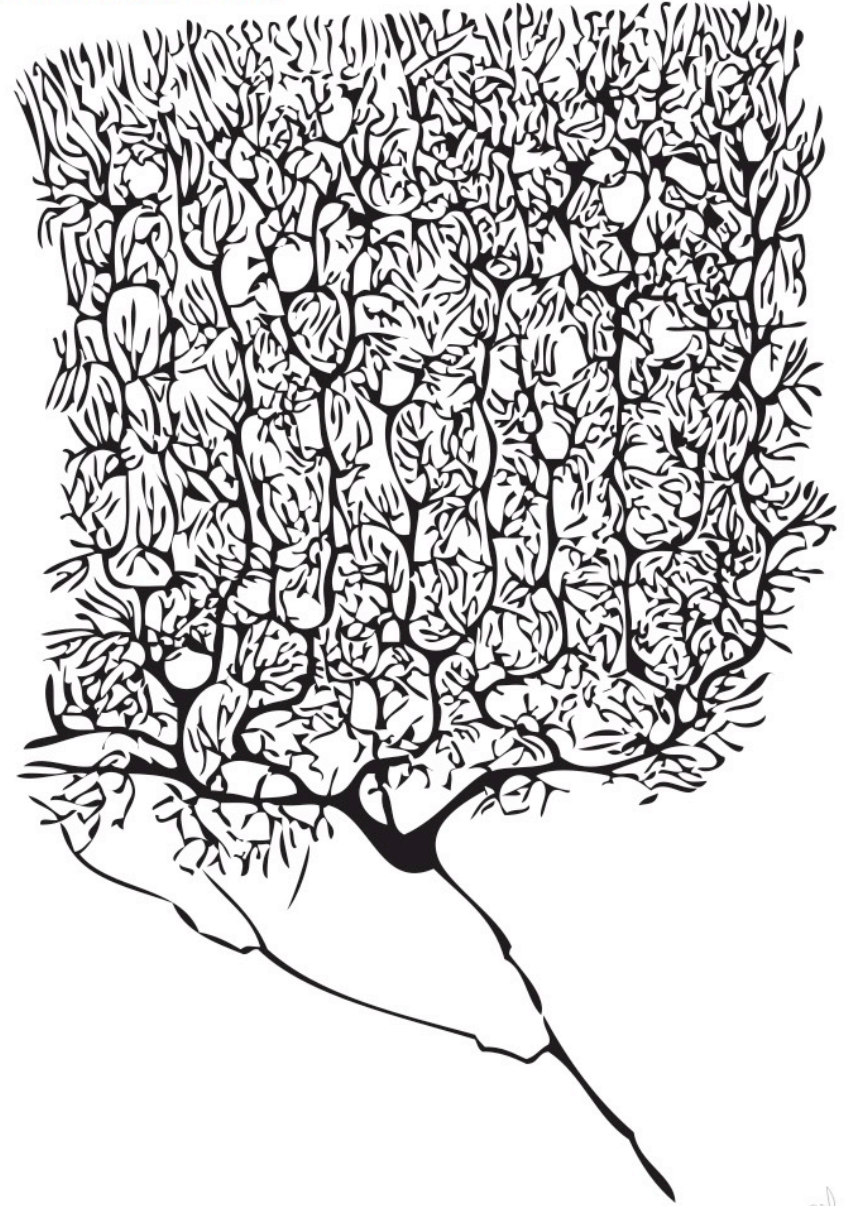


Cellular Electrodynamics

Santiago Ramón y Cajal (1852-1934)



Instructor:

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Website:

