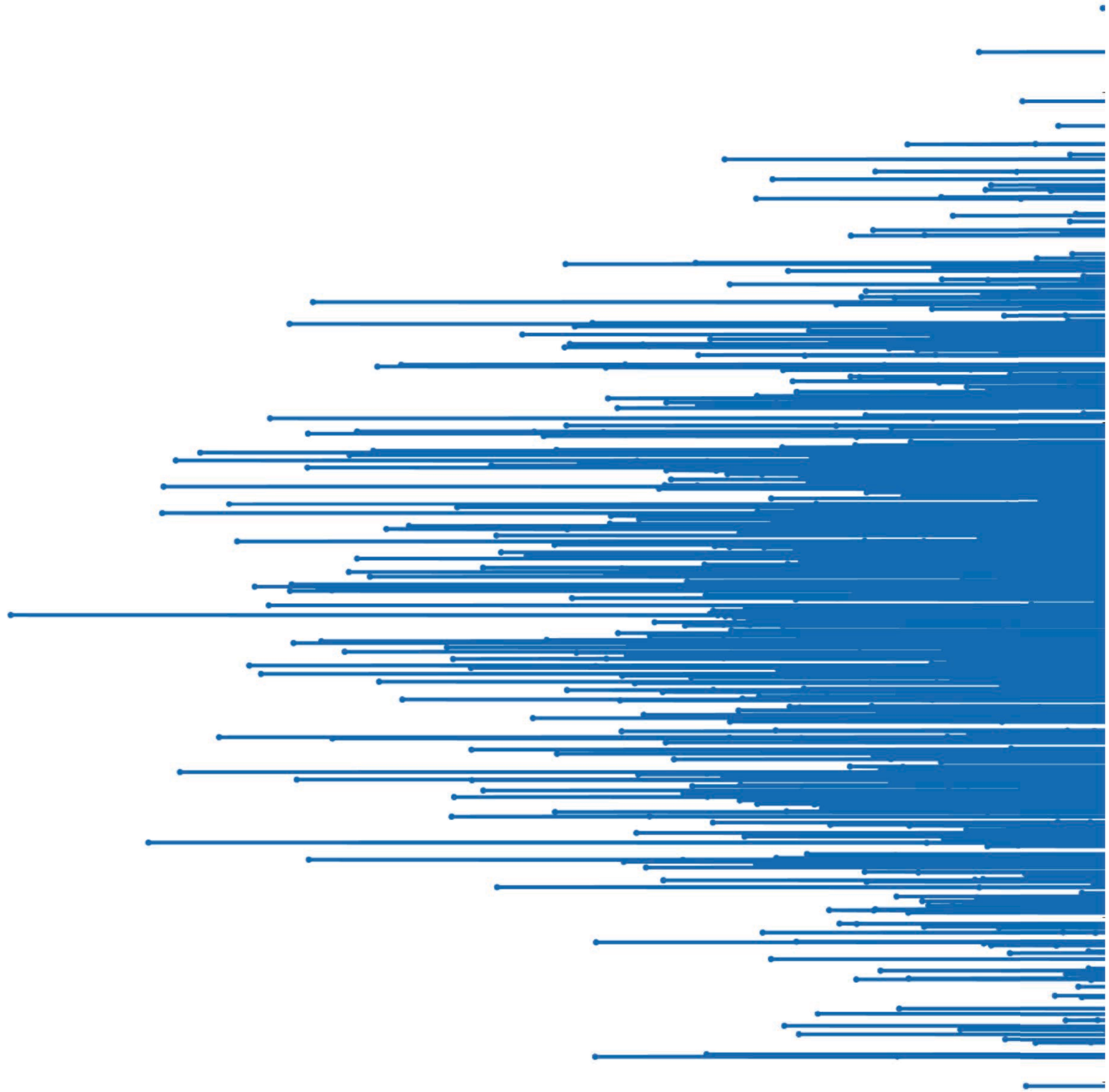
PHYS 2010 (W20)
Classical Mechanics



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Tutorial III

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Ref. (re images):
Knudsen & Hjorth (2000), Kesten & Tauck (2012)

A mass M slides without friction on the roller coaster track shown in Fig. 1.4. The curved sections of the track have radius of curvature R. The mass begins its descent from the height h. At some value of h, the mass will begin to lose contact with the track. Indicate on the diagram where the mass loses contact with the track and calculate the minimum value of h for which this happens.

(*Wisconsin*)

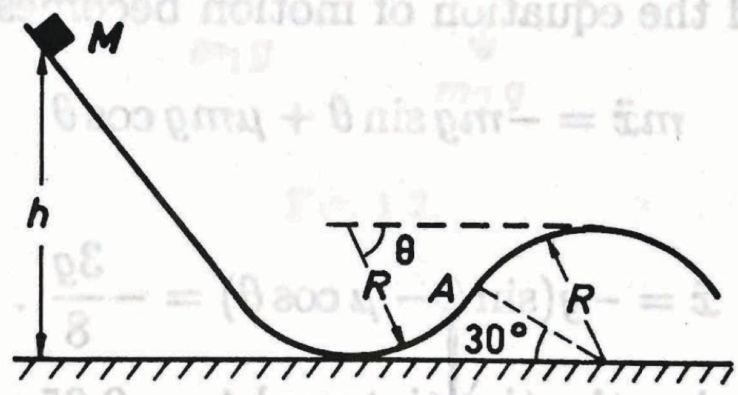


Fig. 1.4.

Show that $\vec{F} = 2xy\vec{i} + xy\vec{j}$ cannot be a gradient vector field.

