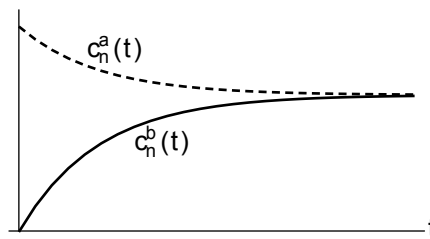**HW 2 SOLUTIONS**

Questions

1. A solute n diffuses through a membrane that separates two compartments that have different initial concentrations. The concentrations in the two compartments as a function of time, $c_n^a(t)$ and $c_n^b(t)$, are shown in the figure below. The volumes of the two compartments are V_a and V_b. Is $V_a > V_b$ or is $V_a < V_b$? Explain.


$\Rightarrow$ **Solution (5 pts):**

Since the concentration in compartment a changes less than that in compartment b, compartment a has the larger volume. More formally, we have $c_n^a(0) - c_n^a(\infty) < c_n^b(\infty) - c_n^b(0)$. But $c_n^a(\infty) = c_n^b(\infty)$ and $c_n^b(0) = 0$. Combining these relations yields $c_n^a(0) < 2c_n^a(\infty)$. Let n_n^a be the number of moles of n in compartment a at time $t = 0$. Then,

$$\frac{n_n^a}{V_a} < 2\frac{n_n^a}{V_a + V_b}$$

from which it follows that $V_a + V_b < 2V_a$ which implies that $V_b < V_a$

2. All cells are surrounded by a cell membrane. The cytoplasm of most cells contains a variety of organelles that are also enclosed within membranes. Assume that a spherical cell with radius $\mathbf{R = 50\mu m}$ contains a spherical organelle called a vesicle, with radius $r = 1\mu m$, as shown in the following figure.
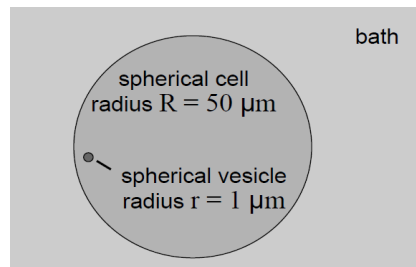
Assume that the membranes surrounding the cell and vesicle are uniform lipid bilayers with identical compositions and the same thickness $d = 10$ nm. Assume that solute X is transported across both the cell

and vesicle membrane via the dissolve and diffuse mechanism. Assume that X dissolves equally well in the bath and in the aqueous interiors of the vesicle and cell. Assume that the solute X dissolves 100 times less readily in the membrane (i.e. the partitioning coefficient is 0.01). Assume the diffusivity of X in the membranes in $10^{-7}cm^2/s$. Initially, the concentration of X is zero inside the cell and inside the vesicle. At time $t = 0$, the cell is plunged into a bath that contains X with concentration 1 mmol/L.

$\Rightarrow$ **Solution:**
There are three interesting time constants. The first is the steady-state time constant for the membrane (which is the same for both the vesicle and cell). It is given by

$$\tau_{ss} = \frac{d^2}{\pi^2 D}$$

where D is the diffusivity of the solute in the membrane and d is the thickness of the membrane. Substituting,

$$\tau_{ss} = \frac{(10 \text{ nm })^2}{\pi^2 \times 10^{-7} cm^2/sec} \approx 1\mu sec$$

The second is the equilibrium time constant for the cell,

$$\tau_{EQ}|_{cell} = \frac{dV_c}{AkD}$$

Where V_c is the volume of the cell and k is the partition coefficient (here we have ignored the effect of the volume of the bath, because it is so large, and the volume of the vesicle, because it is so small). Thus

$$\tau_{EQ}|_{cell} = \frac{10 \text{ nm } \times \frac{4}{3}\pi R^3}{4\pi R^2 \times 0.01 \times 10^{-7} cm^2/s} = \frac{10 \text{ nm } \times \frac{R}{3}}{0.01 \times 10^{-7} cm^2/s} = \frac{10 \text{ nm } \times 50\mu m/3}{0.01 \times 10^{-7} cm^2/s} \approx 1.7 \text{ s}$$

