$$\int f(\mathrm{math})\, g(\mathrm{biology})\, d(\mathrm{pedagogy})$$

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Math Instruction Colloquium

Educational Dilemma?

"Dramatic advances in biological understanding, coupled with equally dramatic advances in experimental techniques and computational analyses, are transforming the science of biology. ... Even though most biology students take several years of prerequisite courses in mathematics and the physical sciences, these students have too little education and experience in quantitative thinking and computation to prepare them to participate in the new world of quantitative biology. At the same time, advanced physical science students who become interested in biological phenomena can find it surprisingly difficult to master the complex and apparently unconnected information that is the working knowledge of every biologist."

Bialek & Botstein (2004)

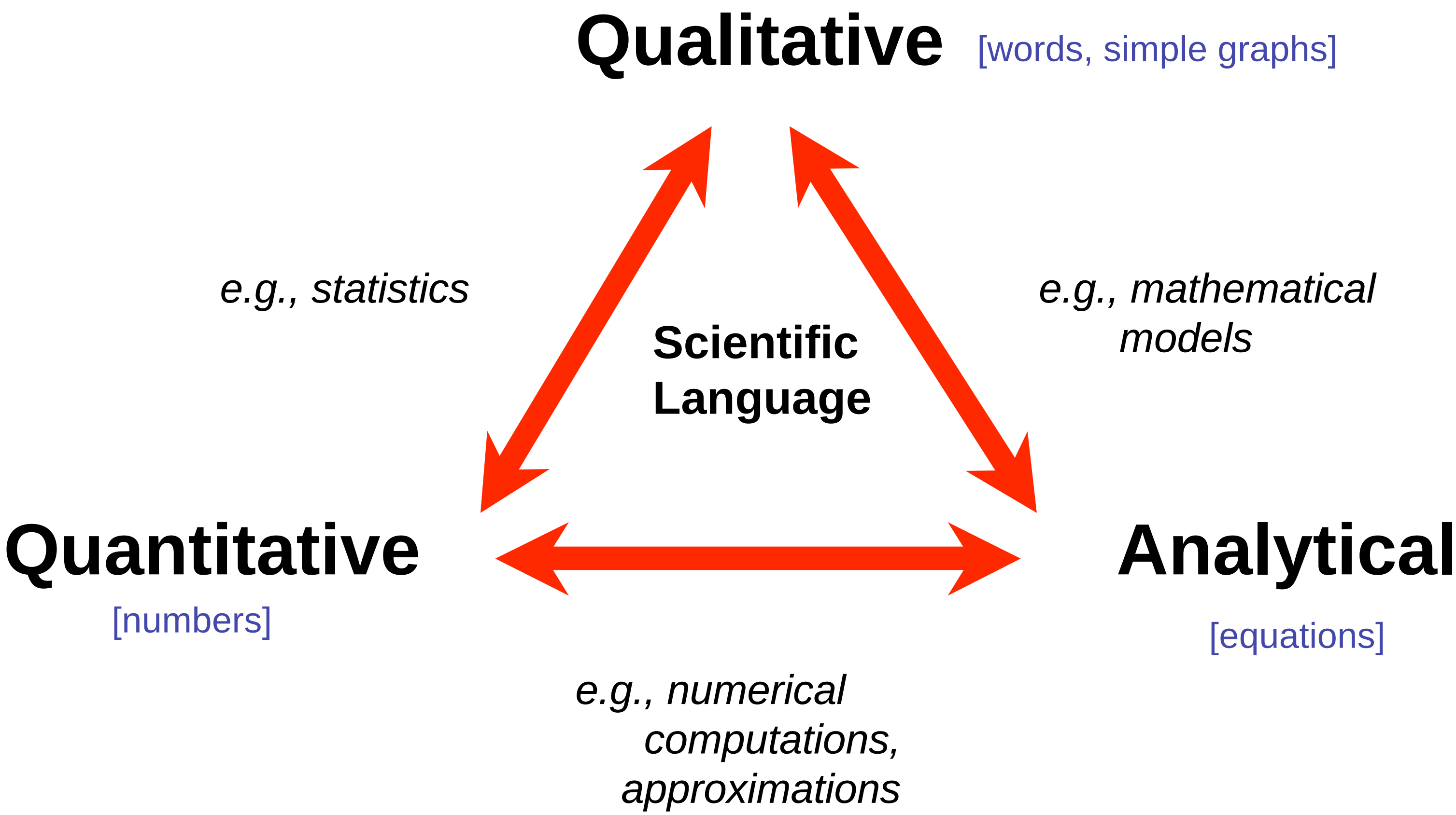
Bio-Math Committee (Funding: VIGRE, HHMI)