<http://www.yorku.ca/cberge/4080W2020.html>

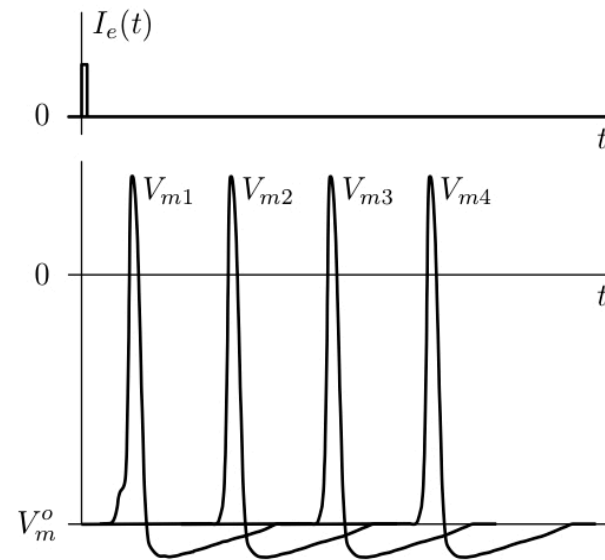
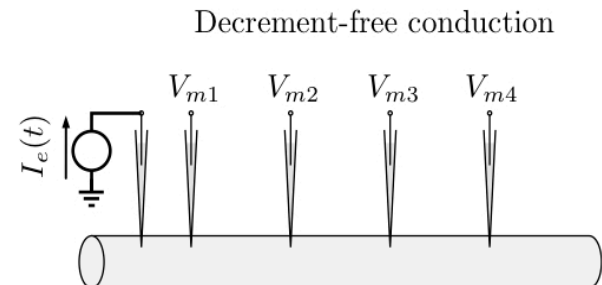
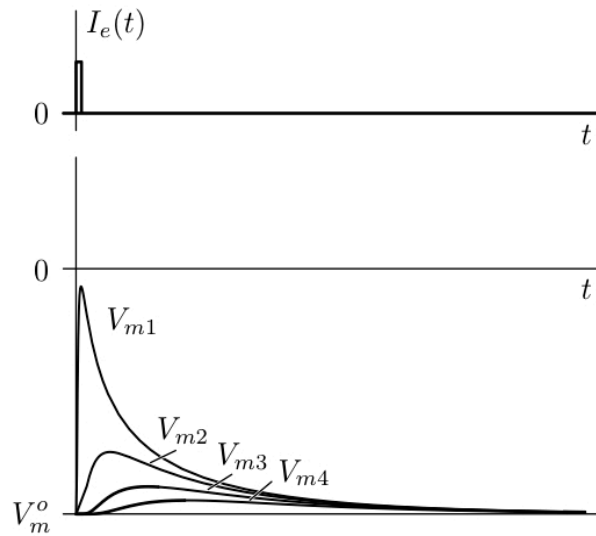
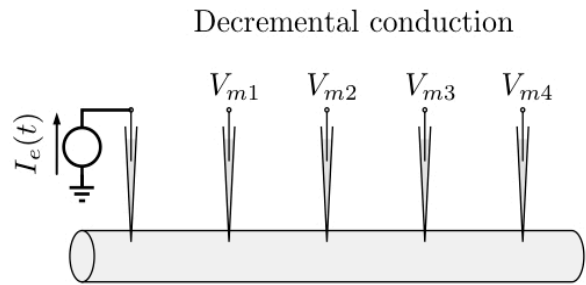
York University
Winter 2020

BPHS 4080 Lecture 12

Reference/Acknowledgement:

- TF Weiss (Cellular Biophysics)
- D Freeman

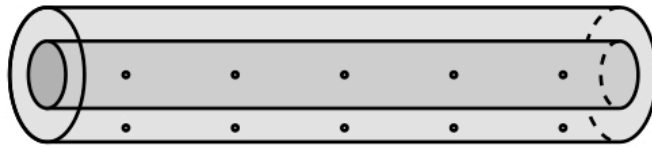
A small, stylized signature or logo located in the bottom right corner of the slide, consisting of a few overlapping, curved lines.



Model for electrically large cells → **Core-Conductor Model** (starting point)

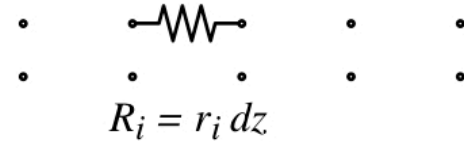
Note: Applicable regardless of whether or not cell is electrically excitable

Core Conductor Model

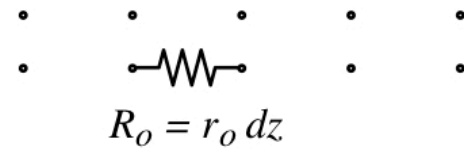
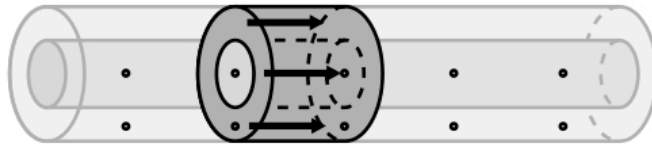


