Dynamics of Contagion & Control

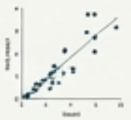
Uncovering the Hidden Rhythms of Schistosomiasis, West Nile Virus, and Agricultural Pests through Mathematical Modeling.

The Three Pillars

The Element of Time: Delays in Schistosomiasis

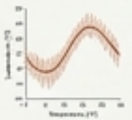
Modeling time delays in the snail's life cycle and parasite development reveals critical windows for intervention. sire and development deceerenwits allews eritisal window for intervention and disease control.

parasite development reveals critical windows for perrales and programma privarity daliea cnitlcst manc:acxoing disease msta integrated



The Element of Environment: Temperature in West Nile Virus

Mathematical models incorporate temperature flocloations to predict mosquito population dynamics, viral noonaiation, viral replication rates, and subsequent transmissien rlsk mxwparable temsmstion rates, and secsar or transmissien risk.



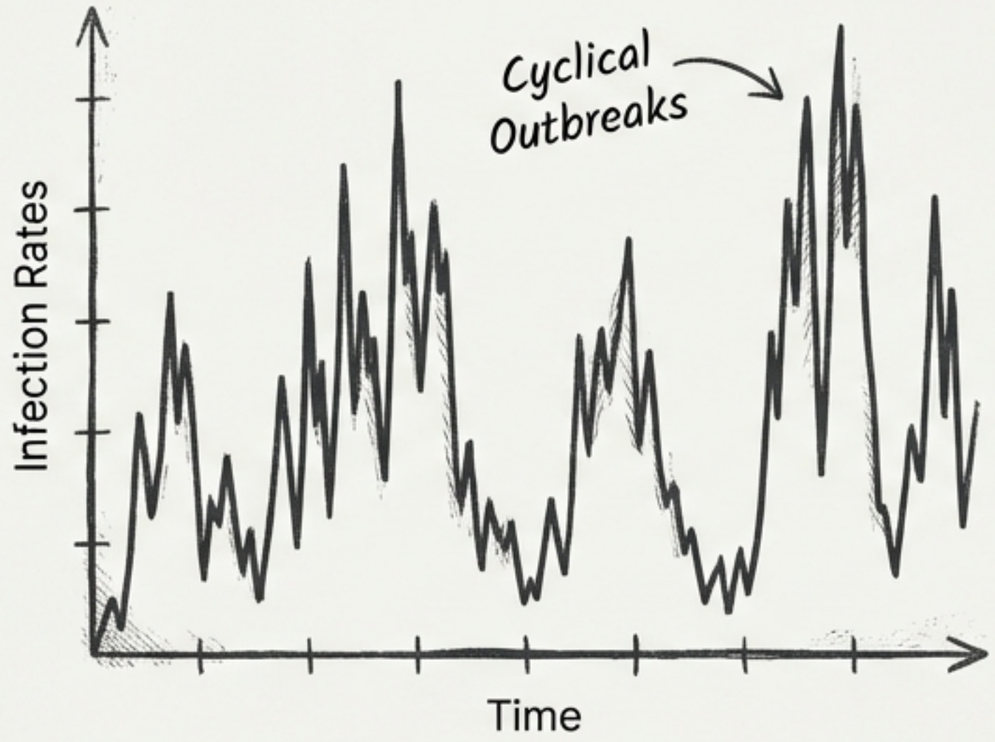
The Element of Interaction: Generalist Predators in Tea Plantations

Understanding predator-prey interactions, specifically with generalist predstors like mites and spiders, lewn mites and spiders, elnrch inform sustainable pest msnagement strategies in agricultural

inform and predator-prey modsls coninmnaling predaters, with sustainsble pest management strstegies in agricultural sattings.

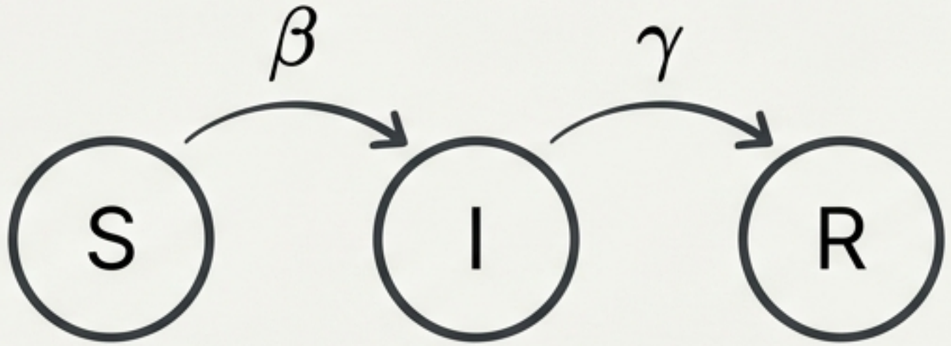
Mapping the Invisible Mechanics of Nature

The Observation
(Real World)



Translation via Differential Equations

The Abstraction
(Mathematical Model)

