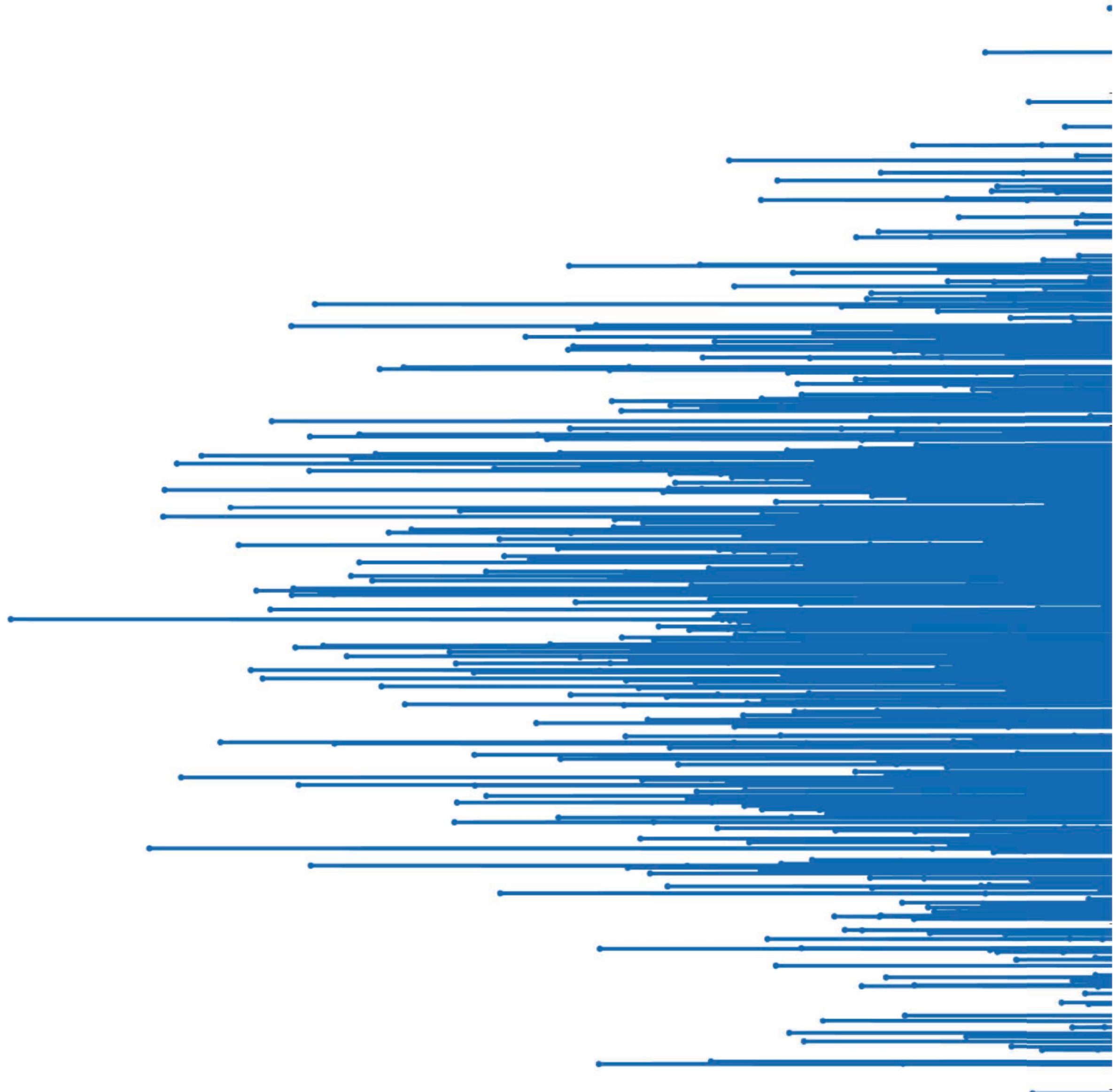
PHYS 2010 (W20)
Classical Mechanics



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Tutorial V

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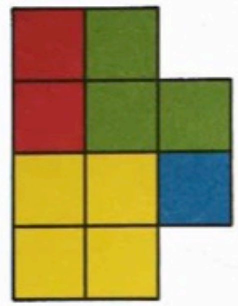
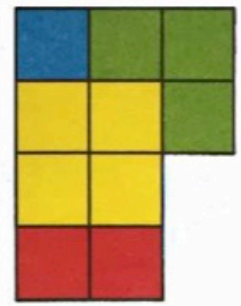
Ref. (re images):
Knudsen & Hjorth (2000), Kesten & Tauck (2012)

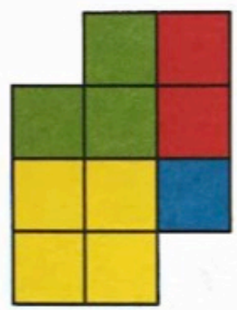
16. One-to-Four



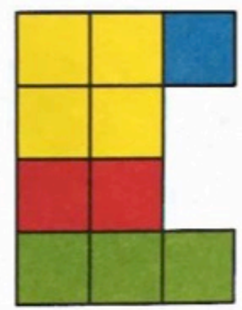
A



B



C



D

Which pattern does not belong?