MISSION: The primary purpose of the Bio-Math Committee is to advise the Mathematics Department at the University of Arizona in the design of novel biology-oriented content for three of its courses that are recommended for students who plan majors in one of the College of Science Biology Departments.

- Calculus/Differential Equations (Math 250 A&B)
- *{Calclus-based}* Statistics (Math 363)

Other Players:
- Integrated Science Program, University of Arizona (aprr.web.arizona.edu/data/084/A6zISxxzBSxzxxx.html)
- Bio 2010 (National Research Council)
- Math and Bio 2010 (Mathematical Association of America)
- iPlant Collaborative (National Science Foundation)

Preliminary Set of Definitions:



Preliminary Assessment

What weak points can we identify early on? How can we use that information to shape course development?

- Pre-test designed to give before/after 250 A

Math 250A
Draft Pre-Test

1. Let $f(t) = \dfrac{Ce^t}{1 + Ce^t}$, where $C \neq 0$.

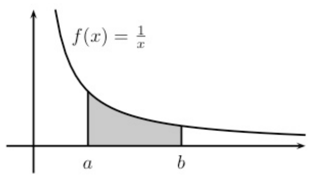
   (a) Find $\lim_{t \to \infty} f(t)$.

   (b) For what value of t is $f(t) = \frac{1}{2}$?

   (c) How does changing the value of C affect the rate of convergence in part (a)?

2. Which is bigger, $\dfrac{1}{\sqrt{\frac{5}{3}}}$ or $\dfrac{1}{\sqrt{\frac{5}{10}}}$?

3. Give possible values of a and b so that the area of the shaded region is 1.



4. Consider a population of cells with a doubling time of about an hour. Suppose you start with one cell and watch the population grow over time.

   (a) Which of the following expressions best describes the number of cells N after p hours?

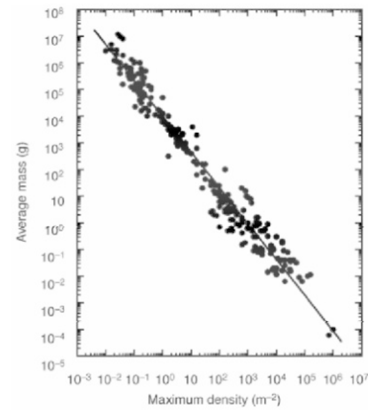
   i. $N = 2p$
   ii. $N = \log(p^2)$
   iii. $N = 2^p$
   iv. $N = e^{2p}$

   (b) What is the order of magnitude of the number of cells after approximately 10 hours?

   i. 10
   ii. 100
   iii. 1000
   iv. 10000

5. The graph below illustrates a relationship between the average mass M of plants and their maximum density D in their natural habitats[1]. Which of the statements below best describes the relationship between M and D?

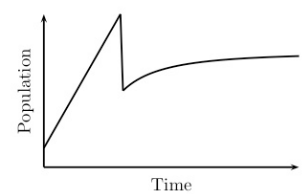
   i. If D is multiplied by 100, then M is multiplied by 1000.
   ii. If D is multiplied by 100, then M is divided by 1000.
   iii. If D is multiplied by 1000, then M is multiplied by 100.
   iv. If D is multiplied by 1000, then M is divided by 100.