$|- dz -|$

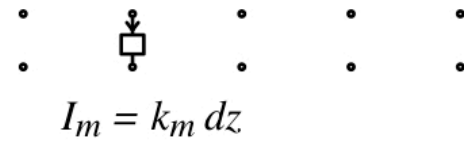
Current through inner conductor



Current through outer conductor



Current through membrane



Core Conductor Model

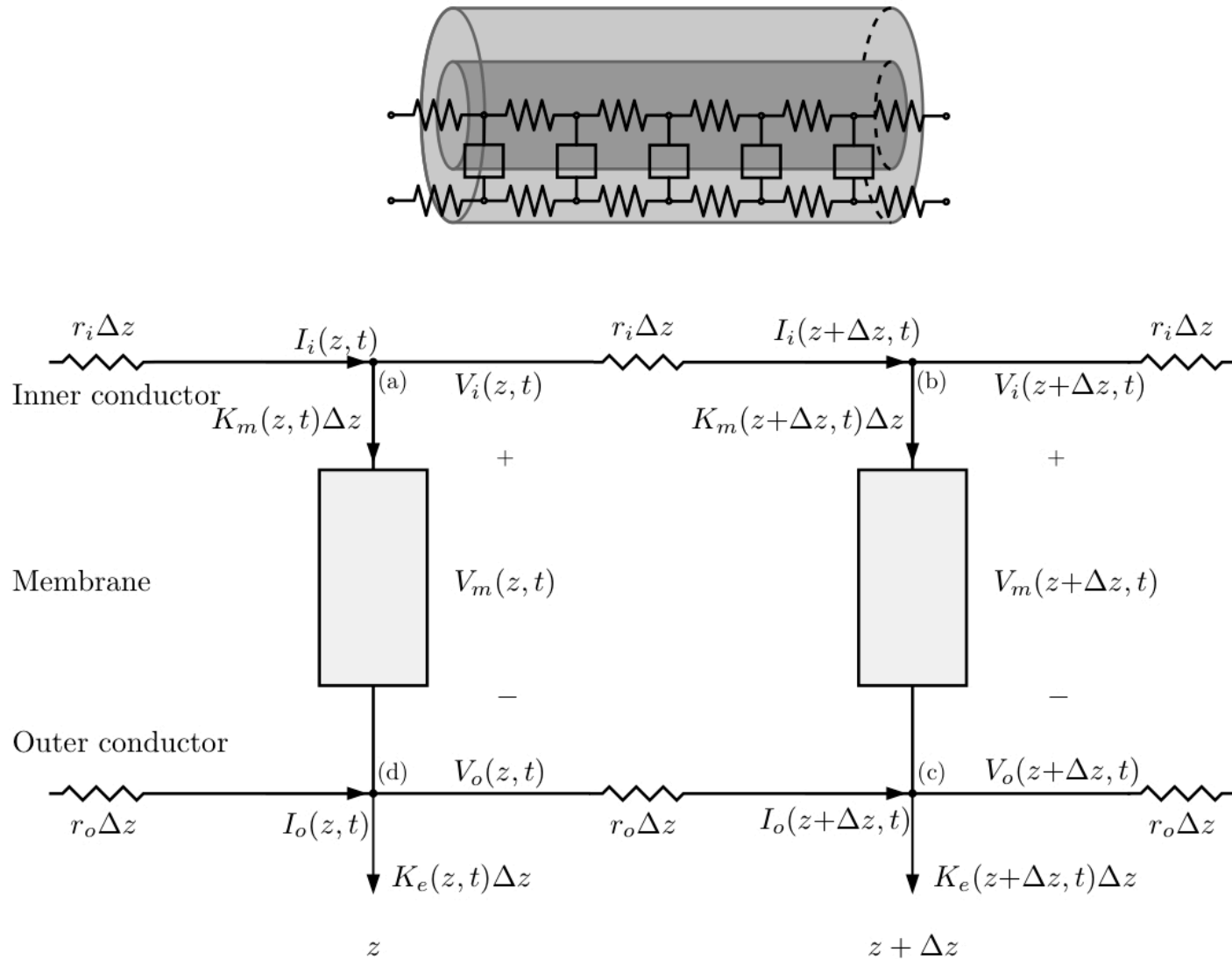


Figure 2.7

