**York University**
**ISCI Integrated Assignment I: Rocket Science**
**Due Oct. 21, 2016 at 5 PM**

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# 1 Overview

This assignment focuses around the theme of *rocket science*. That is, the basic concepts relevant to sending a rocket to, oh say, Mars. Questions will deal with basic kinematics, conservation of momentum as the basic principle of rocket thrust, deriving the "rocket equation", and aspects of gaseous ignition inside a rocket engine. Also, there will be computational components that will require you to code in Matlab. You will be asked several questions, all of which are tied together yet approach the basic idea from different viewpoints. You will need to turn in a typeset report* that addresses the questions posed below. Each team will submit a single report, uploaded as a single pdf. One member of each team should upload one file on Moodle (Week 4: "Integrated Assignment 1: Rocket Science") by Friday October 21 at 5PM. Any late submissions must be emailed to `isci@yorku.ca`. Make sure that any relevant figures, references, Matlab code, etc... are included in the body of the report.

## 1.1 Grading

This is a *team project*. That is, you are expected to work together, equally. As such, your final grade will stem from a weighted distribution of two components:

- Team grade – You will be assigned a grade based upon the final report handed in. All members of the group will receive the same grade for this component.
- Individual grade – The *equally* part matters. Thus, your fellow team members will quantitatively assess your participation (and you in turn will do so for them). Attached to the end of this document is a form you will need to fill out. Hand this in on the day your report is due.

## 1.2 Lateness

1. Unfortunately, some deadlines in the *real world* are quite harsh and allow no room for lateness. Given such, this assignment will be subject to a lateness penalty.
2. The grade penalty for a late assignment will be multiplied by a *lateness factor*

$$L = 0.3e^{-t/4} + 0.7e^{-t/72}$$

where $t$ is the number of hours late. See figure for the lateness factor plotted as a function of time. Notice that the maximum grade for a report that is more than ONE DAY LATE is less than 50%.

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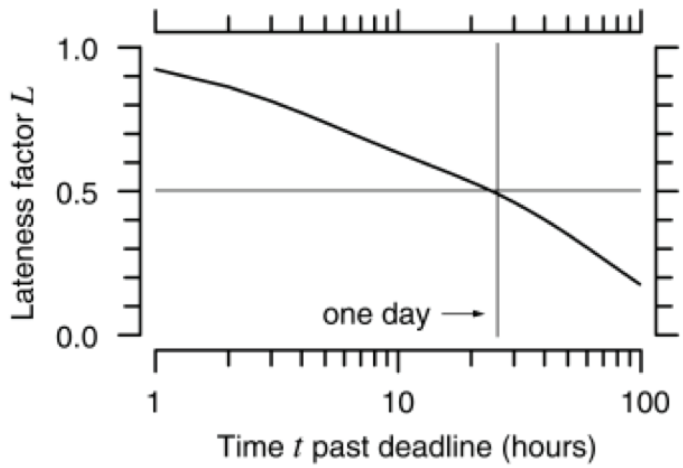
*For those seeking extracurricular reward (i.e., a small degree of extra credit here), you are encouraged to learn LaTeXfor writing your report. A bit of a steep learning curve, but once you start using it, you'll likely never go back to Microsoft Word!




# 2 Problems

1. (6 pts) An important consideration in launching a rocket is *escaping* the gravitational potential energy of earth.

   (a) Briefly explain what gravitational potential energy is, and what the *escape speed* ($v_{esc}$) represents.

   (b) Consider a rocket that is launched vertically upward at $\sqrt{2}v_{esc}$ from planet Earth, whose mass is $M_E$ and approximate radius is $R_E$. Derive an expression for its speed as a function of the distance from the planet [i.e. $v(r)$]. [*Hint: Remember conservation of energy!.*]

   (c) Find numerical values for $M_E$ and $R_E$, and write a Matlab script to plot $v(r)$. Make sure to clearly comment your code, and that your plot is clearly labeled.