A B C D

Ex.

A load of mass m lies on a perfectly smooth plane, being pulled in opposite directions by springs 1 and 2, whose coefficients of elasticity are k_1 and k_2 respectively (Fig. 60). If the load be forced out of its state of equilibrium (by being drawn aside), it will begin to oscillate with period T. Will the period of oscillation be altered if the same springs be fastened not at points A_1 and A_2, but at B_1 and B_2? Assume that the springs are subject to Hooke's law for all strains.

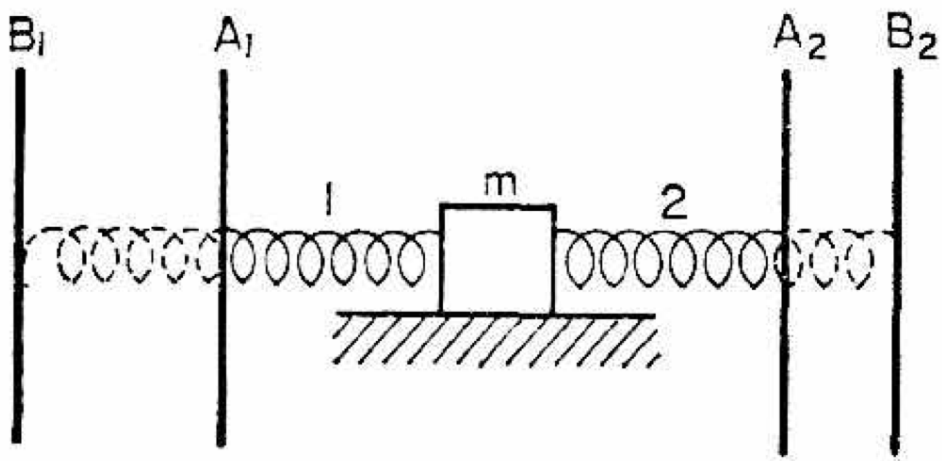


Fig. 60

Shaskol'Skaya & El'Tsin (1963)

Problem

(a) If $z = Ae^{j\theta}$, deduce that $dz = jz\, d\theta$, and explain the meaning of this relation in a vector diagram.

(b) Find the magnitudes and directions of the vectors $(2 + j\sqrt{3})$ and $(2 - j\sqrt{3})^2$.

French (1971)

Problem

Find the equivalent electrical circuit for the hanging mass–spring shown in Figure 3-17a and determine the time dependence of the charge q in the system.

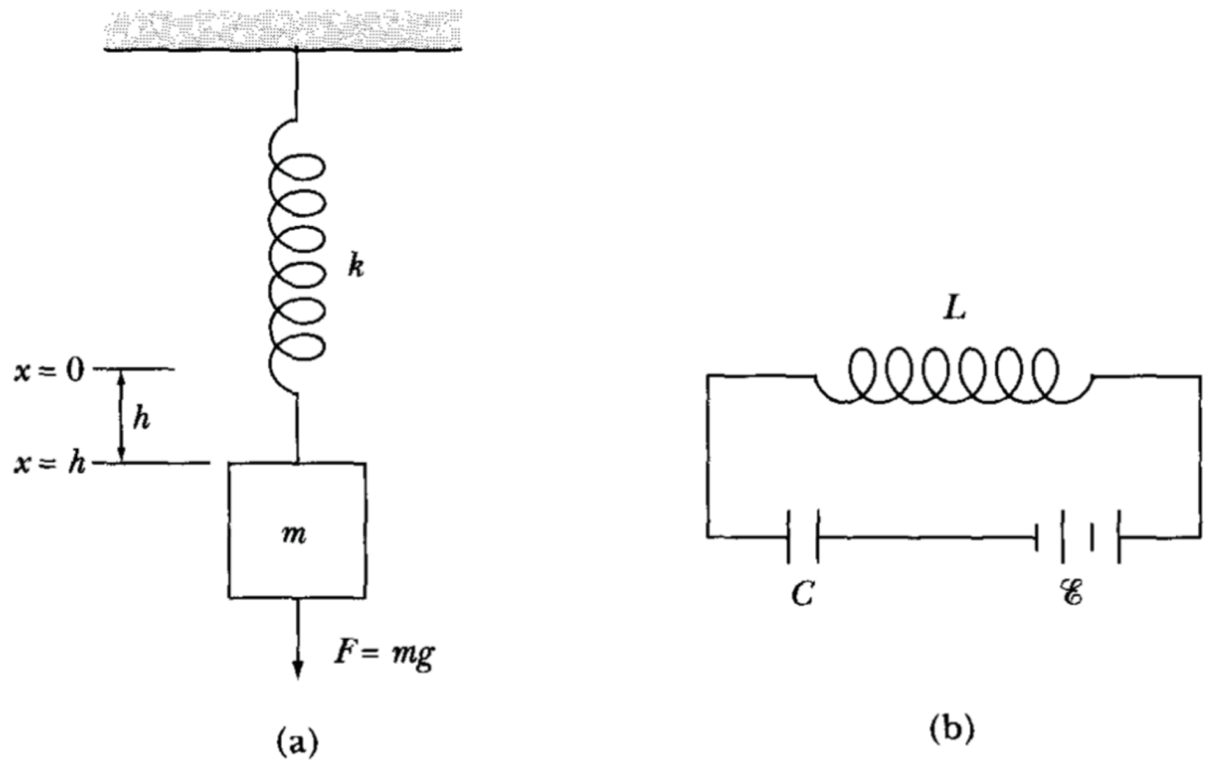
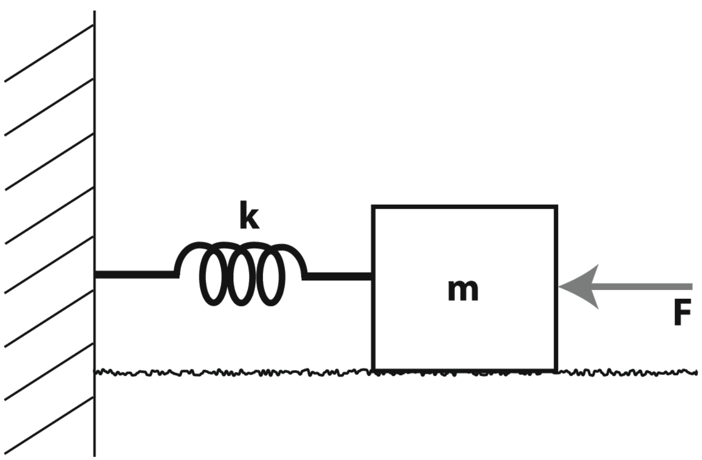