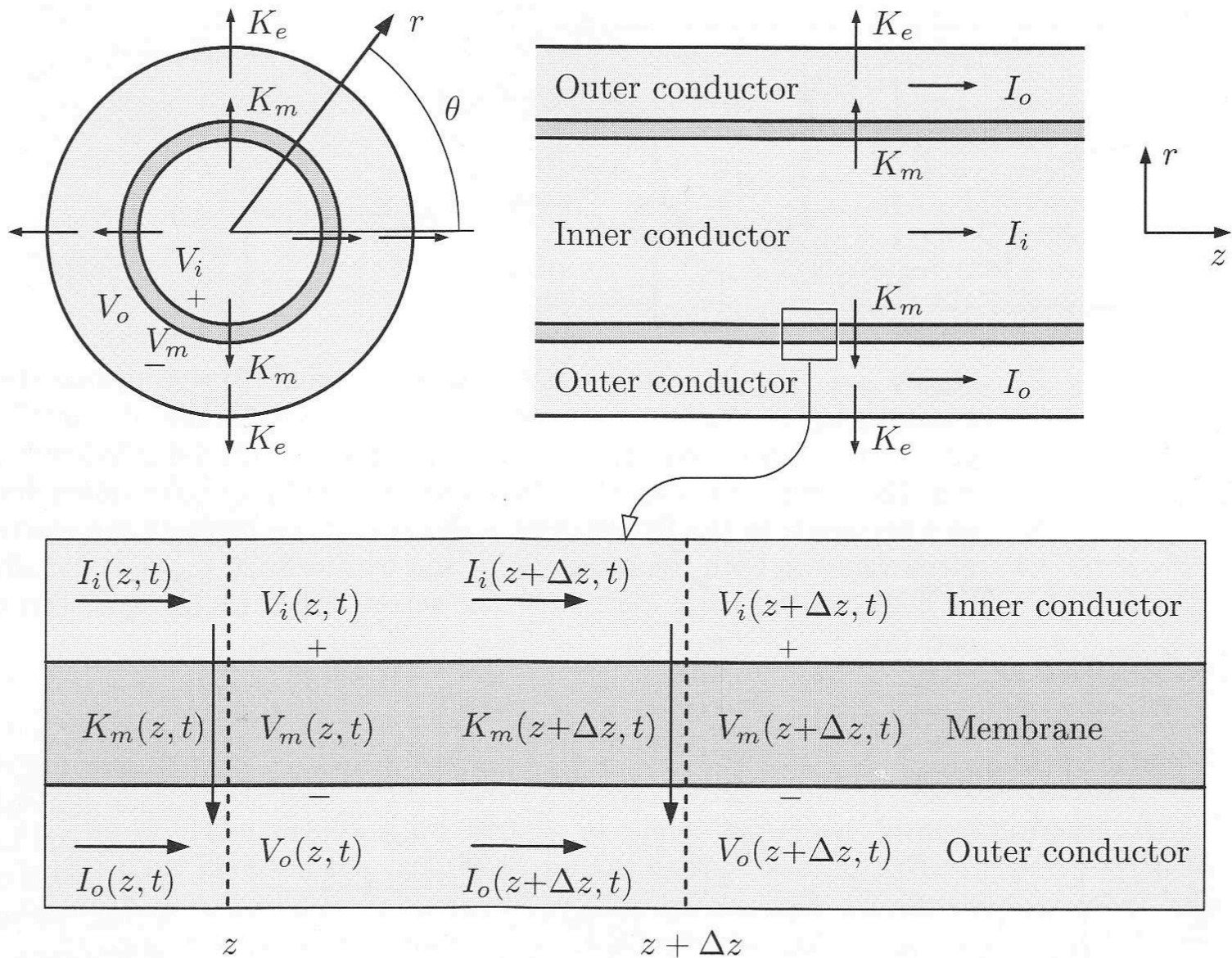


Figure 2.6 Geometry of the core conductor model of a cylindrical cell.

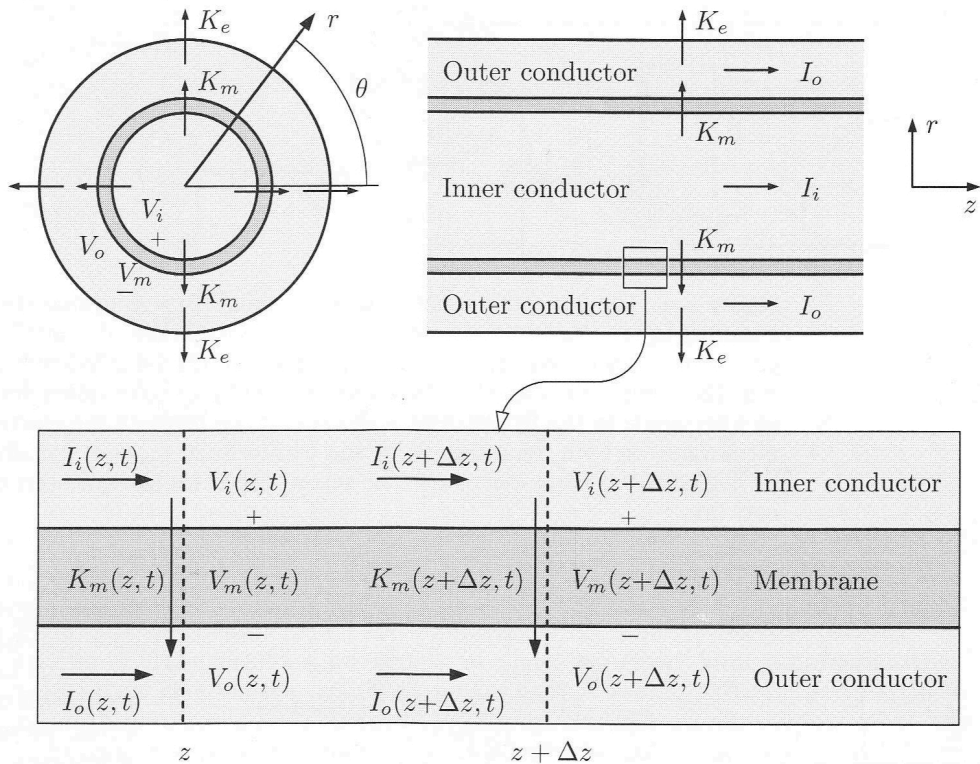


Figure 2.6 Geometry of the core conductor model of a cylindrical cell.