The third is the equilibrium time constant for the vesicle,

$$\tau_{EQ}|_{vesicle} = \frac{dV_v}{AkD}$$

Where V_v is the volume of the vesicle (here we have ignored the effect of the volume of the cell because it is so large). Thus

$$\tau_{EQ}|_{vesicle} = \frac{10 \text{ nm } \times \frac{4}{3}\pi r^3}{4\pi r^2 \times 0.01 \times 10^{-7} cm^2/s} = \frac{10 \text{ nm } \times \frac{r}{3}}{0.01 \times 10^{-7} cm/s} = \frac{10 nm \times 1\mu m/3}{0.01 \times 10^{-7} cm^2/s} \approx 33 \text{ ms}$$

a. Estimate the time that is required for the concentration of X in the cell to reach 0.5 mmol/L. Find a numerical value or explain why it is not possible to obtain a numerical value with the information that is given.

$\Rightarrow$ **Solution (5 pts):**
Since $\tau_{EQ}|_{cell} >> \tau_{ss}$, the concentration $c_c(t)$ in the cell will increase nearly exponentially with time with a time constant equal to $\tau_{EQ}|_{cell}$,

$$c_c(t) \approx (1 \text{ mmol/L}) \times (1 - e^{-t/\tau_{EQ}|_{cell}})$$

The concentration $c_c(t)$ will reach half its initial value when

$$(0.5 \text{mmol/L}) = (1 \text{ mmol/L}) \times (1 - e^{-t/\tau_{EQ}|_{cell}})$$

so

$$t = -\ln(0.5\tau_{EQ}|_{cell}) = 0.69 \times 1.7 \text{ s} \approx 1.2 \text{ s}$$

b. Estimate the time that is required for the concentration of X in the vesicle to reach 0.5 mmol/L. Find a numerical value or explain why it is not possible to obtain a numerical value with the information that is given.

$\Rightarrow$ **Solution (5 pts):**
Since $\tau_{EQ}|_{vesicle} \ll \tau_{EQ}|_{cell}$ the concentration inside the vesicle is nearly equal to that in the cell. Therefore, the anwser to part b is the same as the answer to part a

$$t \approx 1.2 \text{ s}$$

3. Two solutions of an uncharged solute S have volumes $V_1 = 100cm^3$ and $V_2 = 50cm^3$ and are separated by a thin membrane (area $A = 25cm^2$) permeant to S and impermeant to water.
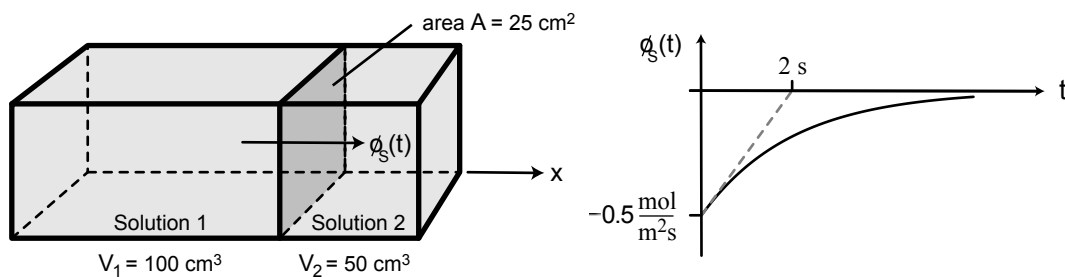
The flux of S through the membrane obeys Fick's law for membranes. At time $t = 0$, the concentration of S in solution 1 is $c_1(0) = 100mol/m^3$. The initial concentration of S in solution 2 is not known. The flux of S through the membrane in the positive x direction is found to be an exponential function of time as shown in the plot.

a. Determine the concentration $c_1(t)$ of S in solution 1 and the concentration $c_2(t)$ of S in solution 2 as functions of time, assuming that the solutions are well-stirred. Sketch $c_1(t)$ and $c_2(t)$ on suitably labeled axes.

b. Determine numerical values for the final concentrations of S: $c_1(\infty)$ and $c_2(\infty)$. If it is not possible to determine numerical values, list the other information that would be needed to determine a numerical value.

