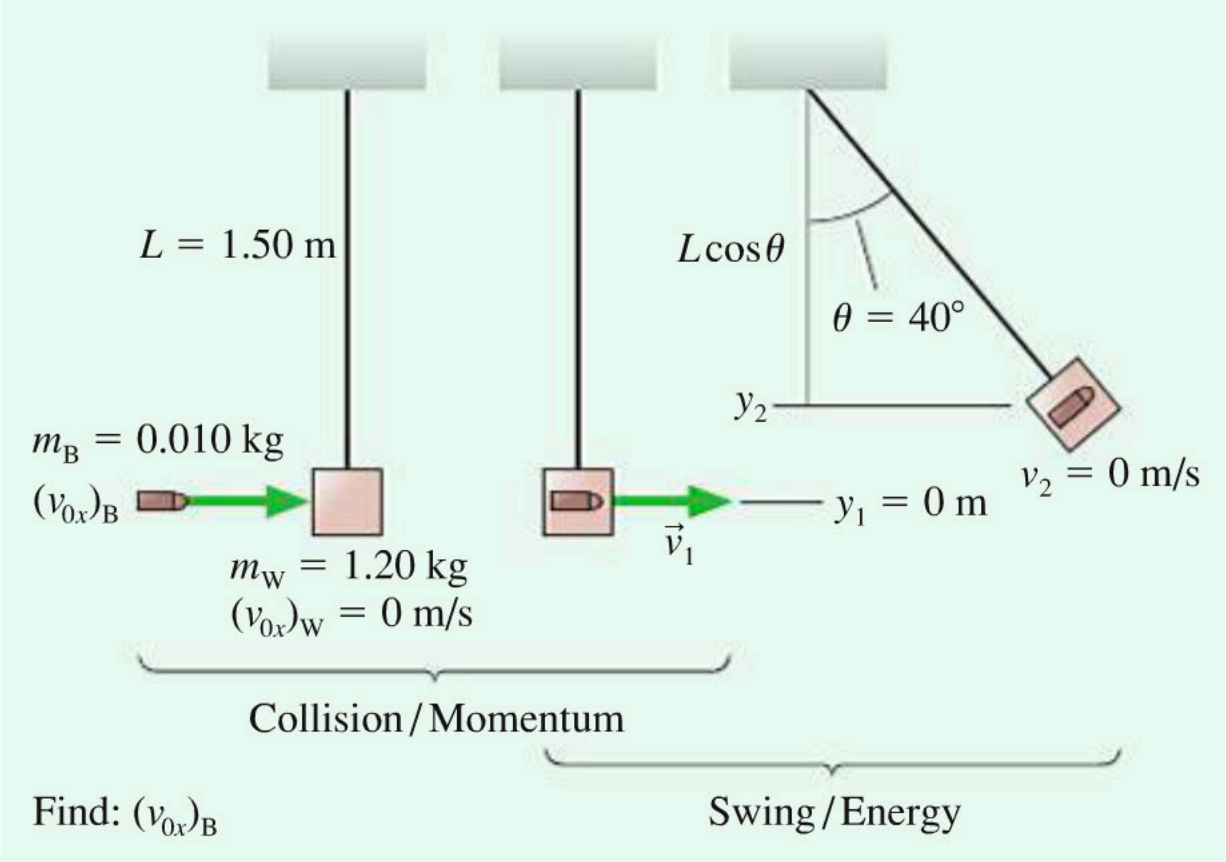

320 m/s

→ Relatively easy problem through the lens of conservation of momentum and energy

Knight (2013)