PHYS 1010 1^{st} lecture

Kinematics = Description of motion

Example in 2D (2 dimensions):

watch how an object slides on a perfect frictionless table. It was set in motion by a tap.

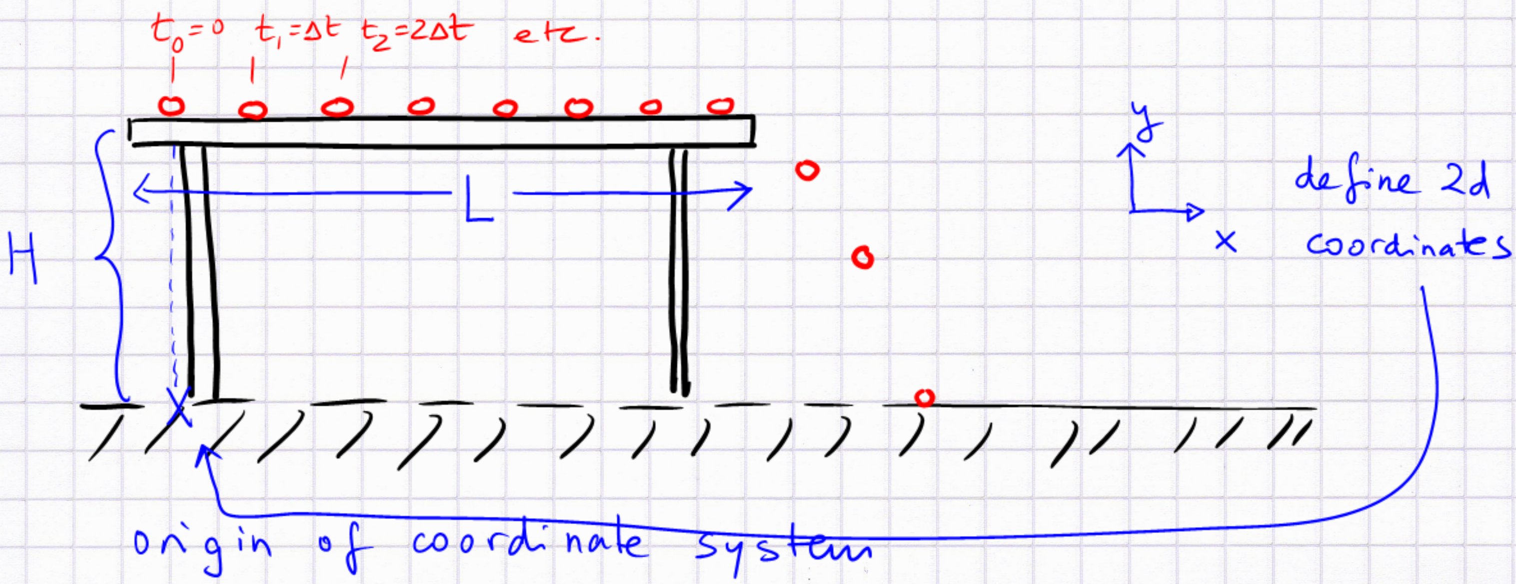
It follows uniform (constant-velocity) motion while on the table (gravity is cancelled by a normal force)

→ as it reaches the edge - gravity kicks in

x - motion continues y-motion = free fall

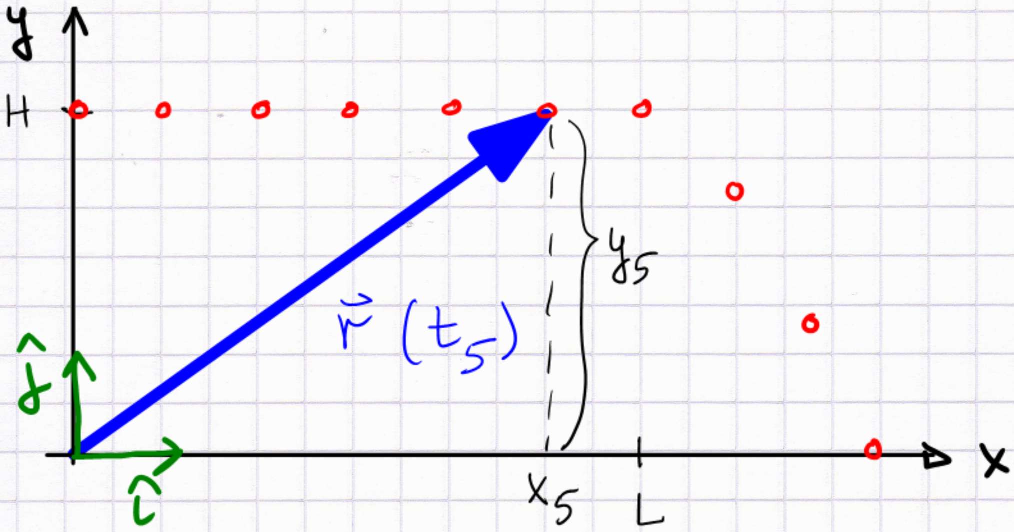
Describe the motion as a sequence of snapshots at times $t_n = n \cdot \Delta t$