⇒ **Solution (5 pts):**
Since the flux $\phi_s(t)$ is out of compartment 1, the concentrationn $c_a(t)$ is given by

$$c_1(t) = c_1(0) - \frac{A}{V_1} \int_0^t \phi_s(t) dt$$

Substituting $c_1(0) = 100$ mol/m^3, $A = 25$ cm^2, $V_1 = 100$ cm^3, and $\phi_s(t) = -0.5\ e^{t/2}$ mol/(m^{2}s) and integrating yields

$$c_1(t) = 125 - 25\ e^{-t/2}\ \text{mol/m}^3$$

Thus the final value of $c_1(t)$ is

$$c_1(\infty) = 125\ \text{mol/m}^3$$

Since the final value of $\phi_s(t)$ is zero, the final value of $c_2(t)$ must equal the final value of $c_1(t)$

$$c_2(\infty) = 125\ \text{mol/m}^3$$

Continuity of S implies that $c_1(t)V_1 + c_2(t)V_2$ equals a constant. We can evaluate the constant at $t \to \infty$. The result is $c_1(t)V_1 + c_2(t)V_2 = 125(V_1 + V_2)$. Solving for $c_2(t)$ yields

$$c_2(t) = 125 + \frac{V_1}{V_2} 25\ e^{-t/2}\ \text{mol/m}^3 = 125 + 50\ e^{-t/2}\ \text{mol/m}^3$$
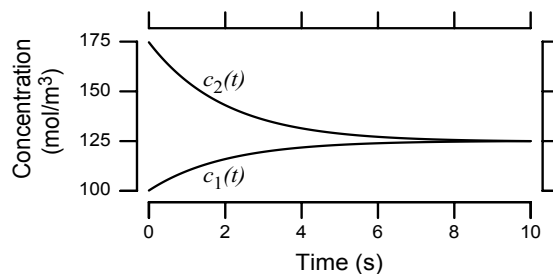
These concentrations are plotted above.

```
┌─────────────────────────────────────────────────────────┐
│   Body fluid              │          Dialysis            │
│  (including blood)        │            fluid             │
│         V                 ┼── J_s        V'              │
│                           │                              │
│            C              │        C'                    │
│                      Membrane                            │
└─────────────────────────────────────────────────────────┘
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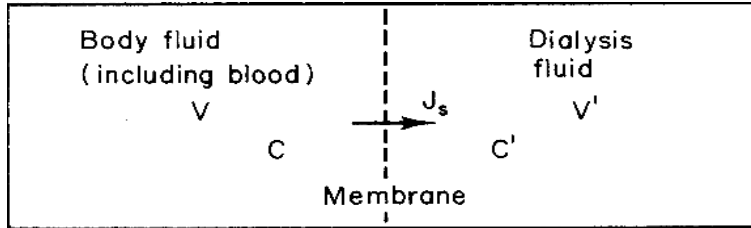
4. Dialysis is commonly used in hospitals to remove urea from a patient's plasma when their kidneys begin to fail. It is known that along with being found in the plasma, urea can also be found in the interstitial brain fluid and the cerebrospinal fluid in the same concentration as the plasma.

If the solute transport by diffusion is given as $J_s = \omega RT(C - C')$, where C denotes the concentration of the solute in the blood, C' denotes the concentration in the dialysis fluid, ω denotes the Solute Permeability (in $mol \cdot N^{-1} \cdot s^{-1}$), R denotes the Gas Constant and T denotes the temperature of the system (in Kelvins):

   a. Determine the rate of change of the number of solute molecules N and the rate of change of the concentration C of the solute in the blood (assuming that the surface area of the membrane can be denoted as S)

   $\Rightarrow$ **Solution (2 pts):**

   If the surface area of the membrane can be denoted as S, then the rate of change of the number of solute molecules N can be expressed as:

$$\frac{dN}{dt} = -N_{avgr}S\omega RT(C - C')$$

   If the solute is well mixed in the body fluid compartment, then $N = CV$, and this equation can be denoted as:

$$\frac{dC}{dt} = \frac{-S\omega RT}{V}(C - C')$$

   b. Determine the solution to the differential equation for the rate of change of the solute concentration C; what is the time constant?

   $\Rightarrow$ **Solution (2 pts):**

   Using the above equation for the change in concentration as a function of time and putting it into standard form, the following is received:

$$\frac{dC}{dt} + \frac{S\omega RTC}{V} = \frac{S\omega RTC'}{V}$$

5