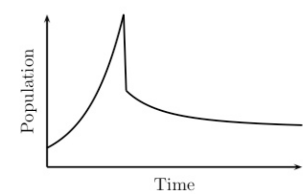
6. Which of the graphs below best illustrates the following scenario?

   A population first grows exponentially, and then a catastrophic event occurs and half of the population dies. After that, the population continues to grow, but much more slowly, and eventually saturates.

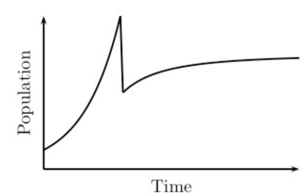
i.



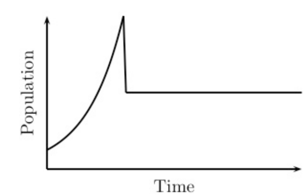
iii.



ii.



iv.



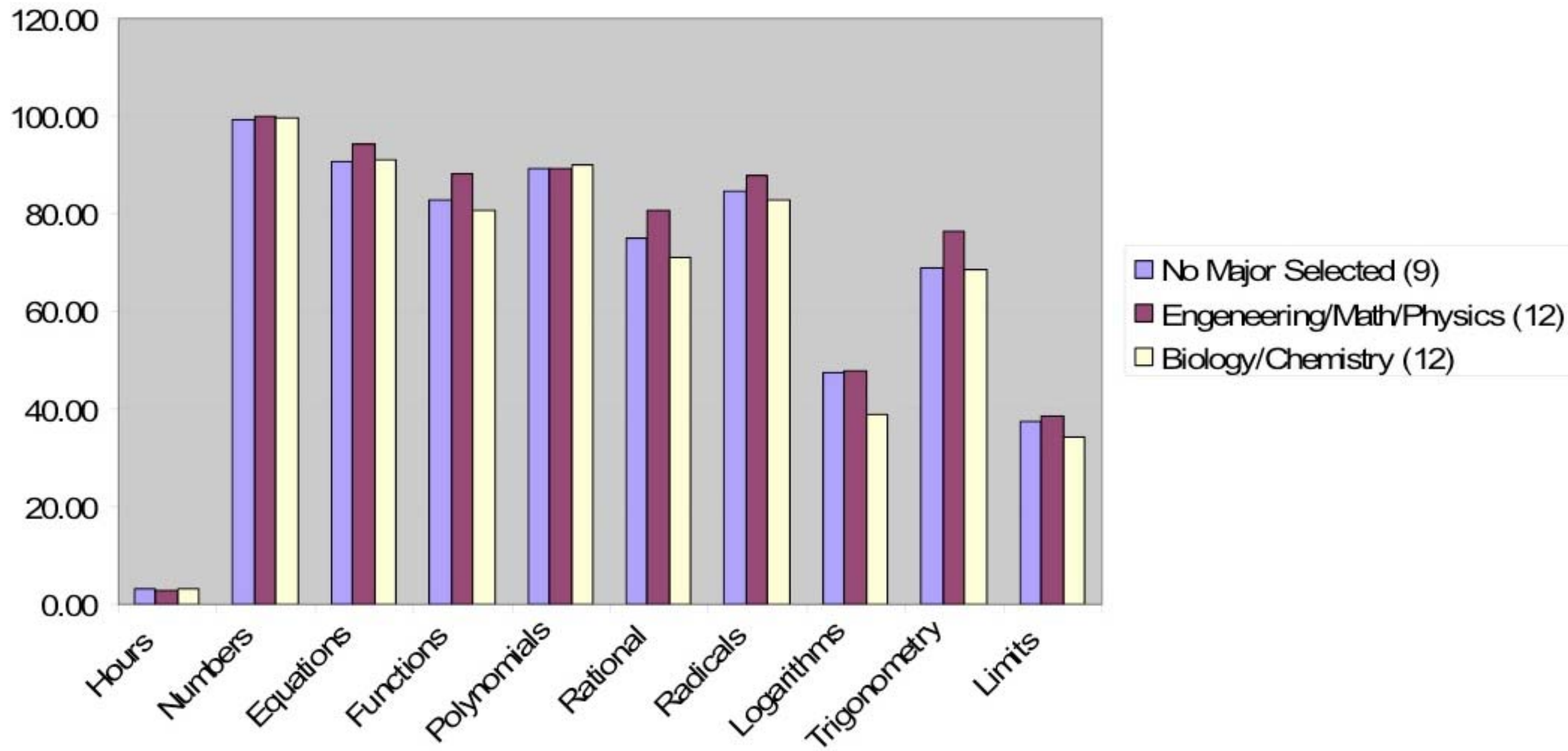
[1] B.J. Enquist et al., *Nature* **395** (1998), pp. 163-165.

