Due Date: Feb. 7, 2020 1:30 PM

Questions

1. A cell in a bath is subjected to changes in extracellular osmolarity the flux of the volume can be described by the equation $\phi = \mathcal{L}_V RT(C_{\Sigma}^o - C_{\Sigma}^i)$. A step change in the osmolarity of the bath produces a change in the normalized volume of the cell shown in the waveform v(t) depicted below in Figure 1a. In Figure 1b, there are twelve possible normalized concentration waveforms c(t).



Assume that all solutes are impermeant and that the cell acts as an ideal osmometer such that $\mathcal{V}_c = \frac{N_{\Sigma}^i}{C_{\Sigma}^o} + \mathcal{V}'_c$, where \mathcal{V}'_c represents the osmotically inactive portion of the total cell volume \mathcal{V}_c ; it can be assumed that the active portion of the total cell volume is greater than the osmotically inactive portion.

a) From Figure 1b, which of the twelve concentration waveforms can describe the change of the normalized volume waveform in Figure 1a? Explain the choices made and the assumptions involved.

b) If the temperature of the cell and bath system was decreased by 10° C, determine mathematically if the initial slope of the normalized volume, $\frac{dv(t)}{dt}$ at $t = 0^+$, and the final value of the normalized volume $v(\infty)$ are either increasing, staying the same, or decreasing; ensure to explain the reasoning behind the solution

2. A spherical cell is subjected to four different aqueous solutions of impermeant solutes and its equilibrium radius is measured as shown in Figure 1. The isotonic radius is 80 μ m.



Figure 1: A spherical cell placed in four different solutions of impermeant solutes. The circles represent cross-sectional diagrams of the cell, and the numbers below the circles indicate the measured radii.

The four solutions are: 150 mmol/L NaCl, 200 mmol/L CaCl₂, 800 mmol/L sucrose, and 150 mmol/L xylose. The latter two solutes are sugars. You may assume that the intracellular quantity of solute does not change during these measurements.

- a) Determine the compositions of Solutions 1-4
- b) Find the total quantity of intracellular solute.
- c) What fraction of the isotonic volume of the cell is due to osmotically active water?

3. Glucose is transported into the body by enterocytes, which are absorptive epithelial cells that line the small intestine. The following figure shows a schematic representation of an enterocyte.



The cell membrane has an apical part that separates the interior of the cell from the lumen of the intestine and a basal part that separates the interior of the cell from extracellular space on the serosal

side. ϕ_a represents the flux of glucose from the lumen of the intestine through the apical part of the cell membrane and into the cell. ϕ_b represents the flux of glucose from the cell through the basal part of the cell membrane and into the extracellular serosal space.

Transport through the apical part of the cell membrane, which faces the lumen of the small intestine, is coupled to the transport of Na⁺. Transport through the basolateral membrane of the cell, which faces the serosal side, is via a glucose carrier. Assume that the glucose carrier in the basolateral part of the cell can be represented by the simple, symmetric four-state carrier model. Let K represent the dissociation constant for the binding of glucose to the carrier, and let ϕ_{max} represent the maximum flux through the carriers in the basolateral part of the membrane. Let A_a and A_b represent the areas of the apical and basolateral membranes, respectively. Let V represent the volume of the cell. Assume that A_a, A_b, and V are constant with respect to time. Assume that glucose is not produced, consumed, or bound by any intracellular mechanism.

a) In the steady-state the concentration of glucose in the cell is constant. Determine a relation that ϕ_a and ϕ_b must satisfy in the steady state.

b) Determine a relation between ϕ_b and c_s^i and c_s^o dictated by the transport properties of the basolateral membrane, where c_s^o is the extracellular concentration of glucose on the serosal side of the membrane.

c) Assume that the flux ϕ_a from the lumen of the intestine into the cell is constant and that the concentration of glucose on the serosal side is zero, $c_s^o = 0$. Using the results of parts a) and b), determine an expression for $c_s^i(\infty)$ in terms of $\phi_a, \phi_{max}, K, A_a$, and A_b .

d) $c_s^i(\infty)$ is plotted versus ϕ_a/ϕ_{max} for $c_s^o = 0, K = 5 \text{ mmol/L}$ and for $A_a/A_b = 25$ in the following figure.



i) Explain the *physical significance* of the value of $c_s^i(\infty)$ when $\phi_a = 0$.

ii) Note from the figure that $c_s^i(\infty)$ increases rapidly as ϕ_a/ϕ_{max} increases from 0 to 0.04. Give a *physical interpretation* for this result.

iii) For $\phi_a/\phi_{max} > 0.04$, $c_s^i(\infty) < 0$. What is the *physical significance* of this result?

4. Three compartments are separated from each other by semi-permeable membranes, as illustrated in the following figure.



Each compartment contains well-stirred solutions of sodium, potassium, and chloride ions, with concentrations indicated in the figure (in mmol/L). The membrane between compartment 1 and 2 is permeant to sodium ions only, and its specific electrical conductivity G_{Na} is 5 mS/cm². The membrane between compartment 2 and 3 is permeant to chloride ions only, and its specific electrical conductivity G_{Cl} is 2 mS/cm². Both membranes have areas A = 10 cm². The temperature T is such that $RT/(F \log e) = 60$ mV.

a) Sketch an electrical circuit that represents the steady-state relation between current and voltage for the three compartments. Label the nodes that correspond to compartments 1, 2, and 3. Include the switch in your sketch. Label I_m , V_m and the conductances.

b) Let V_1 and V_2 represent the steady-state potentials in compartments 1 and 2 with reference to compartment 3 when the switch is open. Calculate numerical values for V_1 and V_2

c) Compute the steady-state value of the current I_m when the switch is closed.

	Concentration (mmol/L)	
	Inside	Outside
Potassium	150	15
Sodium	15	150

An electrode is inserted into the cell and connected to a current source so that the current through the cell membrane is I_m . The steady-state voltage across the cell membrane V_m is determined as a function of the current as shown in the following figure.

Assume that: (1) the cell membrane is permeable to only K^+ and Na^+ ions; (2) the Nernst equilibrium potentials are $V_n = (60/z_n) \log_{10} (c_n^o/c_n^i)$ (mV); (3) ion concentrations are constant; (4) active transport processes make no contribution to these measurements.

a) Determine the equilibrium potentials for sodium and potassium ions, V_{Na} and V_K .

b) What is the resting potential of the cell with these ionic concentrations?

c) With the current I_m adjusted so that $V_m = V_K$, what is the ratio of the sodium current to the total membrane current, I_{Na}/I_m ?



d) What is the total conductance of the cell membrane \$\mathcal{G}_m = \mathcal{G}_{Na} + \mathcal{G}_K\$?
e) Determine \$\mathcal{G}_{Na}\$ and \$\mathcal{G}_K\$.