**PHYS 1420 (Fall 2019) - Midterm Exam (B)**

**Name:** SOLS

**Student Number:**

**Instructions:**

- **Read all instructions carefully.**
- Clearly write your name and student number above **BEFORE** you start the exam. Also, have your student ID out and on your desk (you may be asked for an invigilator to see it before/during/after the exam).
- Once the test begins, the instructor and invigilators will not be able to answer questions. You will need to interpret things as best you can and answer accordingly.
- Show all work clearly in order to get full credit. Points can be taken off if it is not clear to see how you arrived at your answer (even if the final answer is correct).
- Calculators can be used for this exam. Use of phones/tablets/computers/smart watches/etc... is not permitted.
- Sketch all relevant graphs and explain all relevant mathematics. Circle/box your final answers.
- Please keep your written answers brief; be clear and to the point.
- Feel free to use scratch paper (some is included at the back, feel free to detach it). You will not be graded upon what is on the scratch paper, though you must turn it in with your exam.
- You are allowed a formula sheet (8.5x11 in) to bring with you. It must be a single hard copy sheet of paper (though you can write on front and back). **You must turn such in with your exam.**
- This test has 3 problems (plus an extra credit problem) and is worth 100 points. It is your responsibility to make sure that you have done all the problems!
- Make sure to turn your test in as requested at the end of the exam period. Failure to do such can lead to a failing grade.

**1.** (*30 points*)
For the following questions, circle the appropriate choice for True or False. **No explanation in necessary.**

- [*True*] *or* *False* – A falling body asymptotically approaches a *terminal velocity*.
- *True* *or* [*False*] – A change in energy is always positive.
- [*True*] *or* *False* – For a thrown ball, it is possible to have the velocity and acceleration vectors to point in the same direction at the instant the ball is thrown.
- [*True*] *or* *False* – Uniform motion can effectively be described as "unchanging change".
- *True* *or* [*False*] – The product of two vectors is always a scalar.
- [*True*] *or* *False* – The basic idea underlying rocket thrust is a. reduction of mass and b. conservation of momentum.
- *True* *or* [*False*] – *Stiction* refers to the friction of sliding bodies.
- *True* *or* [*False*] – Kinetic friction tends to be greater than static friction.
- [*True*] *or* *False* – Riemann sums can be generalized to "slices" in 3-D.
- *True* *or* [*False*] – A cylinder falling "end down" and another falling "side down" will have the same drag coefficients.
- *True* *or* [*False*] – Work is the energy transferred between systems via a continuous force.
- *True* *or* [*False*] – If a plane flying at constant velocity drops a package, it will be directly over the package when it hits the ground.
- *True* *or* [*False*] – Velocity is the integral of displacement with respect to time.
- [*True*] *or* *False* – Newton's 2nd law is $\mathbf{F} = \frac{d\mathbf{p}}{dt}$ where $p$ is momentum.
- [*True*] *or* *False* – If the rate of change of a function is proportional to itself, then the function is exponential.
- [*True*] *or* *False* – For uniform circular motion, the acceleration vectors all point in the same direction.
- *True* *or* [*False*] – A feather and a rock both move towards Earth with the same acceleration.
- [*True*] *or* *False* – A ball dropped versus throw up both move towards Earth with the same acceleration.
- *True* *or* [*False*] – Arc length is $r\theta^2$, where $r$ is the radius and $\theta$ is the angle extended.
- [*True*] *or* *False* – Work has the same units as energy.
- [*True*] *or* *False* – When compressing a spring, the work done on the spring is positive.
- *True* *or* [*False*] – Integrals can arise from Riemann sums when the rectangles get infinitesimally large.

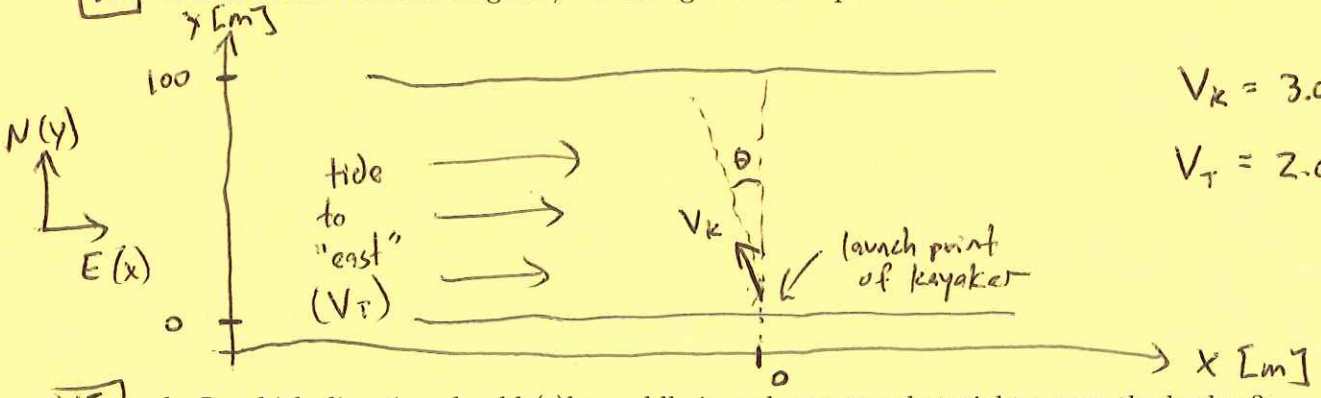
- *True* or *False* – A falling object experiencing drag will asymptotically approach terminal velocity.