The motion is sampled with a frequency (frame rate) of $f = \frac{1}{\Delta t}$



Now formulate a mathematical description

2dim. position vector for discrete times t_n, $n=0,1,2,\ldots$



unit vectors

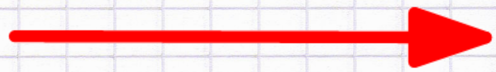
$t_5 = 5\,\Delta t$

$$\vec{r}\,(t_5) = x_5\,\hat{i} + y_5\,\hat{j}$$

$x_5 = x(t_5)$

$y_5 = y(t_5)$

Note: $x_5\,\hat{i}$ is the vector



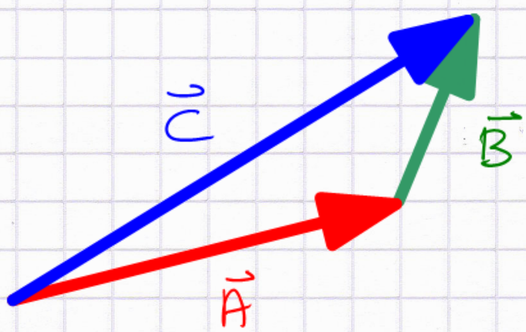
($\hat{i}$ stretched by a factor of x_5)

← $y_5\,\hat{j}$ is $\hat{j}$ stretched by a factor y_5

Vector addition:

graphically: move the tail of the 2nd vector to the tip of the first

resultant: tail of first → tip of second



$$\vec{C} = \vec{A} + \vec{B}$$

Algebraic vector addition:

$$\vec{A} = A_x \hat{\imath} + A_y \hat{\jmath} \qquad \vec{B} = B_x \hat{\imath} + B_y \hat{\jmath}$$

$$\vec{C} = (A_x + B_x)\,\hat{\imath} + (A_y + B_y)\,\hat{\jmath}$$

(add the components separately)

Keep in mind: $\vec{C} = \vec{A} + \vec{B}$

represents more than one scalar equation:

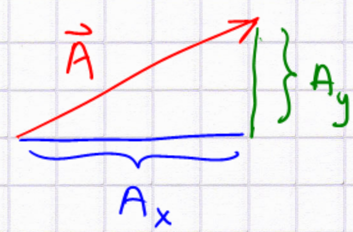
$$C_x = A_x + B_x \quad \text{and} \quad C_y = A_y + B_y$$

If the LHS of an eqn is a vector, the RHS also must be a vector

Magnitude of a vector: (geometry: its length)

$$A \equiv |\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

(from Pythagoras)



Length of $\vec{A}$ is A

DO NOT CONFUSE $\vec{A}$ and A !

Position vector $\vec{r}(t)$ for all times,

not just at sampling times $t_n = n\,\Delta t$

This is an abstraction. 300 yrs ago Newton, Leibniz and others came up with this powerful idea:

Rather than just looking at physically measurable times $n \cdot \Delta t$ where Δt is a timing interval

[seconds, milliseconds (10^{-3} s), microseconds (10^{-6} s), nanoseconds (10^{-9} s),

picoseconds (10^{-12} s), femtoseconds (10^{-15} s), attoseconds (10^{-18} s) ...]

femtoseconds: chemical reactions at molecular level

attoseconds: atomic "reaction" times

...: nuclear processes

frequencies: $Hz = \frac{1}{s}$, kHz, MHz, GHz, THz, PHz

kHz	MHz	GHz	THz	PHz
kilo	Mega	Giga	Tera	Peta
10^3	10^6	10^9	10^{12}	10^{15}

define the trajectory for continuous time $t \in \mathbb{R}$

($\Delta t = 0$ limit or infinite sampling rate):

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$$

$x(t)$ and $y(t)$ are real variables that are functions of a continuous variable. They are differentiable.