To solve this linear first-order equation, find the integrating factor from the standard form such that:

$$\mathbf{I} = \mathbf{e}^{\int \frac{\mathbf{S}\omega\mathbf{RT}}{\mathbf{V}}\mathbf{dt}} = \mathbf{e}^{\frac{\mathbf{S}\omega\mathbf{RT}}{\mathbf{V}}\mathbf{t}}$$

By multiplying the standard form of the equation by the integrating factor, it can be seen that the left-hand side of the resulting equation is automatically the derivative of the integrating factor and y. Below, D denotes the arbitrary integrating factor, which can be solved by using the steady state solution where $C = C'$:

$$\left(\frac{\mathbf{dC}}{\mathbf{dt}} \cdot \mathbf{e}^{\frac{\mathbf{S}\omega\mathbf{RT}}{\mathbf{V}}\mathbf{t}}\right) + \left(\mathbf{e}^{\frac{\mathbf{S}\omega\mathbf{RT}}{\mathbf{V}}\mathbf{t}} \cdot \frac{\mathbf{S}\omega\mathbf{RTC}}{\mathbf{V}}\right) = \left(\mathbf{e}^{\frac{\mathbf{S}\omega\mathbf{RT}}{\mathbf{V}}\mathbf{t}} \cdot \frac{\mathbf{S}\omega\mathbf{RTC'}}{\mathbf{V}}\right)$$

$$\therefore \frac{\mathbf{d}}{\mathbf{dt}}\left(\mathbf{Ce}^{\frac{\mathbf{S}\omega\mathbf{RT}}{\mathbf{V}}\mathbf{t}}\right) = \mathbf{AC'e}^{\frac{\mathbf{S}\omega\mathbf{RT}}{\mathbf{V}}\mathbf{t}}$$

$$\mathbf{Ce}^{\frac{\mathbf{S}\omega\mathbf{RT}}{\mathbf{V}}\mathbf{t}} = \int \mathbf{AC'e}^{\frac{\mathbf{S}\omega\mathbf{RT}}{\mathbf{V}}\mathbf{t}}\mathbf{dt}$$

$$\mathbf{C(t)} = \mathbf{C'} + \mathbf{De}^{\frac{-\mathbf{S}\omega\mathbf{RT}}{\mathbf{V}}\mathbf{t}}$$

$$\therefore \mathbf{C(t)} = \mathbf{C'} + (\mathbf{C_0} - \mathbf{C'})\mathbf{e}^{\frac{-\mathbf{t}}{\tau}} \text{ where } \tau = \frac{\mathbf{V}}{\mathbf{S}\omega\mathbf{RT}}$$

c. During dialysis conducted at room temperature, typical cellophane wrappers are used to introduce the dialysis fluid. If 40 liters of dialysis solution is used, and the wrapper has an area of $3m^2$ and a solute permeability of $2.45 \cdot 10^{-9} mol \cdot N^{-1} \cdot s^{-1}$, how long would the dialysis session run?

$\Rightarrow$ **Solution (1 pt):**

Assuming standard conditions for ambient temperature and pressure where temperature equals 298.15 K:

$$\therefore \tau = \frac{40 \cdot 10^{-3} \text{ m}^3}{(3 \text{ m}^2) \cdot (2.45 \cdot 10^{-9} \text{mol} \cdot \text{N}^{-1} \cdot \text{s}^{-1}) \cdot (8.314 \cdot \frac{\text{J}}{\text{m K}}) \cdot (298.15 \text{ K})} \approx 2195.47 \text{ s} \approx 36.59 \text{ min}$$

5. The following figure shows an experimental apparatus for testing a semipermeable membrane. Volumes V_1 and V_2 contain well-stirred aqueous solutions of glucose and NaCl, respectively. These volumes are separated by a membrane that is permeable only to water. Assume that the pistons are ideal (i.e. they are frictionless, and faithfully transmit the pressure P_1 and P_2 to V_1 and V_2 respectively). Also assume that effects of gravity are negligible.
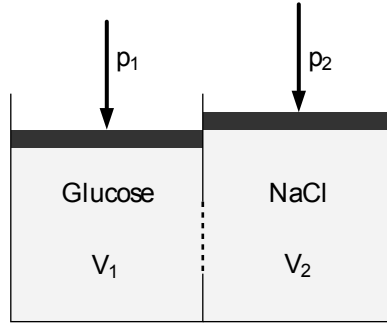
a. When the system is in equilibrium, what will be the relation between the hydraulic pressure (P_1 and P_2 and solute concentrations in each compartment?

⇒ **Solution (5 pts):**

At osmotic equilibrium

$$\mathbf{p_1 - p_2 = \pi_1 - \pi_2 = RT(C_\Sigma^1 - C_\Sigma^2) = RT(C_{glucose} - C_{Na^+} - C_{Cl^-})}$$