Preliminary Assessment

-Pre-test designed to give before/after 250 A

- ALEKS

ALEKS Scores



(weak points: limits, logarithms)

⇒ ***No distinct advantage for initial major choice in scores***

Preliminary Assessment

- Pre-test designed to give before/after 250 A

- ALEKS (weak points: limits, logarithms)

- Interview: Practical application of mathematical knowledge to a biological problem

Student Video

Major Difficulties:

- What does *rate* actually mean? $$S + E \overset{k_{\pm 1}}{\leftrightharpoons} C$$

- Law of Mass Action $$\frac{ds}{dt} = -k_1 se + k_{-1} c$$

Challenge 1: *Educators*

Differences in perception Re: *mathematics*

math faculty ⟷ life sciences faculty

"the science that draws necessary conclusions"
- B. Peirce

'quantitative literacy'
(i.e., a tool or language)

Need to be '*mindful of cultural differences across departments*'

- HHMI external review committee

<u>Challenge 1:</u> *Strategies for Educators*

1. Incorporate more biological content into math courses

   (via a broad range of contextual and realizable applications)

Mid-modiolar section of a guinea pig cochlea stained with hematoxylin and eosin. Note the absolute and relative differences in dimensions of structures in different cochlear turns. This calibration bar is 200 microns long.

<u>Ex.1 (Integral Calculus)</u>

Estimate the volume of the cochlear duct (or the endolymphatic space, labeled SM). Clearly state any assumptions made.

Challenge 1: *Strategies for Educators*

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   (via a broad range of contextual and realizable applications)

$$S + E \underset{}{\overset{k_{\pm 1}}{\leftrightharpoons}} SE \xrightarrow{k_2} P + E$$

Ex.2 (Differential Equations)

Using the Law of Mass Action, derive the Michaelis-Menten Law:

$$s' = -\frac{v_{\mathrm{max}}\, s}{K_{\mathrm{m}} + s}$$

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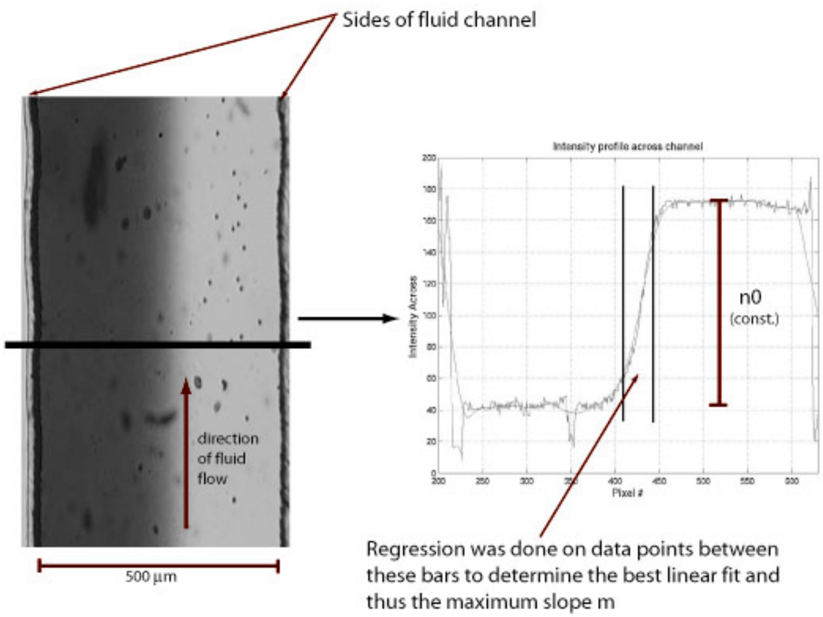