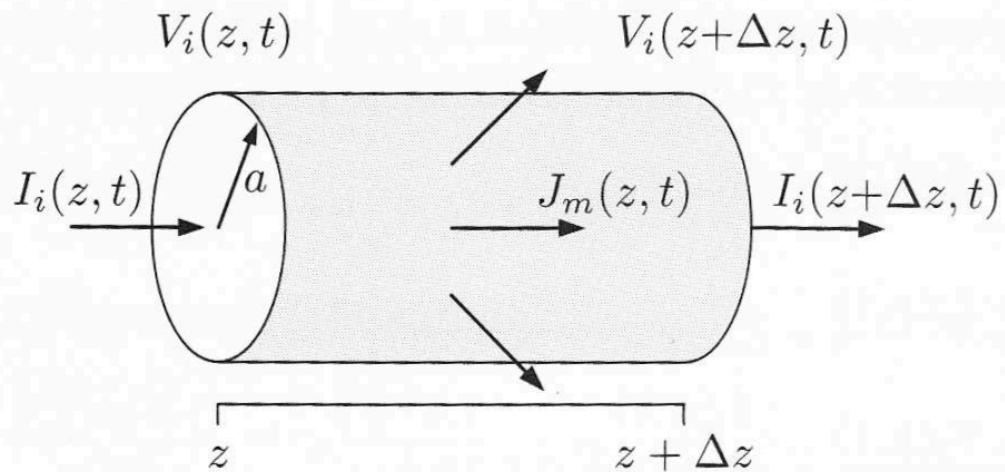


Figure 2.8

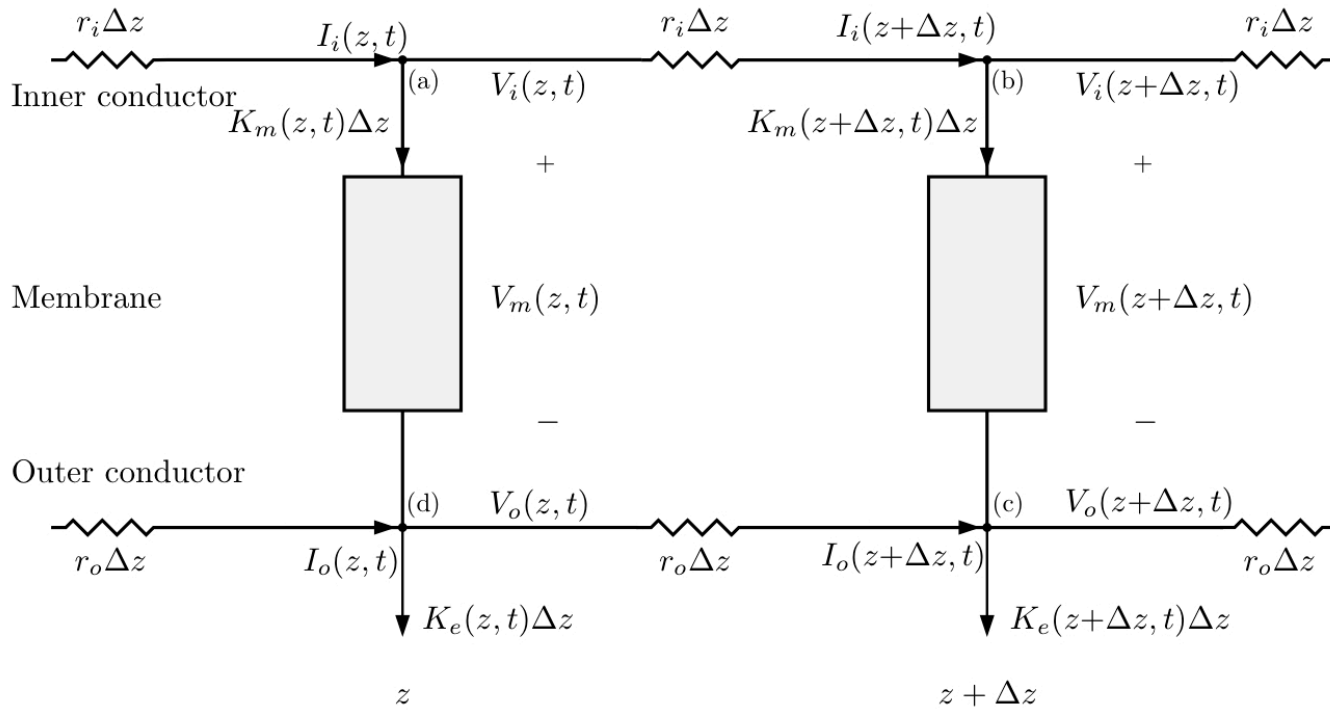
- The cell membrane is a cylindrical boundary that separates two conductors of electric current, the intracellular and extracellular solutions, which are assumed to be homogeneous and isotropic and to obey Ohm's law.
- All the electrical variables have cylindrical symmetry, i.e., all the electrical variables are independent of θ .
- A circuit theory description of currents and voltages is adequate. That is, the quasi-static terms of Maxwell's equations are sufficient, and electromagnetic radiation effects are negligible.
- Currents in the inner and outer conductors flow in the longitudinal direction only. Current flows through the membrane in the radial direction only.