b. Volumes V_1 and V_2 are initially equal, with $V_1 = V_2 = 1L$. At time $t = 0$, the concentration of glucose in compartment 1 is 0.01 mol/L and the concentration of NaCL in compartment 2 is 0.01 mol/L. If $P_1 = P_2$, what is the final volume of compartment 2? Sketch V_2 as a function of time for $t \geq 0$.

⇒ **Solution (5 pts):**

The total volume of water is $V_1 + V_2 = 2$ L. Since $p_1 = p_2$, at osmotic equilibrium $C_\Sigma^1(\infty) = C_\Sigma^2(\infty)$. Initially, $C_\Sigma^1(0) = 0.01$ mol/L and $C_\Sigma^2(0) = 0.02$ mol/L. The number of moles of glucose is 0.01 mol and of Na^+ plus Cl^- is 0.02 mol. Therefore at osmotic equilibrium

$$\frac{0.01}{\mathbf{V_1}} = \frac{0.02}{\mathbf{V_2}} \text{ and } \mathbf{V_1 + V_2 = 2}$$

These two equations are solved by taking the reciprocal of the equation on the left and substituting the equation on the right to obtain,

$$\mathbf{100V_1 = 50V_2 = 50(2 - V_1)}$$

Solving these equations yields $V_1 = 2/3$L and $V_2 = 4/3$ L. The volume can be found as a function of time from the relation

$$-\frac{1}{\mathbf{A}}\frac{d\mathbf{V_1(t)}}{dt} = \mathcal{L}_v\mathbf{RT}(\mathbf{C_\Sigma^2 - C_\Sigma^1})$$

7

Note that $V_1(t) + V_2(t) = 2L$, $N_\Sigma^1 = 0.01$ mol and $N_\Sigma^2 = 0.02$ mol so that

$$\frac{dV_1(t)}{dt} = \frac{1}{\tau}\left(\frac{1}{V_1(t)} - \frac{2}{2 - V_1(t)}\right)$$

Where $\tau = 100/(\mathcal{L}_v RTA)$ and $V_2(t) = 2 - V_1(t)$. This differential equation can be integrated numerically to yield the solution shown in figure 1. The differential equation for the volume is nonlinear and the solution shows monotonic, but non-exponential changes in volume.
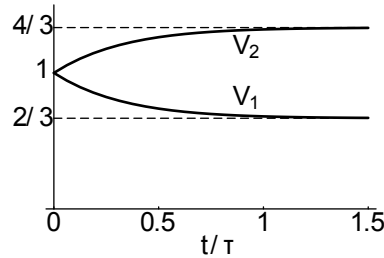

Figure 1: Change in volume with time. the volumes are expressed in liters

6. The water content of plants is very high (up to 90% by weight) but this water is in flux; water is absorbed through the roots, rises as sap, and evaporates from the leaves. The total water content of a plant can be replaced many times per hour. The mechanisms that determine water flow include: gravity, osmosis, and capillarity. In this problem we will consider the effect of gravity and osmosis. Assume that water flow is steady and is due to the gravitational, osmotic, and *other* forces. Let the pressure due to gravity be p_g; and the osmotic pressure be π. Let the pressure due to other sources in the trees be p_o (e.g., capillary forces); assume these sources are not appreciable in the medium that is in contact with the roots. Consider two trees:

1. The General Sherman giant sequoia (*Sequoia gigantea*) which is located in Sequoia National Park in California stands 272 feet high and has a diameter of 36.5 feet at its base; you can drive your car through a tunnel that has been cut through the base. The tree was estimated to be 3,800 years old.

2. The Red mangrove (*Rhizophora mangle*) grows in the tropics to a height of as much as 80 feet. It is found in tidal creeks and estuaries and it can grow with its roots in seawater. Yet the sap has the composition of fresh water.

The compositions of the relevant media are given below

Ion	Concentration mmol/L			
	Giant Sequoia		Red Mangrove	
	soil	sap	seawater	sap