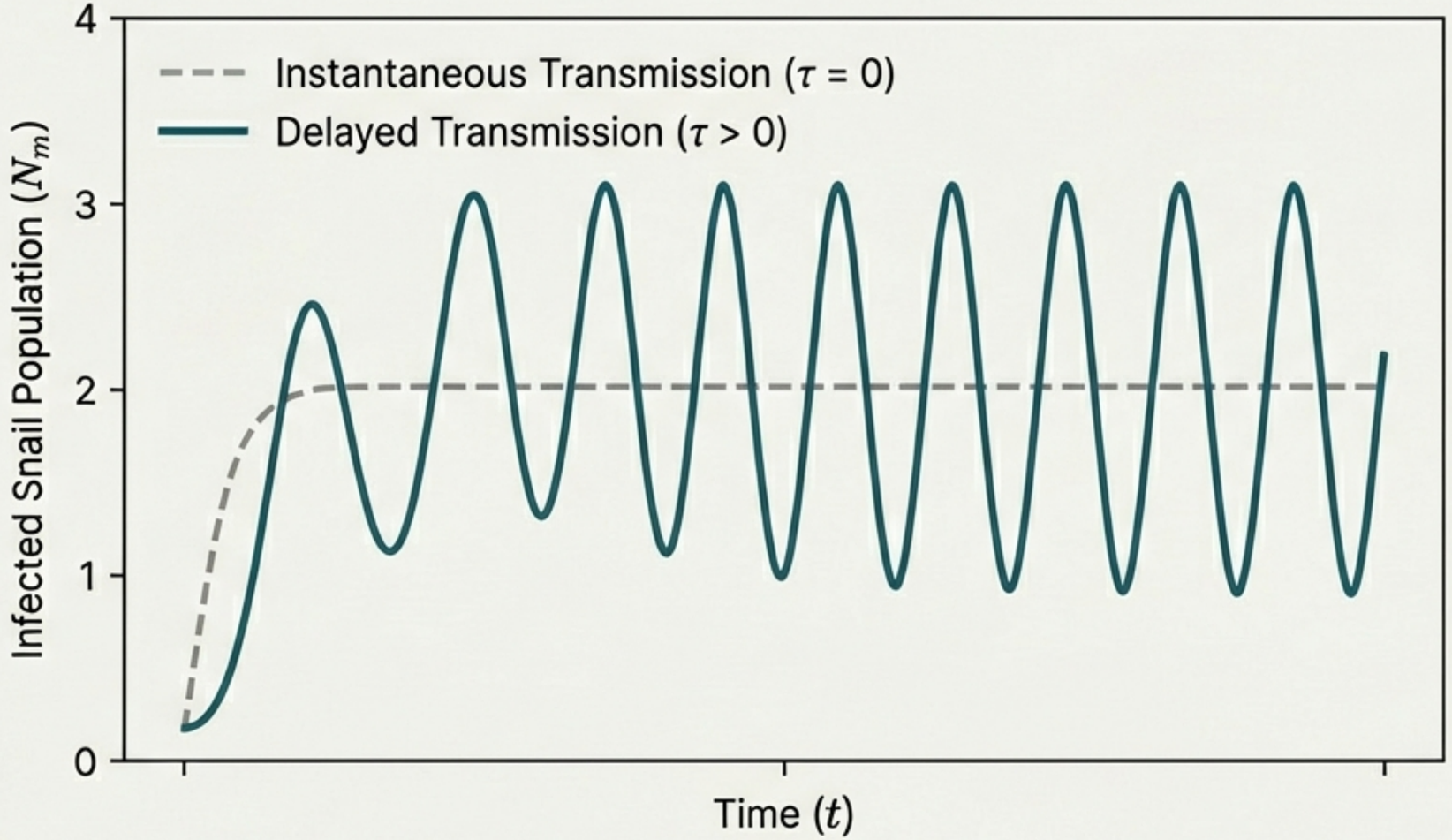


$$\frac{dS}{dt} = -\beta SI$$

Complexity Ladder: Simple Delays → Environmental Forcing → Multi-species Interaction

The Cost of Waiting: Schistosomiasis & Time Delays

Shan, Gao, Zhu (2011)