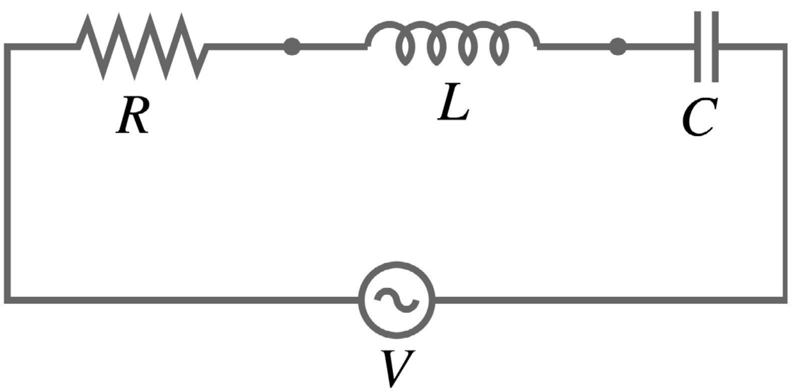
FIGURE 3-17 Example 3.4 (a) hanging mass–spring system; (b) equivalent electrical circuit.

RLC circuit = Damped, Driven Harmonic Oscillator

Mechanical		Electrical
F (force)	←→	V (potential)
v (velocity)	←→	I (current)
x (position)	←→	q (charge)
m (mass)	←→	L (inductance)
b (damping)	←→	R (resistance)
k (spring)	←→	$1/C$ (capacitance)

state variables





Problem

Consider the series RLC circuit shown in Figure 3-18 driven by an alternating emf of value $E_0 \sin \omega t$. Find the current, the voltage V_L across the inductor, and the angular frequency ω at which V_L is a maximum.

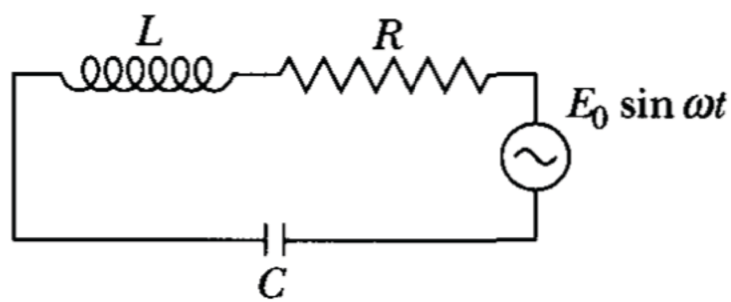


FIGURE 3-18 Example 3.5. RLC circuit with an alternating emf.

Problem

A simple harmonic oscillator consists of a 100-g mass attached to a spring whose force constant is 10^4 dyne/cm. The mass is displaced 3 cm and released from rest. Calculate **(a)** the natural frequency ν_0 and the period τ_0, **(b)** the total energy, and **(c)** the maximum speed.

Problem

Allow the motion in the preceding problem to take place in a resisting medium. After oscillating for 10 s, the maximum amplitude decreases to half the initial value. Calculate **(a)** the damping parameter β, **(b)** the frequency ν_1 (compare with the undamped frequency ν_0), and **(c)** the decrement of the motion.

<u>Problem</u>

Consider a simple harmonic oscillator. Calculate the *time* averages of the kinetic and potential energies over one cycle, and show that these quantities are equal. Why is this a reasonable result? Next calculate the *space* averages of the kinetic and potential energies. Discuss the results.

Hint: Recall that

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$