■ At a given longitudinal position along the cell, the inner and outer conductors are equipotentials, so the only variation in potential in the radial direction, r , occurs across the membrane.

The variables used to describe the electrical properties of a cylindrical cell are defined as follows:

- $I_o(z, t)$ is the total longitudinal current flowing in the positive z -direction in the outer conductor (A).
- $I_i(z, t)$ is the total longitudinal current flowing in the positive z -direction in the inner conductor (A).
- $J_m(z, t)$ is the membrane current density flowing from the inner conductor to the outer conductor (A/m^2).
- $K_m(z, t)$ is the membrane current per unit length flowing from the inner conductor to the outer conductor (A/m).
- $K_e(z, t)$ is the current per unit length due to external sources applied in a cylindrically symmetric manner (A/m). Inclusion of this current allows us to represent the current applied through external electrodes to the cell surface. A similar term could be added to represent the current supplied by an internal electrode (see Problem 2.6).
- $V_m(z, t)$ is the membrane potential, which is a positive quantity when the inner conductor has a positive potential with respect to the outer conductor (V).
- $V_i(z, t)$ is the potential in the inner conductor (V).
- $V_o(z, t)$ is the potential in the outer conductor (V).
- r_o is the resistance per unit length of the outer conductor (Ω/m).
- r_i is the resistance per unit length of the inner conductor (Ω/m).
- a is the radius of the cylindrical cell.