Note: Unintended repeat

- *True* or *False* – Conservation on energy indicates that energy does not change.
- *True* or *False* – Centripetal acceleration ($a$) is related to the square of the angular velocity.
- *True* or *False* – Parabolas are related to cubic functions.
- *True* or *False* – Einstein's theory of general relativity is an approximation of Newton's law of gravity.
- *True* or *False* – The First Law of Thermodynamics embodies the distinction between "good" versus "bad" energy.
- *True* or *False* – Rolling friction tends to be larger than kinetic or static friction, thus giving tires their "grip".
- *True* or *False* – Drag forces can only be proportional to velocity or its square.

**2.** (*35 points*)
A kayaker needs to paddle north across a 100-m-wide harbor. The tide is going out, creating a tidal current that flows to the east at 2.0 m/s. The kayaker paddles with a constant speed of 3.0 m/s.

10 a. Sketch the relevant diagram, indicating the main quantities of interest.



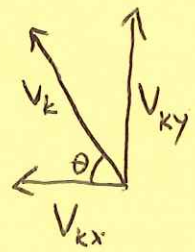

15 b. In which direction should (s)he paddle in order to travel straight across the harbor?

☑ Note that $V_k$ is relative to the water. (S)he is being swept east, so must paddle to the west (i.e. have a negative x-component) in addition to a North (i.e. +y) component, thereby heading straight north

☑ To paddle straight north (re Earth), $V_{kx} = -2.0$ m/s. And since $|V_k| = 3.0$ m/s:

$$V_{kx}^2 + V_{ky}^2 = V_k^2 = (3.0)^2 = 9.0 = (-2.0)^2 + V_{ky}^2$$

$$\rightsquigarrow V_{ky}^2 = 5.0 \rightarrow V_{ky} = \sqrt{5.0} \simeq 2.2 \text{ m/s}$$




$$\theta = \tan^{-1}\left(\frac{V_{ky}}{V_{kx}}\right) = \tan^{-1}\left(\frac{2.24}{2}\right) \simeq \boxed{41.8^\circ}$$

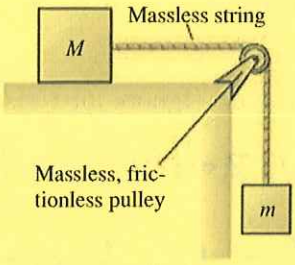
10 c. How long will it take the kayaker to cross?

$V_{ky} = 2.24$ m/s and there is 100 m to cross:

$$t = \frac{100}{2.24} \simeq \boxed{44.7 \text{ s}}$$

Note that this is 35 points

**3.** (~~35~~ *points*) Consider a mass $M$ connected to another mass $m$ via a mass-less string and mass-/friction-less pulley as shown in the figure below. Do not assume one of the masses is relatively larger than the other.



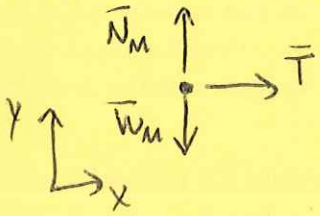
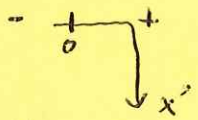





Figure 1: Mass $m$ connected to another mass $M$.

Note: Could also make into a 1-D problem by creating x'-axis as follows:




$W_m = mg$

**25** a. Consider the surface to be frictionless. Find an expression for the tension $T$ in the string.

- Both blocks have the same acceleration (call it $a$).
- For the hanging block, Newton's 2nd yields: $\sum F_m = T - W_m = -ma$
- For the block on the table: $(\sum F_m)_x = T = Ma$ and $(\sum F_m)_y = N_M - W_m = 0 \rightarrow N_M = Mg$
- Combining the two equations to eliminate $a$, we have:

$$a = \frac{T}{M} \rightarrow T - mg = -m\left(\frac{T}{M}\right) \rightarrow T\left(1 + \frac{m}{M}\right) = mg \Longrightarrow T = \frac{mg}{\left(1 + \frac{m}{M}\right)} = \frac{Mmg}{M+m}$$

**10** b. Describe concisely how your answer would change if there was friction. Make sure to clearly convey all possible cases.

- Acceleration of two blocks still the same, but two conditions re friction

1) Static friction could prevent blocks from moving (i.e. $a = 0$) if $\mu_s \geq \frac{m}{M}$ (since static friction would dominate over m-induced tension)

2) Kinetic friction would <u>increase</u> the tension in the rope as

$$T = g\,\frac{Mm}{m+M}\left(1 + \mu_k\right)$$

**Extra Credit (10 Points):**

An airplane carries relief supplies for a person stranded on an island. Since the island is too small to land on, the pilot drops the supplies as she flies (horizontally) over the island. Assume the plane is flying at altitude $h$ and with speed $v_p$.

**a.** How long does the package spend in the air. Ignore air resistance.

- Assume package is released w/ zero y-velocity at height $y_0 = h$:

$$y_f = y_0 + v_{0y}t - \frac{1}{2}gt^2 = h - \frac{1}{2}gt^2 = 0 \quad \text{(assumes "ground" is } y=0\text{)}$$

$$h - gt^2 = 0 \longrightarrow \boxed{t = \sqrt{\frac{2h}{g}}}$$

**b.** Where should the pilot release the package?

- Assume $x_0 = 0$ (i.e. $x$-coord. of plane at $t=0$ is zero)

$$x_f = x_0 + v_{0x}t = 0 + v_p\left(\sqrt{\frac{2h}{g}}\right)$$

$\longrightarrow$ release at distance (before island) $\boxed{x = v_p\sqrt{\frac{2h}{g}}}$

**c.** How would your answer to part a change if you included air resistance?

- Package would fall more slowly due to drag and horizontal velocity would decrease due to more resistance
  $\longrightarrow$ more time in air

- Plane should thus release package later (due to relatively decreased horizontal velocity)