   (d) Like the above problem, Example 8.5 (pg.136) of your physics text (Wolfson) states "*A rocket is launched vertically upward at 3.1 km/s*". Briefly explain conceptually what is wrong with the basis of these problems.

   [*Hint: Concepts from Wolfson ch.8 will likely be useful here.*]

2. (8 pts) Estimate the energy required to send a rocket of mass $M_o$ to Mars. Clearly state assumptions made and outline all the relevant calculations in detail.

   [*Hint: This is a "Fermi-type" problem. There are many paths one could take and possible considerations to make, such as gravitational potential energy tied to Earth/Mars/Moon/Sun, engine efficiency, etc. Wolfson ch.8.4 will also be useful (e.g., Eqn.8.6 in particular).*]

3. (4 pts) Rockets move about via *thrust*. That is, say in outer space, there is nothing to "push against" so to induce an external force and thereby accelerate something else is required. So the relevant forces have to come from "inside" the system (and hence thrust). A fundamental principle underlying thrust is the *conservation of momentum* (COM).

(a) State the law of COM†. Identify several different interdisciplinary scenarios where COM plays an important role (e.g., kinesiology, car crashes, etc.).

(b) Explain conceptually why COM is relevant for rockets. For example, the momentum of what? Does a rocket's mass change with time? How is such related back to COM? Feel free to use equations as needed.

(c) Examine Figs.1a and 1b. Outline what parallels there are between the two different scenarios shown there. Additionally, what differences are there?

4. (12 pts) Consider a rocket in deep space (like that in Fig.1b), where there are negligible external forces such as gravity or drag. The rocket, with mass $M$, burns fuel at a constant rate $\mu$ until all the fuel is used up (call that the *burnout time*, or $t_B$). Assume that the velocity $v_{\rm rel}$ of the ejected gas particles relative to the rocket is constant.

   (a) Draw a free-body diagram and set up the equation of motion via Newton's 2nd law). Specifically, you should find that

   $$M\frac{d\mathbf{v}}{dt} = \mathbf{F}_T \tag{1}$$

   where $\mathbf{F}_T$ is the thrust force. How does $\mathbf{F}_T$ depend upon the prescribed variables?

   (b) Determine the change of the velocity [i.e., $v_{tB} - v_0$] of the rocket during the time interval $t \in [0, t_B]$ as a function of the change of the rocket's mass during that interval. Note that your answer does not have an explicit time dependence and velocity depends upon the *mass ratio* (a useful measure of time into the burn of a rocket).

   A common approach to solving this is to use integrals, but you should try to solve it without integrals. Use only your knowledge of derivatives and solutions to the (differential) equation of the form $x' = a + bx$ (which you saw earlier in the semester).

   [*Hint: Your answer should be the so-called* **rocket equation**.]‡

   (c) Comment briefly on how time factors into your answer to the last part. How does the thrust change as a function of time?

   (d) Rearrange your answer to the last part to express the rocket's mass as a function of its velocity.

   (e) A rocket, weighing 30000 lbs. before liftoff, is fired vertically upward. At burnout, it weighs 10000 lbs. Gases are exhausted at a rate of 10 slugs/s, at a velocity of 5000 ft/s (relative to the rocket). Assume both those two quantities are constant. What is the thrust force? Additionally, what is the rocket's speed and kinetic energy at $t_B$?

   (f) At what mass ratio is the kinetic energy ($T$) of the rocket, including fuel, maximal? Calculate the velocity, mass, kinetic energy of the rocket, and the time at which that occurs [assuming $v(0) = 0$].

   (g) In terms of mass ratios, is a rocket like a car? Why or why not?

† Mastering Physics has a useful module along these lines entitled "Momentum Module 10: Linear Momentum Dynamic Study Module"

‡ Kudos if you didn't Google this! But since you likely did, who was Konstantin Tsiolkovsky?

