PHYS 1420 (Fall 2019) - Midterm Exam (C)

Name: SOLS

Student Number:

Instructions:

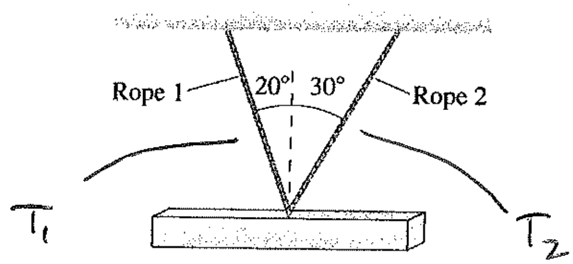
- **Read all instructions carefully.**
- Clearly write your name and student number above **BEFORE** you start the exam. Also, have your student ID out and on your desk (you may be asked for an invigilator to see it before/during/after the exam).
- Once the test begins, the instructor and invigilators will not be able to answer questions. You will need to interpret things as best you can and answer accordingly.
- Show all work clearly in order to get full credit. Points can be taken off if it is not clear to see how you arrived at your answer (even if the final answer is correct).
- Calculators can be used for this exam. Use of phones/tablets/computers/smart watches/etc... is not permitted.
- Sketch all relevant graphs and explain all relevant mathematics. Circle/box your final answers.
- Please keep your written answers brief; be clear and to the point.
- Feel free to use scratch paper (some is included at the back, feel free to detach it). You will not be graded upon what is on the scratch paper, though you must turn it in with your exam.
- You are allowed a formula sheet (8.5x11 in) to bring with you. It must be a single hard copy sheet of paper (though you can write on front and back). **You must turn such in with your exam.**
- This test has 3 problems (plus an extra credit problem) and is worth 100 points. It is your responsibility to make sure that you have done all the problems!
- Make sure to turn your test in as requested at the end of the exam period. Failure to do such can lead to a failing grade.

1. (*30 points*)
For the following questions, circle the appropriate choice for True or False. **No explanation in necessary.**

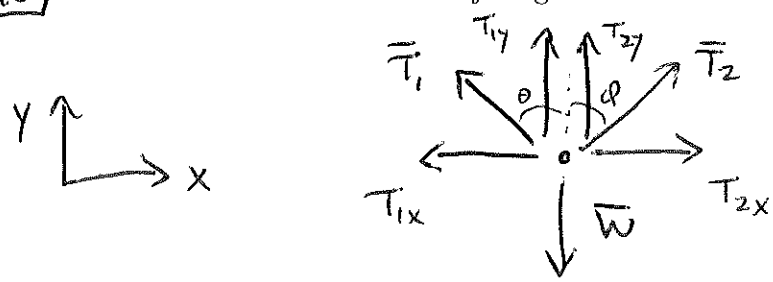
- *True* or *False* – If the rate of change of a function is proportional to itself, then the function is exponential.
- *True* or *False* – For uniform circular motion, the acceleration vectors all point in the same direction.
- *True* or *False* – A feather and a rock both move towards Earth with the same acceleration.
- *True* or *False* – If a plane flying at constant velocity drops a package, it will be directly over the package when it hits the ground.
- *True* or *False* – Velocity is the integral of displacement with respect to time.
- *True* or *False* – Newton's 2nd law is $\mathbf{F} = \frac{d\mathbf{p}}{dt}$ where p is momentum.
- *True* or *False* – Speed and velocity are the same thing.
- *True* or *False* – Centripetal acceleration (a) is related to the square of the angular velocity.
- *True* or *False* – Parabolas are related to cubic functions.
- *True* or *False* – Einstein's theory of general relativity is an approximation of Newton's law of gravity.
- *True* or *False* – A ball dropped versus throw up both move towards Earth with the same acceleration.
- *True* or *False* – Arc length is $r\theta^2$, where r is the radius and θ is the angle extended.
- *True* or *False* – The basic idea underlying rocket thrust is a. reduction of mass and b. conservation of momentum.
- *True* or *False* – *Stiction* refers to the friction of sliding bodies.
- *True* or *False* – Kinetic friction tends to be greater than static friction.
- *True* or *False* – A falling cylinder would feel a larger drag force than a sphere.
- *True* or *False* – A cylinder falling "end down" and another falling "side down" will have the same drag coefficients.
- *True* or *False* – Work is the energy transferred between systems via a continuous force.
- *True* or *False* – A falling body can asymptotically approach a *terminal velocity*.
- *True* or *False* – A change in energy is always positive.
- *True* or *False* – For a thrown ball, it is possible to have the velocity and acceleration vectors to point in the same direction at the instant the ball is thrown.