K^+	1	20	10	20
Na^+	2	0	450	0
Cl^-	30	2	530	2
NO_3^-	3	20	0	20
Ca^{2+}	7	1	10	1
Mg^{2+}	8	0	50	0

a. Draw a figure for both the General Sherman and the Red mangrove; ensure to denote all forces working in each system which aid/suppress the sap in rising within the trees, and to clearly state all assumptions that have been made.

$\Rightarrow$ **Solution (1 pt):**

It can be assumed that the system denoted by both the General Sherman and the Red mangrove can be characterized by a two-chamber system which relies on osmotic pressure (as well as other outside pressure) as the driving force working against gravity and soil osmotic pressure.
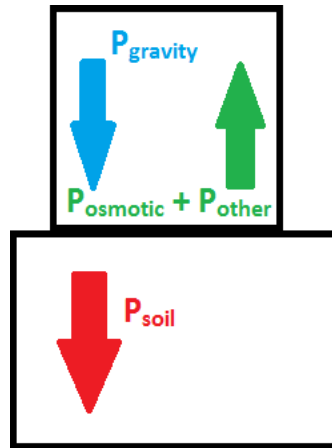
b. Find a numerical bound on p_o (expressed in atmospheres) such that the sap will rise in each tree.

$\Rightarrow$ **Solution (4 pts):**

At the junction between the roots and the soil/seawater, the flux of water from the soil/seawater into the tree root is

$$\Phi_v = \mathcal{L}_v((p_g^s - \pi^s) - (p_o + p_g^t - \pi^t))$$

Where the superscript t is used to denote the tree root and the superscript s to denote either soil or seawater. Let $p_g = p_g^t - p_g^s$. Therefore, for the sap to rise in the tree $\pi^t - \pi^s - p_o - p_g > 0$ which implies that $p_o < \pi^t - \pi^s - p_g$.

b. Which tree requires the larger value of p_o? Explain.

$\Rightarrow$ **Solution (5 pts):**

The pressure due to gravity is $p_g = \rho g h$ where ρ is the density of water, g is the acceleration of gravity, and h is the height of the tree. this can be expressed as follows

$$p_g = (10^3 \text{ kg/ m}^3)(9.8 \text{ N/kg})h = 9.8 \times 10^3 h \text{ Pa}$$

9

The osmotic pressure at 27°C

$$\pi = \text{RTC}_\Sigma = (8.314\,\text{J/(mol K)})(300\,\text{K })\text{C}_\Sigma = 2.49 \times 10^3 \text{C}_\Sigma$$

Where C_Σ is expressed in mol/m^3. Note that concentration in mmol/L is the same as mol/m^3. The table below summarizes the relevant factors. There are a number of interesting factors involved in the rising sap that are illustrated in this problem. To interpret the results intuitively, recall that atmospheric pressure (which results from the weight of the atmosphere on the surface of the earth) corresponds to 10^5 pascals also called an *atmosphere*. Thus, the gravitational pressure due to the height of the tree must be greater in the giant sequoia, which is a taller tree than the red mangrove. Both pressures correspond to several atmospheres. Both of these pressures tend to make the water flow out of the tree root into the soil/seawater. Because, the osmolarity of the soil/seawater exceeds that of sap for both trees, this difference in osmotic pressure also causes water to flow out of the tree. However, this effect is much larger in the red mangrove than in the giant sequoia because the osmolarity of seawater greatly exceeds that of the soil. Thus, the capillary forces that make sap rise must be much greater in the red mangrove than in giant sequoia.

Variable	Sequoia	Mangrove
h(m)	82.9	24.4
p_g (Pa)	8.1×10^5	2.4×10^5
C_Σ^t (mol/m^3)	43	43
C_Σ^s (mol/m^3)	51	1050
π^t(Pa)	1.1×10^5	1.1×10^5
π^t(Pa)	1.3×10^5	2.6×10^6
$\pi^t - \pi^s - p_g$ (Pa)	-8.3×10^5	-2.7×10^6
p_o(Pa)	$< -8.3 \times 10^5$	$< -2.7 \times 10^6$