Assumption & Variables in the Core Conductor Model

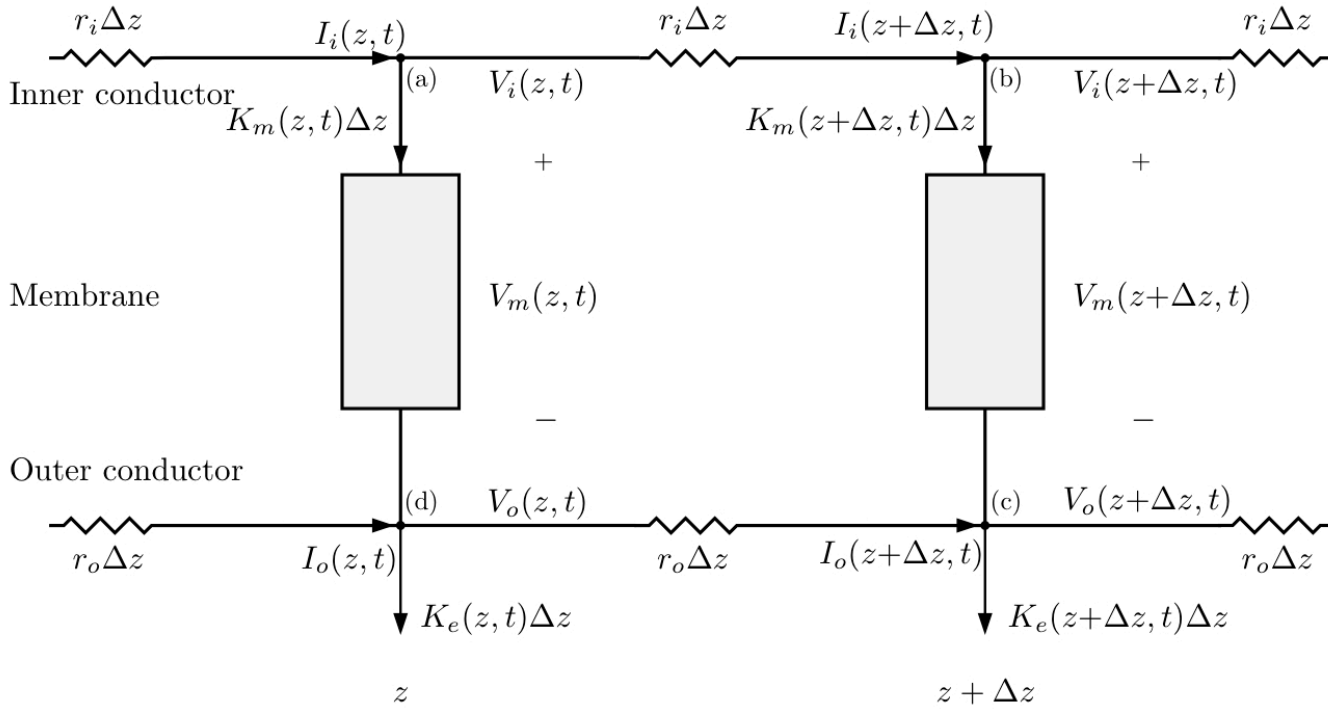


$$\text{KCL at (a) : } I_i(z, t) - I_i(z + \Delta z, t) - K_m(z, t) \Delta z = 0$$

$$\text{KCL at (d) : } I_o(z, t) - I_o(z + \Delta z, t) + K_m(z, t) \Delta z - K_e(z, t) \Delta z = 0$$

$$\text{Ohm's law at (a) - (b) : } V_i(z, t) - V_i(z + \Delta z, t) = r_i \Delta z I_i(z + \Delta z, t)$$

$$\text{Ohm's law at (c) - (d) : } V_o(z, t) - V_o(z + \Delta z, t) = r_o \Delta z I_o(z + \Delta z, t)$$



$$\frac{I_i(z + \Delta z, t) - I_i(z, t)}{\Delta z} = -K_m(z, t) \rightarrow \frac{\partial I_i(z, t)}{\partial z}$$

$$\frac{I_o(z + \Delta z, t) - I_o(z, t)}{\Delta z} = K_m(z, t) - K_e(z, t) \rightarrow \frac{\partial I_o(z, t)}{\partial z}$$

$$\frac{V_i(z + \Delta z, t) - V_i(z, t)}{\Delta z} = -r_i I_i(z + \Delta z, t) \rightarrow \frac{\partial V_i(z, t)}{\partial z}$$

$$\frac{V_o(z + \Delta z, t) - V_o(z, t)}{\Delta z} = -r_o I_o(z + \Delta z, t) \rightarrow \frac{\partial V_o(z, t)}{\partial z}$$

Core – Conductor Equations

$$\frac{\partial I_i(z, t)}{\partial z} = -K_m(z, t)$$

$$\frac{\partial I_o(z, t)}{\partial z} = K_m(z, t) - K_e(z, t)$$

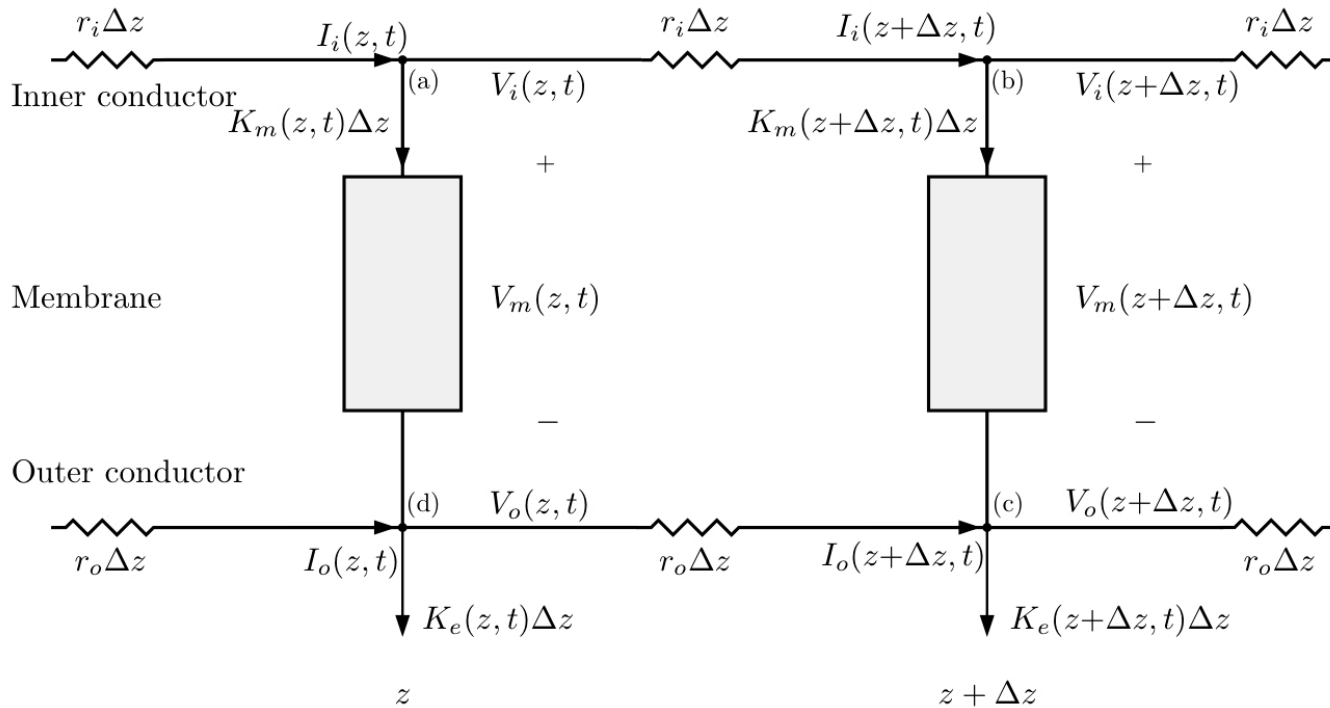
$$\frac{\partial V_i(z, t)}{\partial z} = -r_i I_i(z, t)$$