- $\boxed{\textit{True}}$ *or* *False* – Uniform motion can effectively be described as "unchanging change".
- *True* *or* $\boxed{\textit{False}}$ – The product of two vectors is always a scalar.
- $\boxed{\textit{True}}$ *or* *False* – Work has the same units as energy.
- $\boxed{\textit{True}}$ *or* *False* – When compressing a spring, the work done on the spring is positive.
- *True* *or* $\boxed{\textit{False}}$ – Integrals can arise from Riemann sums when the rectangles get infinitesimally large.
- $\boxed{\textit{True}}$ *or* *False* – A ball rolling on a curved track is a 1-D problem.
- $\boxed{\textit{True}}$ *or* *False* – The First Law of Thermodynamics embodies the distinction between "good" versus "bad" energy.
- *True* *or* $\boxed{\textit{False}}$ – Rolling friction tends to be larger than kinetic or static friction, thus giving tires their "grip".
- *True* *or* $\boxed{\textit{False}}$ – Drag forces can only be proportional to velocity or its square.

2. (*35 points*)
The 1000 kg steel beam in is hanging from the ceiling at height h as shown below. It is supported by two ropes.



$\boxed{10}$ a. Draw the associated free-body diagram. Make sure to clearly label all relevant forces.



$\theta = 20°$
$\varphi = 30°$

$\boxed{25}$ b. What is the tension in each rope?

$\bar{W} = -mg\hat{y} = -9800\hat{y}$ N

For equilibrium, we must have: $T_{1x} + T_{2x} = 0 \longrightarrow T_{1x} = -T_{2x}$ (1)

$T_{1y} + T_{2y} + W_y = 0 \longrightarrow T_{1y} + T_{2y} = -W$ (2)

From (1): $T_{1x} = T_1 \sin\theta = T_{2x} = T_2 \sin\varphi$

So $T_1 \sin\theta = T_2 \sin\varphi$ or $T_1 = T_2 \dfrac{\sin\varphi}{\sin\theta}$ (3)

From (2): $T_{1y} + T_{2y} = T_1\cos\theta + T_2\cos\varphi = mg = 9800$ (4)

Two eqns. w/ two unknowns, so use (3) to plug into (4):

$$T_1\cos\theta + T_2\cos\varphi = T_2\left[\frac{\sin\varphi}{\sin\theta}\cos\theta + \cos\varphi\right] = 9800$$

$$\longrightarrow T_2 = \frac{9800}{\left[\left(\frac{\sin 30}{\sin 20}\right)\cos 20 + \cos 30\right]} \simeq 4376 \text{ N}$$

$$T_1 = T_2 \frac{\sin 30}{\sin 20} \simeq 6397 \text{ N}$$

$\Longrightarrow$ $\boxed{T_1 = 6397 \text{ N} \quad T_2 = 4376 \text{ N}}$

3. (*35 points*) Consider the scanning part of a the drive motor and spinning disk of a confocal microscope. Suppose the disk has radius R and is initially at rest. It then speeds up with angular acceleration α.

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a. Determine an expression for the tangential velocity after the disk has rotated through an angle $\Delta\phi$.

2 We know: disk radius (R), const. angular accel. (α) and initially at rest ($\omega_0 = 0$)

Angular velocity: $\omega = \int \alpha\, dt = \alpha t + \omega_0 = \alpha t$

let this = 0 for simplicity

Ang. position: $\theta = \int \omega\, dt = \int \alpha t\, dt = \frac{1}{2}\alpha t^2 + \theta_0 = \Delta\phi$

So $\Delta\phi = \frac{1}{2}\alpha t^2 \longrightarrow t = \sqrt{\frac{2\Delta\phi}{\alpha}}$

Now $\omega = \alpha t = \alpha\sqrt{\frac{2\Delta\phi}{\alpha}} = \sqrt{2\alpha\Delta\phi}$

