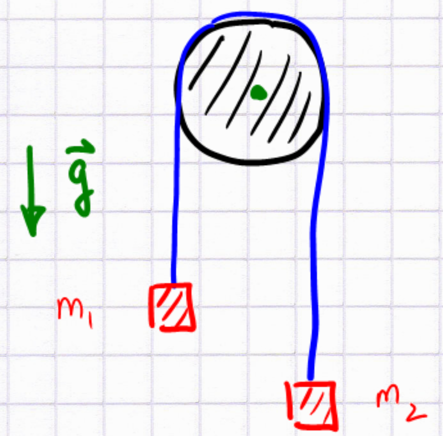
Inertia

Galileo observed, Newton formulated a first law of motion: objects (masses) which are force-free move with constant velocity (Newton's first law)

Example of force-free environment:

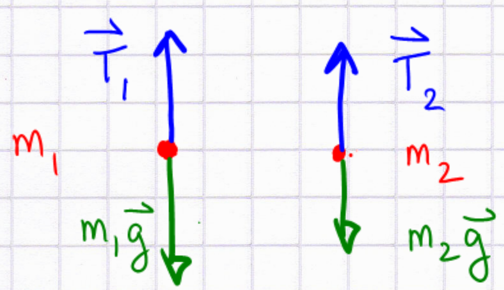
Atwood machine → idealize: massless pulley, frictionless massless, non-stretch rope (string)



For $m_1 = m_2$ the system is force-free: once set in motion: it moves with constant velocity

Now understand it with $m_1 > m_2$

A two-body problem, idealize masses as points



The tension force has the same magnitude $T_1 = T_2$

why? otherwise the string (rope) would stretch

NB: a string (rope) re-directs the tension force.

If $m_1 = m_2$, then $T_1 = m_1 g$ and $T_2 = m_1 g$,

and both masses are force-free. Once set into motion, they move with constant and equal speed.

Newton's 2^{nd} law : $m\vec{a} = \vec{F}_{net}$

Here m is the inert mass (specifying how an object to which we apply force resists motion)

- The same force applied to different objects causes different accelerations depending on the inert mass of the object. (hammer blow demonstration)

Apply this to the two masses in the Atwood machine:
$(m_1 > m_2)$

m_1 : $m_1\vec{a} = m_1\vec{g} + \vec{T}_1$ $(= \vec{F}_{net})$

Use $\downarrow = \hat{j}$ for m_1 (downward acceleration is positive,
$|T| = |\vec{T}_1| = T > 0$

① $m_1 a_1 = m_1 g - T$ $g = 9.8\ m/s^2$

For m_2 we use $\uparrow \hat{j}$. This will allow us to set $\boxed{a_1 = a_2}$ (no stretch in the string)
$\underbrace{a_1 = a_2}_{= a}$

② $m_2 a_2 = T - m_2 g$

①' $m_1 a = m_1 g - T$

$T = m_1(g - a)$

②' $m_2 a = T - m_2 g$

$T = m_2(g + a)$

Combine 1' and 2'

$$m_1 (g - a) = m_2 (g + a)$$ and solve for a

$$m_1 g - m_1 a = m_2 g + m_2 a$$

$$(m_1 + m_2) a = (m_1 - m_2) g$$

$$\boxed{a = \frac{m_1 - m_2}{m_1 + m_2} g}$$

Observations : 1) $m_1 = m_2 \quad \therefore \quad a = 0$

(constant velocity)

2) $m_2 = m, \quad m_1 = m + \Delta m$:

$$a = \frac{\Delta m}{2m + \Delta m} g$$

Small acceleration → easily observed (when $\Delta m \ll m$)

equal-mass + glider demo

Re-write : $\underbrace{(2m + \Delta m) a}_{\text{acceleration of an object with inertia } (2m+\Delta m)} = \underbrace{\Delta m\, g}_{\text{small gravitational net force}}$

The small mass difference Δm yields a net gravitational force which is used to accelerate a big inert mass, namely $(2m + \Delta m)$

We accelerate the pulley "for free" - its mass is negligible.