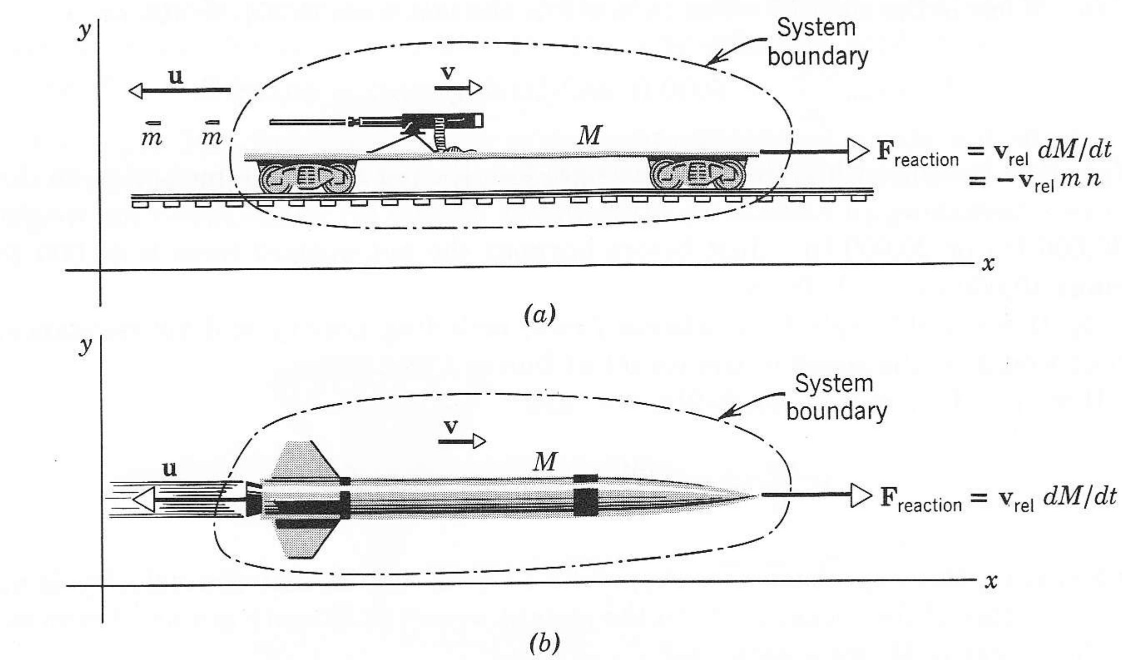



**Fig. 9–11** (*a*) Example 8. A machine gun is fixed to a car that rolls with negligible friction. The gun fires bullets of mass $m$ at a rate (number per unit time) $n$, the velocity of the bullets with respect to the gun being $\mathbf{u} - \mathbf{v}$. At the instant shown some bullets have already left the system. The velocities indicated for the car and the bullets are those that would be measured by an observer in a reference frame fixed to the rails as shown. The reaction force on the system is $\mathbf{F} = -mn\mathbf{v}_{\text{rel}} = (dM/dt)\mathbf{v}_{\text{rel}}$. (*b*) A rocket moves through space with negligible external forces. Gas particles are ejected from the exhaust, the particles having a velocity $\mathbf{u} - \mathbf{v}$ with respect to the rocket. The rate at which mass is expelled at the exhaust is $-dM/dt$. The reaction force on the rocket is $\mathbf{F} = (dM/dt)\mathbf{v}_{\text{rel}}$. The velocities indicated for the rocket and exhaust gases are relative to the ground.

Figure 1: From Resnick and Halliday (1966).

(h) Now consider the rocket in the *constant* gravity field near Earth, where the goal is to get off the surface. Draw the relevant free-body diagram. Set up the equation of motion (i.e., what modifications to Eqn.1 need to be made?). Which condition has to be satisfied to allow the rocket to take off?

[*Hint: Ch.7.8 of Hawkes (Physics for Scientists and Engineers: An Interactive Approach) might be helpful! Try heading over to Steacie!*]

5. (6 pts) This question builds off your derivation of the rocket equation in the last part and asks you to computationally explore several aspects via Matlab. Along with your answers, you should turn in any relevant code. Make sure it is concise and commented (so to make clear what is what).