7. The following figure shows the design of a miniature pump that can be implanted in the body to deliver a drug. No batteries are required to run this pump!
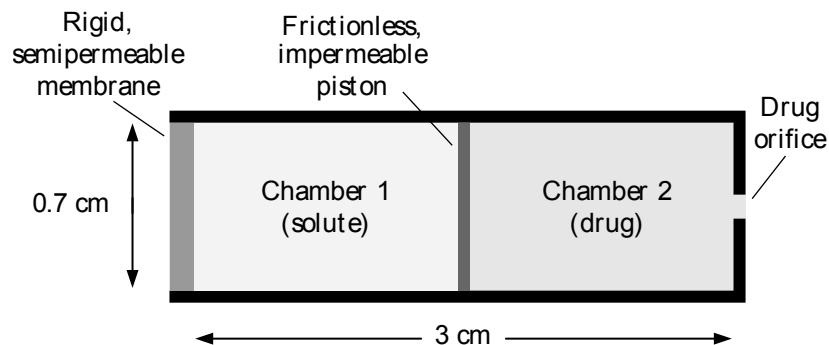
The pump contains two cylindrical chambers filled with incompressible fluids: the two chambers together have a length of 3 cm and a diameter of 0.7 cm. Chamber 1 is filled with a solution whose concentration is 10 mol/L; the osmolarity of this solution greatly exceeds that of body fluids. Chamber 2 is filled with the drug solution. The two chambers are separated by a frictionless, massless, and impermeable piston.

The piston moves freely and supports no difference in hydraulic chambers. The pump walls are rigid, impermeable and cylindrical with an orifice at one end for delivering the drug and a rigid, semipermeable membrane at the other end. The orifice diameter is sufficiently large that the hydraulic pressure drop across this orifice is negligible and sufficiently small so that the diffusion of drug though the orifice is also negligible. The semipermeable membrane is permeable to water only, and not permeable to the solute. Assume the $T = 300$ K.

a. Provide a discussion of 50 words or fewer for each of the following:

i. What is the physical mechanism of drug delivery implied by the pump design?

$\Rightarrow$ **Solution (5 pts):**

Because the concentration of solute in Chamber 1 is much larger than the total solute concentration in the body fluids, the osmotic pressure $\pi = RTC_\Sigma^1$ is larger than the external osmotic pressure. Because of the properties of the piston and drug-delivery orifice, no hydraulic pressure difference exists. Flux of water into Chamber 1, the piston moves to the right and the drug solution is forced out of the orifice at the same volume flow rate.

ii. What is (are) the source(s) of energy for pumping the drug?

$\Rightarrow$ **Solution (5 pts):**

Chemical energy is stored in the high solute concentration in Chamber 1. This store of energy is dissipated as water flows in and dilutes the solution

iii. Assume there is an adequate supply of drug in the pump for the lifetime of the implanted subject and this it is necessary to provide a constant rate of drug delivery. Which fundamental factors limit the useful lifetime of this pump in the body?

$\Rightarrow$ **Solution (5 pts):**

If it is assumed that all the solute in Chamber 1 is in solution initially, then the concentration will decrease as water flows in. The concentration will not change much if $\Phi_v A T_L \ll V_1$, where T_L is the time that the device is in operation. Thus, for a given volume $\Phi_V A$, $T_L \ll V_1/(\Phi_V A)$. Alternatively, if enough undissolved solute is in Chamber1 to keep the solution saturated until all the drug is delivered, then the pump could operate at a constant rate throughout its life.

b. When implanted in the body, the pump delivers the drug at a rate of 1 μL/h. Find the value of hydraulic conductivity $\mathcal{L}_V$, of the semipermeable membrane.

$\Rightarrow$ **Solution (5 pts):**

The volume flux is $\mathcal{L}_v RTC_\Sigma^1$ and $\Phi_v A = 1\mu L/h$. Therefore,

$$\mathcal{L}_v = \frac{\Phi_v A}{RTC_\Sigma^1 A}$$