$$\frac{\partial V_o(z, t)}{\partial z} = -r_o I_o(z, t)$$

$$V_m(z, t) = V_i(z, t) - V_o(z, t)$$

$$\frac{\partial V_m(z, t)}{\partial z} = \frac{\partial V_i(z, t)}{\partial z} - \frac{\partial V_o(z, t)}{\partial z} = -r_i I_i(z, t) + r_o I_o(z, t)$$

$$\frac{\partial^2 V_m(z, t)}{\partial z^2} = -r_i \frac{\partial I_i(z, t)}{\partial z} + r_o \frac{\partial I_o(z, t)}{\partial z} = r_i K_m(z, t) + r_o (K_m(z, t) - K_e(z, t))$$



THE Core – Conductor Equation

Assumptions/geometry above, along with Kirchoff's & Ohm's Laws lead us to the...

$$\frac{\partial^2 V_m(z, t)}{\partial z^2} = (r_o + r_i)K_m(z, t) - r_o K_e(z, t)$$

→ Relates spatial change in transmembrane potential to current flowing through the membrane

Some Implications

Consider no external electrodes
(i.e., $K_e = 0$):

$$K_m(z, t) = \frac{1}{r_o + r_i} \frac{\partial^2 V_m(z, t)}{\partial z^2}$$

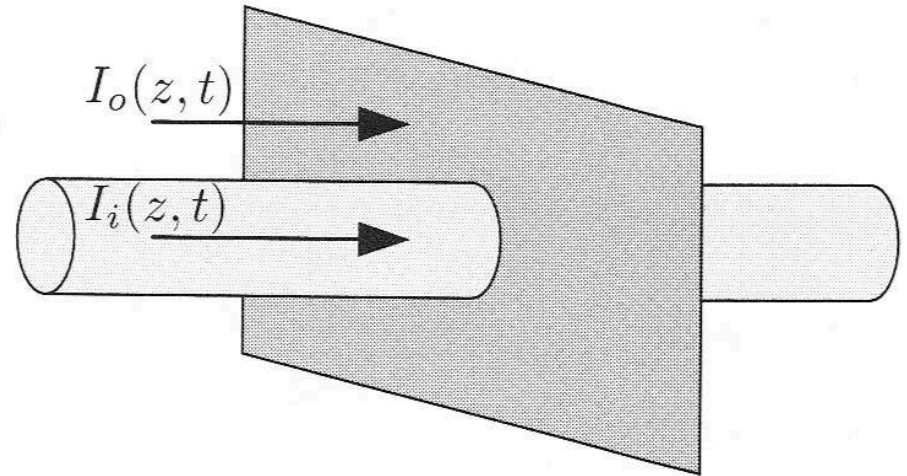


Figure 2.9

Conservation of charge requires: $I_i(z, t) + I_o(z, t) = 0$

Core – Conductor Equations

$$\frac{\partial I_i(z, t)}{\partial z} = -K_m(z, t)$$

$$\frac{\partial I_o(z, t)}{\partial z} = K_m(z, t) - K_e(z, t)$$

$$\frac{\partial V_i(z, t)}{\partial z} = -r_i I_i(z, t)$$

$$\frac{\partial V_o(z, t)}{\partial z} = -r_o I_o(z, t)$$

$$I_o(z, t) = \frac{1}{r_o + r_i} \frac{\partial V_m(z, t)}{\partial z}$$