Ex.3 (multiple subjects)

Develop a project with a laboratory component w/ data collection to provide motivation, [e.g., microfluidics to measure diffusion]

http://ocw.mit.edu/OcwWeb/Electrical-Engineering-and-Computer-Science/6-021JFall-2004/CourseHome/index.htm

Challenge 1: *Strategies for Educators*

1. Incorporate more biological content into math courses
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2. Develop effective mathematics *refresher* sessions for life sciences faculty

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2. Develop effective mathematics *refresher* sessions for life sciences faculty

3. Consider how mathematical content can be integrated into higher division/graduate biology courses

PHYSIOLOGY 472/572, FALL 2008

Schedule - ***Subject to change***

#	Date	Day	Topic
1	Aug. 26	Tu	Introduction, diffusion in one dimension, partial derivatives.
2	Aug. 28	Th	Conservation of mass, diffusion equation (1D) - MatLab exercise
3	Sep. 2	Tu	Membrane transport: passive, facilitated, Michaelis-Menten kinetics
4	Sep. 4	Th	Membrane transport: active, ion channels
5	Sep. 9	Tu	Differential equations, wave equations
6	Sep. 11	Th	Nerve impulse conduction: Hodgkin-Huxley model
7	Sep. 16	Tu	Nerve impulse conduction: Hodgkin-Huxley model - MatLab exercise
8	Sep. 18	Th	Diffusion in 3D, gradient, divergence, Laplace operators
9	Sep. 23	Tu	Oxygen transport: diffusion, Krogh cylinder model
10	Sep. 25	Th	Oxygen transport: red blood cells, hemoglobin, convective transport
11	Sep. 30	Tu	Continuum mechanics, fluids and solids, stress and strain
12	Oct. 2	Th	Fluid mechanics concepts
13	Oct. 7	Tu	Flow in tubes, Poiseuille's law
14	Oct. 9	Th	Circulatory system, propagation of the arterial pulse
15	**Oct. 14**	Tu	**Mid-term exam**
16	Oct. 16	Th	Matrices, linear algebra, eigenvalues
17	Oct. 21	Tu	Compartmental models: transport, pharmacokinetics, impulse response
18	Oct. 23	Th	Compartmental models: circulatory system
19	Oct. 28	Tu	Complex numbers, damped harmonic oscillator - MatLab exercise
20	Oct. 30	Th	The middle ear, traveling waves, the inner ear
21	Nov. 4	Tu	Population dynamics: exponential and logistic growth
22	Nov. 6	Th	Population dynamics: coupled linear systems
	Nov. 11	Tu	No class (Veterans Day)
23	Nov. 13	Th	Population dynamics: nonlinear dynamics, limit cycles, chaos
24	Nov. 18	Tu	Infectious diseases
25	Nov. 20	Th	Infectious diseases: SIR models - MatLab exercise
26	Nov. 25	Tu	Modeling techniques: regression
	Nov. 27	Th	No class (Thanksgiving)
2728	Dec. 2	Tu	Modeling techniques: parameter identification - MatLab exercise
	Dec. 4	Th	Student presentations
29	Dec. 9	Tu	Student presentations
30	Dec. 11	Th	Review session.
	Dec. 16	Tu	**8am - 10 am. Final Exam**

3. Consider how mathematical content can be integrated into higher division/graduate biology courses

- Develop *modules* for existing courses (Bio 2010)

- Explore new roles for technology in classroom

(e.g., students writing code and running simulations in class)