$\omega = \frac{V}{R} \longrightarrow \boxed{V = R\sqrt{2\alpha\Delta\phi}}$

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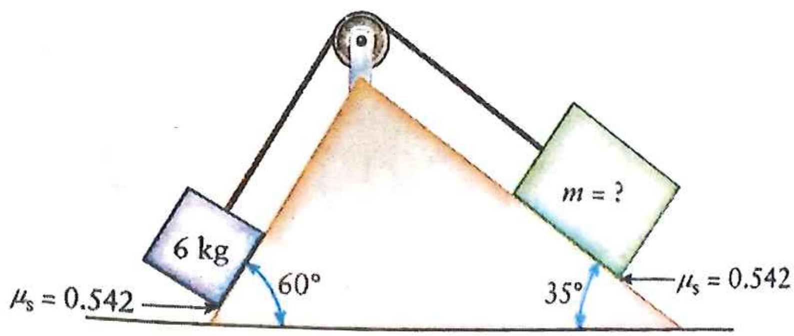
b. Similarly, determine an expression (in terms of $\Delta\phi$ and any other relevant quantities) for the centripetal acceleration.

$$\boxed{a = \frac{V^2}{R} = 2R\alpha\Delta\phi}$$

Extra Credit (10 Points):

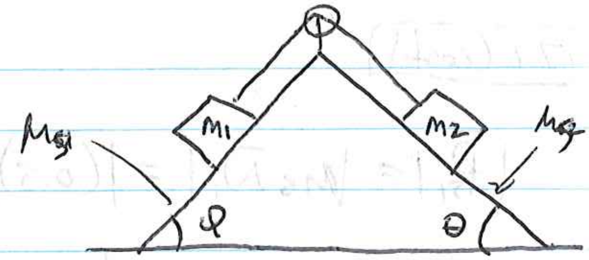
Two blocks are connected to each other by a massless string over a frictionless pulley. The mass of the block on the left incline ($\equiv m_L$) is 6.00 kg. Assuming the coefficient of static friction μ_S equals 0.542 for all surfaces, find the range of values of the mass of the block on the right incline ($\equiv m_R$) so that the system is in equilibrium.



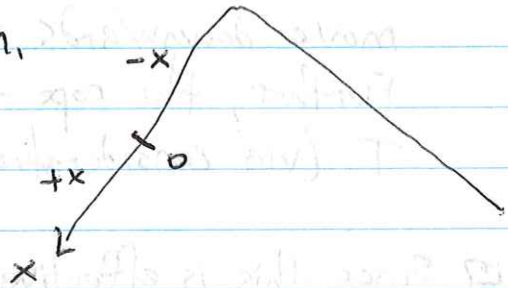
See
Attached

K+S 82

$m_1 = 6.00\,kg$, $\mu_s = 0.542$
$\varphi = 60°$, $\theta = 35°$

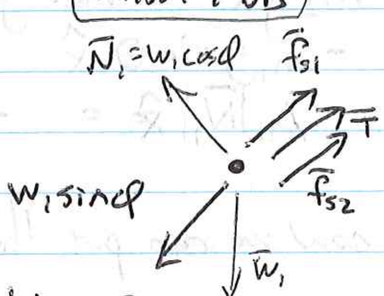


- Need to find range of m_2 vals. such that the system is in equilibrium (i.e. all net forces on each block are zero)
- To do this, let's focus on m_1 under two conditions: when m_2 is just about too small such that m_1 slides downwards (call that m_{2L}) and when m_2 is just about too large such that m_1 starts getting pulled upwards ($\equiv m_{2H}$). Further, since this is a 1-D problem like before, we'll adapt the coord system to the right and drop the explicit vector notation (i.e. will just down $\hat{x}$ vector components and the sign takes care of direction)



m_{2L} Here, static frictional forces will act along $-\hat{x}$, along w/ the tension (due to the weight of m_{2L})

Mass 1 FBD



$g = 9.8\ m/s^2$

$W_1 = m_1 g$

$W_{2L} = m_{2L} g$

$$\Sigma F_{m_1} = W_1 \sin\varphi - \mu_s W_1 \cos\varphi - W_{2L} \sin\theta - \mu_s W_{2L} \cos\theta = 0$$

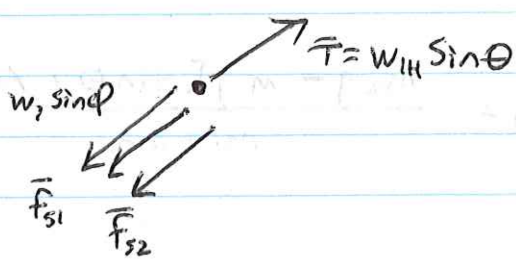
Zero because we are just at the point where the block exp. no net force.