   (a) Write a code that plots the velocity of the rocket as a function of the mass ratio for several different exhaust velocities ($v_{\rm rel}$).

   (b) Pick a set of parameters (e.g., $v_{\rm rel}$) and determine at what point the rocket's velocity exceeds that of the exhaust velocity. Comment (e.g., How does such depend upon $v_{\rm rel}$? Do the relevant values seem *special* in some way?). Additionally, how would such appear to an observer on the ground?

   [*Hint: A book entitled* **Rocket and Spacecraft Propulsion** *by Martin J.L. Turner might be helpful. You should be able to find a soft copy via York's subscription to SpringerLink (if you have trouble, ask a librarian in Steacie!).*]

6. (10 pts) The Rocketdyne $F_1$ is NASAs most powerful liquid fueled rocket engine. It was the engine that propelled the Saturn V rocket to the moon. The F1 burns a mixture of kerosene and liquid oxygen. Kerosene is a mixture of hydrocarbons, but it can be represented as primarily $C_{14}H_{30}$. The combustion temperature is 3545 K.

   The mixture is sprayed into the combustion chamber at a rate of 3521 kg/s (788 kg s$^{-1}$ of kerosene, 2733 kg s$^{-1}$ of liquid oxygen). Combustion occurs so rapidly that you can assume the gaseous products are produced instantaneously. Given the law of conservation of mass, $dm/dt$ must therefore be 3521 kg s$^{-1}$. The exhaust gases exit through a 1.23 m diameter nozzle at a desired exhaust velocity, $C$ (note the different variable name relative to previous problems), of 340 m s$^{-1}$.

   You can assume that the pressure at the nozzle is constant and equal to the pressure in the chamber. Thus, the exhaust gas is expanding through the nozzle under constant pressure, and hence doing work on the surroundings.

   (a) Write a balanced chemical equation for the combustion reaction at 3545 K. Dont forget to include states of all reactants and products.

   (b) What is the molar mass of the exhaust gas mixture?

   (c) Does the engine need oxygen in the (external) air to function?

   (d) Derive an equation expressing the pressure of the gas mixture in the combustion chamber to $C$, $T$, $A$ (nozzle cross-sectional area), $M$, and $m'$ (which is $dm/dt$). Assume the exhaust gas mixture behaves ideally (a very good assumption at such high temperature).

(e) Using the equation you derived along with the parameters given in this problem, calculate the pressure in the combustion chamber required to produce the desired exhaust velocity.

(f) Derive an equation that relates the power (time derivative of work) of the exhaust gases to $C$. Avoid using the symbol $P$ for power since it can be confused for pressure. Use the Greek letter $\phi$.

(g) Use your equation to calculate the power (in megawatts) of the $F_1$ rocket engine.

7. (5 pts) Watch video of a classic rocket launch (Apollo 8):
`https://www.youtube.com/watch?v=FzCsDVfPQqk`.

   (a) Describe from a interdisciplinary point-on-view the various phenomena you see throughout the video. Concisely describe several aspects that are or are not consistent with your preceding analysis (e.g., the twisting of the gases about the rocket around the 41 s point).

   (b) Comment on certain quotes (e.g., "The sideways shaking was unbelievable. The vibration was so intense, you couldn't see the instrument panel.") and how they are or are not consistent with assumptions made in the preceding problems. Put another way, comment upon the simplifying assumptions made throughout your analysis.

**Your Name:** ____________________________________________________ **Group #:**

Assess the contribution of each member in your group towards the completion of the project with a grade between 1 and 5.  Do not show your grading to the other members in your group.

Note that filling out this form (including the comments section) clearly and legibly is part of your grade as well. This form is due on the same day as your group project report.

1 = no effort, 5 = crucial and large amount of work done by the individual.

| Name (exclude yourself) | Discussion of ideas/concepts relevant to the assignment | Input into determining solutions to the problems | Writing up the report | Comments (specific things she/he contributed & did well, etc) |
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Your expected grade for the project (out of 100):