Challenge 1: *Strategies for Educators*

1. Incorporate more biological content into math courses
   (via a broad range of contextual and realizable applications)

2. Develop effective mathematics *refresher* sessions for life sciences faculty

3. Consider how mathematical content can be integrated into higher division/graduate biology courses

4. Advising (e.g., practical relevance of Math 363 vs. 223)

Challenge 1: Caveat

Differences in perception Re: *biology*

life sciences faculty ⟷ math faculty

an evolutionary science
(e.g., first ask *how*, then *why*)

"science fiction"

<u>Challenge 2:</u> *Students*

Expectations coming into a mathematics course:

learning/doing math ⟷ (vs) developing *analytical* & *quantitative* reasoning skills

'taught to the test'

critical thinking

<u>Challenge 2:</u> *Strategies for Students* (& Instructors!)

1. *Critical transitions*

   (e.g., high school to university, undergraduate to graduate school)

2. Ask yourself: What do you want to gain from a math class?

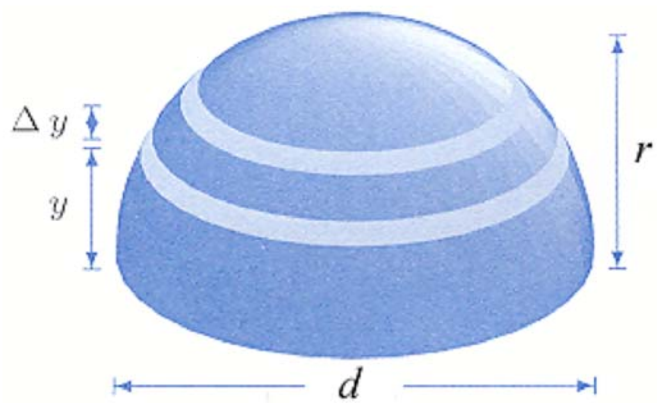
   $\Rightarrow$ <u>*critical thinking* skills</u>

   - Unlike biology, ask <u>both</u> *how* and *why* simultaneously

   - To Math Instructors: Don't dilute the mathematical formalisms

Challenge 2: *Strategies for Students*

Examples To Emphasize Critical Thinking [How & Why]



Formula vs. Concept of Setting Up Slices

Challenge 2: *Strategies for Students*

Examples To Emphasize Critical Thinking [How vs. Why]

$$\int_{-1}^{1} \sqrt{1-\delta^2}\, d\delta$$

Means to solve this integral?

1. Table of integrals
2. Numerically (e.g., calculator)
3. Trigonometric substitution and integration by parts

Challenge 2: *Strategies for Students*

Examples To Emphasize Critical Thinking [How vs. Why]

- 'Word problems'
(and associated jargon)

- *Modeling*

- Dimensional Analysis

Ex. (Ch.0 in Cushing's book)

Suppose a population has a constant per unit death rate (d>0) and a per unit birth rate that is proportional to the population concentration x (with a constant of proportionality denoted by a>0). Using the balance law, write a differential equation for the population concentration $x(t)$.

$$\frac{dx}{dt} = x(ax - d)$$

Summary

Challenges for Educators

- Perceptions Re: *mathematics*
- Cultural differences across departments
- Integrating math content into life sciences courses & vice versa

Challenges for Students

- Expectations coming into/out of math courses $\Rightarrow$ critical thinking skills
- *Critical* transitions

Question to Audience:

How might we adapt a talk of this nature to present to instructors in other departments?

Need to be '*mindful of cultural differences across departments*'

- HHMI external review committee

Bio-Math Committee (past & present)

Carol Bender, Chris Bergevin, Phil Callahan,
Jim Cushing, Nick Ercolani, Nancy Horton,
Joceline Lega, Osamu Miyashita,
Erika Offerdahl, Chad Park, Roy Parker,
Nick Rogers, Koen Visscher, Bruce Walsh,
Joe Watkins, Vicki Wysocki