$$= \frac{1\mu L \times 1/3600 h/s}{\pi \times (0.007/2)^2 \, \text{m}^2 \times 8.314 \, \text{Nm/(mol K)} \times 300 \, \text{K} \times 10 \, \text{mol/L} \times 10^3 \text{L/m}^3}$$

$$= 2.9 \times 10^{-16} \text{m}^3/(\text{Ns})$$

The principle of the osmotic pump is the basis of commercially available systems for drug delivery.

8. This problem will require the use of SoftCell, software used in MATLAB. To access the remote version of MATLAB provided by York University, go to computing.yorku.ca/students/computer-labs/connecting-from-home-webfas/ and follow the instruction specific to your computer type. Next you need to download from the course website the SoftCell software and unzip it into an accessible directory. Next launch MATLAB using webfas and change the directory to the one containing SoftCell by clicking on the ellipsis next to current folder. In the command window now input "softcell" which should launch the SoftCell graphical user interface. We are interested in the random walk module.

    a. Now that the random walk simulator has been launched, go ahead and run a simulation using 100 steps and 50 particles. The number of steps can be changed by via the "Control" panel and the number of particles can be changed by clicking on the parameters button in the "Particle Set #1" panel (you can ignore the columns regarding region 2 and 3, ensure the initial distribution is set to pulsatile, the location at 200 and ensure that you click the update button to initialize your changes). Start the simulation, and after its completion, click the "Hist" button to observe both the expected and actual distribution of the random walk. What shape does the expected distribution appear to be? Does the actual distribution match the expected (make sure to run several simulations to get a better sample size)? Click on the "Graphs" button and view the graph for the mean particle location. Describe and explain the expected mean.

    ⇒ **Solution (5 pts):**

The expected distribution should appear Gaussian. To first order the expected distribution roughly matches the actual distribution.

The mean value for the expected location is a line with zero slope at 200. This is because the random walk is unbiased, so you can expect equal number of particles to move left of the central starting location as you would right.

    b. Next bias the random walk so that one direction is preferred over another. This can be done by varying the "Right Step Prob" in the "Parameters" window. Initially, what distribution do you predict will arise? What distribution arises from the simulation? Can you think of any systems where biased random walks are relevant? Make sure you include relevant information in answering this question (ie particle number, number of steps, biasing probabilities)

⇒ **Solution (5 pts):**

If the step size is chosen small enough, the Gaussian distribution will appear shifted in the direction biased. If the step size is large, then the distribution will hit one of the boundaries and roughly will appear as a half-Gaussian distribution. Many systems can mimic biased random walk the most obvious being a charged particle in an electric potential.