$$\rightsquigarrow m_1 g \left[\sin\varphi - \mu_s \cos\varphi\right] = m_{2L} g \left[\sin\theta + \mu_s \cos\theta\right]$$

$$\Rightarrow \boxed{m_{2L} = m_1 \frac{\sin\varphi - \mu_s \cos\varphi}{\sin\theta + \mu_s \cos\theta}}$$

m_{2H} Now since the block (m_1) would move up if $m_2 > m_{2H}$, the frictional forces acts along $+\hat{x}$

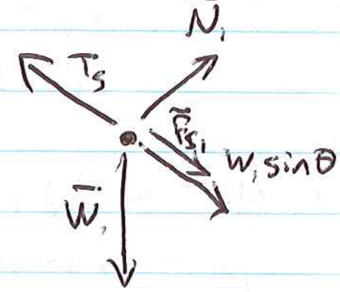
Mass 1 FBD



(Cont)

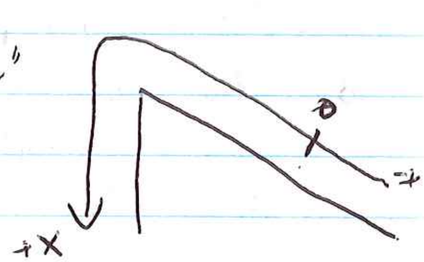
79 (cont)

$|\bar{f}_{s1}| = |\mu_s \bar{N}_1| = |(0.5)(1.0)(-9.8)\sin 30| \simeq 4.9\,N$



→ This tells us that m_2 is heavy enough to overcome the static friction acting on m_1 (because $19.6 > 4.9\,N$) and thus m_2 will move downwards while M_1 moves upwards to the left. Further, the rope tension (T) is not T_s and determining T (via consideration of all net force will reveal a_1)

☐ Since this is effectively a 1-D problem, let's create a coord. system (x) so that all "vector" aspects are dealt w/ via a + or − sign



$\bar{W}_2 = m_2 g \hat{x}$ (where $g = 9.8\ m/s^2$)

$\bar{W}_{1\parallel} = -m_1 g \sin\theta\ \hat{x}$

$\bar{f}_{k1} = -\mu_k |\bar{N}_1| \hat{x} = -\mu_k m_1 g \cos\theta$

☐ Ok, so now we can put the pieces together:

net forces on m_2: $\bar{W}_2 - \bar{T} = m_2 a$

" on m_1: $T - m_1 g \sin\theta - \mu_k m_1 g \cos\theta = m_1 a$

both blocks will have the same acceleration

→ Now add both eqns. together to cancel out T (and all are ref. re $\hat{x}$)

$a(m_1 + m_2) = W_2 - m_1 g \sin\theta - \mu_k m_1 g \cos\theta$

$$\Rightarrow a = \frac{m_2 g - m_1 g[\sin\theta + \mu_k \cos\theta]}{m_1 + m_2} = \frac{(2.0)(9.8) - 9.8[\sin 30 + (0.4)\cos 30]}{3}$$

$\boxed{\simeq 3.8\ m/s^2}$

K+S 82 (cont)

$$\Sigma F_{m_1} = W_1 \sin\varphi + \mu_s W_1 \cos\varphi - W_{1H}\sin\theta + \mu_s W_{1H}\cos\theta = 0$$

$$W_{2H} = m_{2H}g \rightarrow W_1(\sin\varphi + \mu_s\cos\varphi) = m_{2H}g(\sin\theta - \mu_s\cos\theta)$$

$$\Longrightarrow \boxed{m_{2H} = m_1 \frac{\sin\varphi + \mu_s\cos\varphi}{\sin\theta - \mu_s\cos\theta}}$$

So $m_2 \in [m_{2L}, m_{2H}]$ (i.e. $m_2 = m_{2L}$ through m_{2H}) would represent the mass range over which no movement takes place. We can plug in numerical values at this point

$$m_{2L} = (6.00)\frac{\sin 60 - 0.542\cdot\cos 60}{\sin 35 + 0.542\cdot\cos 35} \simeq 3.51 \text{ kg}$$

$$m_{2H} = (6.00)\frac{\sin 60 + 0.542\cos 60}{\sin 35 - 0.542\cos 35} \simeq 52.6 \text{ kg}$$

$\Longrightarrow$ Relevant range of m_2 masses here is $\boxed{m_2 \in [3.51, 52.6] \text{ kg}}$