Kinetic Energy

When a force acts over some displacement Δx it can deliver work. When $W > 0$: $v_f > v_i$, and when $W < 0$: $v_f < v_i$

For a constant force, $F = ma = \text{const}$, we know:

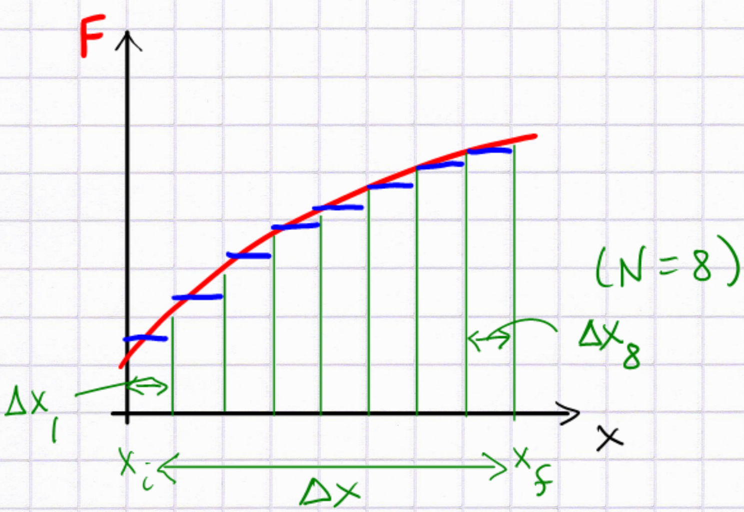
$$v_f^2 = v_i^2 + 2a\Delta x = v_i^2 + 2\frac{F}{m}\Delta x = v_i^2 + \frac{2}{m}W$$

Turn this into an equation that shows what the work does:

$$W = \underbrace{\frac{m}{2}v_f^2}_{\text{KE after}} - \underbrace{\frac{m}{2}v_i^2}_{\text{KE before}} \qquad KE \equiv \frac{1}{2}mv^2$$

What if we have a variable force $F(x)$?

$\rightarrow$ break a large displacement Δx into a sequence of short displacements $\Delta x = \sum_{i=1}^{N} \Delta x_i$



replace $F(x)$ by an appropriate $F(x_i)$ for each small box and calculate the incremental KE changes

assume $\Delta x_1 = \Delta x_2 = \ldots = \frac{\Delta x}{N}$

$$W = \sum_{i=1}^{8} W_i = \sum_{i=1}^{8} F(x_i)\,\frac{\Delta x}{8} = \sum_{i=1}^{8}\left(KE_{i+1} - KE_i\right)$$

$$= (\cancel{KE_2} - KE_1) + (KE_3 - \cancel{KE_2}) + \ldots + (KE_8 - KE_7)$$

etc.

$$= KE_8 - KE_1 \implies KE_f - KE_i$$

Now ask: does the final answer depend on N?

Calculus: $\lim_{N \to \infty}$ defines the area under the curve

$$W = \int_{x_i}^{x_f} F(x)\,dx$$

Computer arithmetic: find an N large enough such that

$\left(\Delta x_i = \frac{\Delta x}{N} \text{ small enough} \right)$

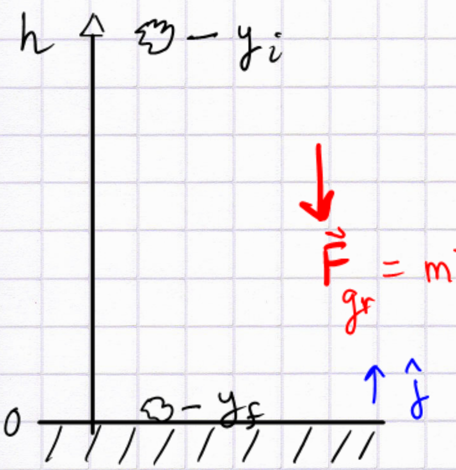
$$W = \sum_{i=1}^{N} F(x_i) \frac{\Delta x}{N}$$

is stable to some number of significant digits

Note how the simpler constant-force example follows:

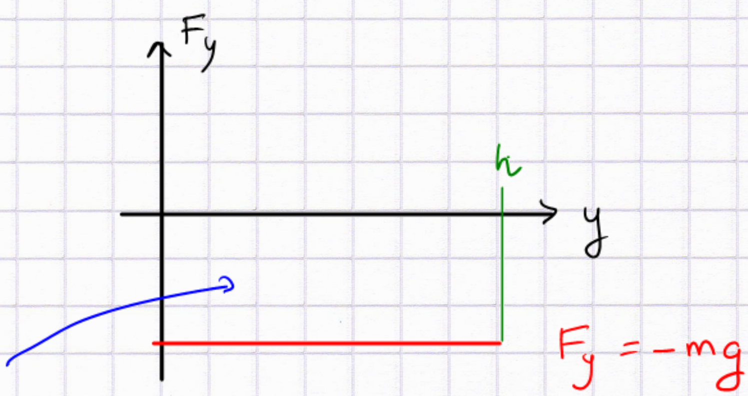
$$W = \frac{F}{N} \Delta x \underbrace{\left(\sum_{i=1}^{N} 1 \right)}_{=N} = F \Delta x$$

falling rock



Note: $\Delta y < 0$ and $F_y < 0$

$\Rightarrow W > 0$



area taken from $0 \rightarrow h$ is negative, but we need the area for going $h \rightarrow 0$ $(\Delta y < 0) \Rightarrow$ positive

in Calculus:

$$W = (-mg)\int_h^0 dy = -(-mg)\int_0^h dy = +mgh$$

This work done by gravity causes change in KE!