The gravitational field, $\vec{F}$, of an object of mass M is given by

$$\vec{F} = -\frac{GM}{r^3}\vec{r}.$$

Show that $\vec{F}$ is a gradient field by finding a potential function for $\vec{F}$.

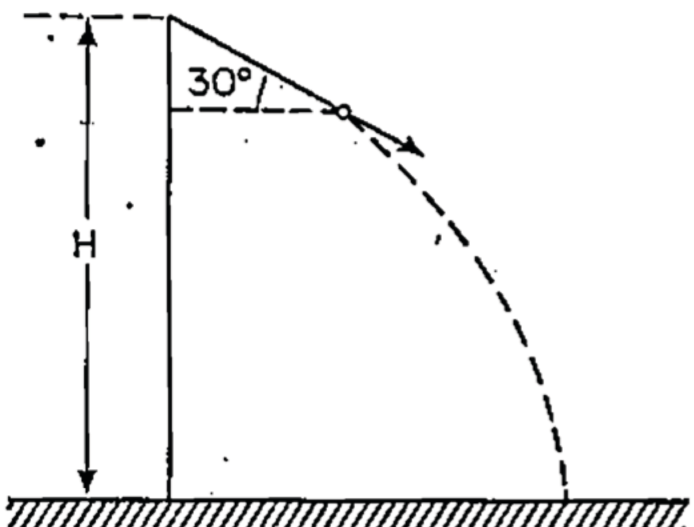


FIG. 43

81. Starting from a height H, a ball slips without friction, down a smooth plane inclined at an angle of 30° to the horizontal (Fig. 43). The length of the plane is $H/3$. The ball then falls on to a horizontal surface with an impact that may be taken as perfectly elastic. How high does the ball rise after striking the horizontal plane?

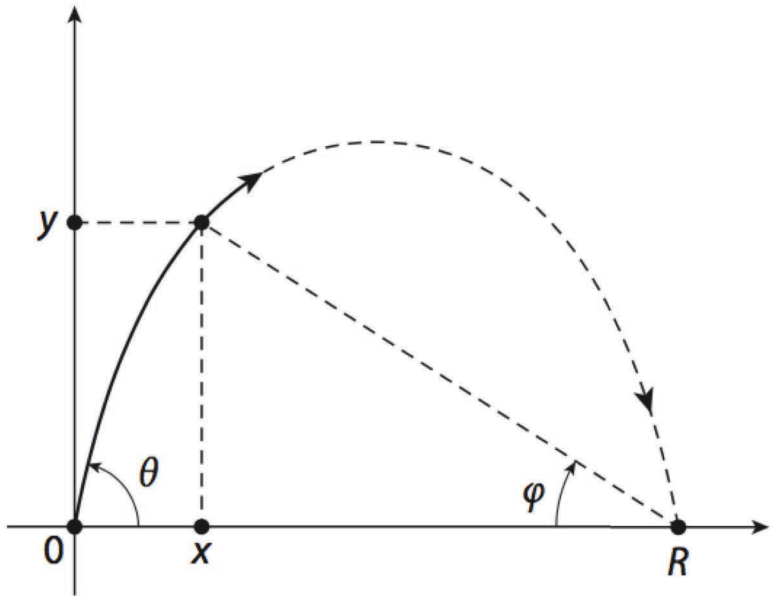
For what values of the constants a, b, and c is the force $\mathbf{F} = \mathbf{i}(ax + by^2) + \mathbf{j}cxy$ conservative?

"Willie Mays, at the crack of the bat, will take a brief look at the flight of the ball, run without looking back, be at exactly the right spot at the right time, and take the ball over his shoulder with a basket catch. How he does it no one knows, certainly not Willie Mays."

Vannevar Bush (1890–1974)

→ So how does an outfielder know where to go to catch a baseball?

May help to neglect air resistance....



Hint – May be useful to dig up:
Chapman, S. "Catching a Baseball," *Am. J. Phys.*, Oct. 1968, pp. 868–870.

Let $\vec{F}$ be the vector field given by $\vec{F}(x, y) = \dfrac{-y\vec{i} + x\vec{j}}{x^2 + y^2}$.

(a) Calculate $\dfrac{\partial F_2}{\partial x} - \dfrac{\partial F_1}{\partial y}$. Does the curl test imply that $\vec{F}$ is path-independent?

(b) Calculate $\displaystyle\int_C \vec{F} \cdot d\vec{r}$, where C is the unit circle centered at the origin and oriented counterclockwise. Is $\vec{F}$ a path-independent vector field?

(c) Explain why the answers to parts (a) and (b) do not contradict Green's Theorem.

82. A bullet of mass m hits a wooden block of mass M, which is suspended from a thread of length l (a ballistic pendulum), and is embedded in it. Find through what angle the block will swing if the bullet's velocity is v (Fig. 44).

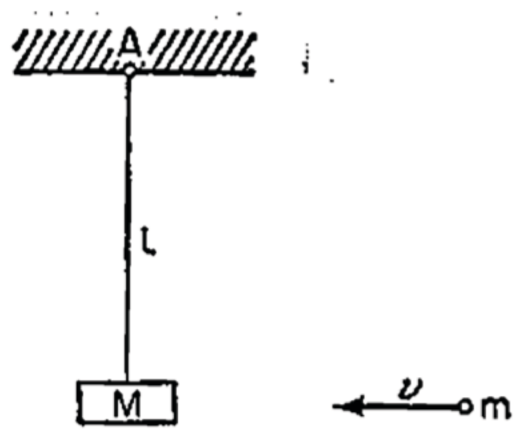


FIG. 44