Math Translator

The Delay Term

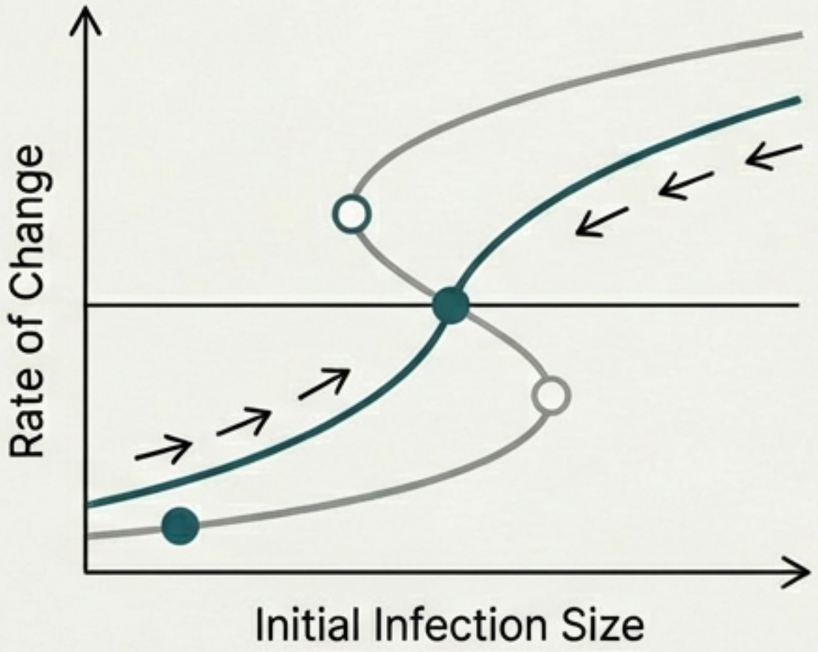
$$N_m(t - \tau)$$

The population state τ days ago. In biology, consequences are rarely instant. This "lag" destabilizes the equilibrium, creating permanent cycles instead of a steady state.

Complexity Ladder: Simple Delays → Environmental Forcing → Multi-species Interaction

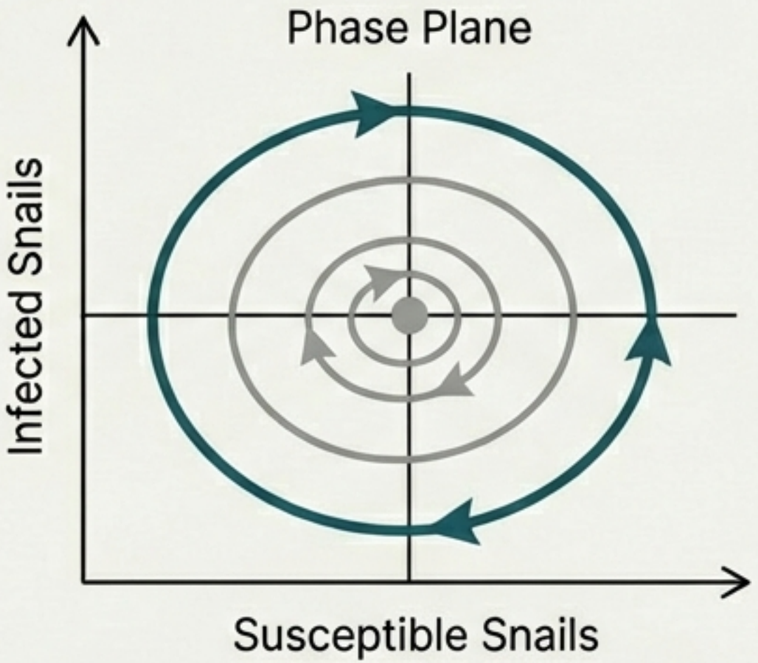
Bifurcations in Snail Populations

Saddle-Node Bifurcation



Initial Size Dependence. If the infection starts below the threshold, it dies out. If above, it becomes endemic.

Hopf Bifurcation



The Periodic Trap. When delay τ exceeds τ_0, the stable population breaks into a limit cycle.

Math Translator

Hopf Bifurcation

=

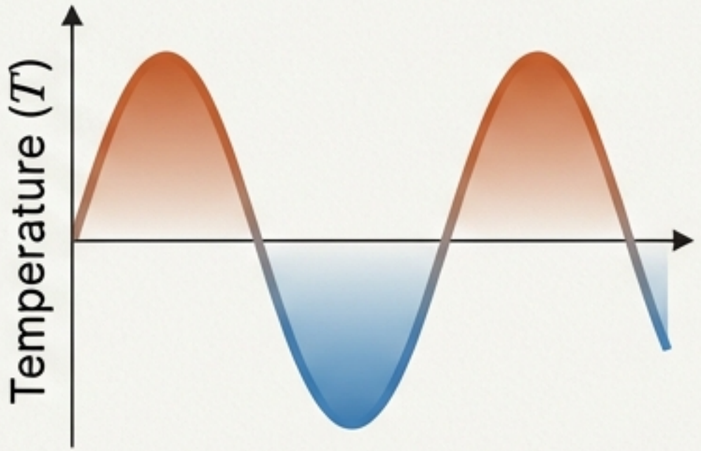
The mathematical moment a system loses its balance and starts vibrating.

Biologically, this represents the shift from a constant disease level to seasonal/periodic outbreaks.

Complexity Ladder: Simple Delays → Environmental Forcing → Multi-species Interaction

The Seasonal Pulse: West Nile Virus & Temperature

Shan, Fan, Zhu (2020)



Ambient Temperature (T)

Maturation Delay (τ)

Temperature Dependent
Hotter = Faster Maturation.
Colder = Slower Maturation.

Cold (< 16°C)
→ Extinction.

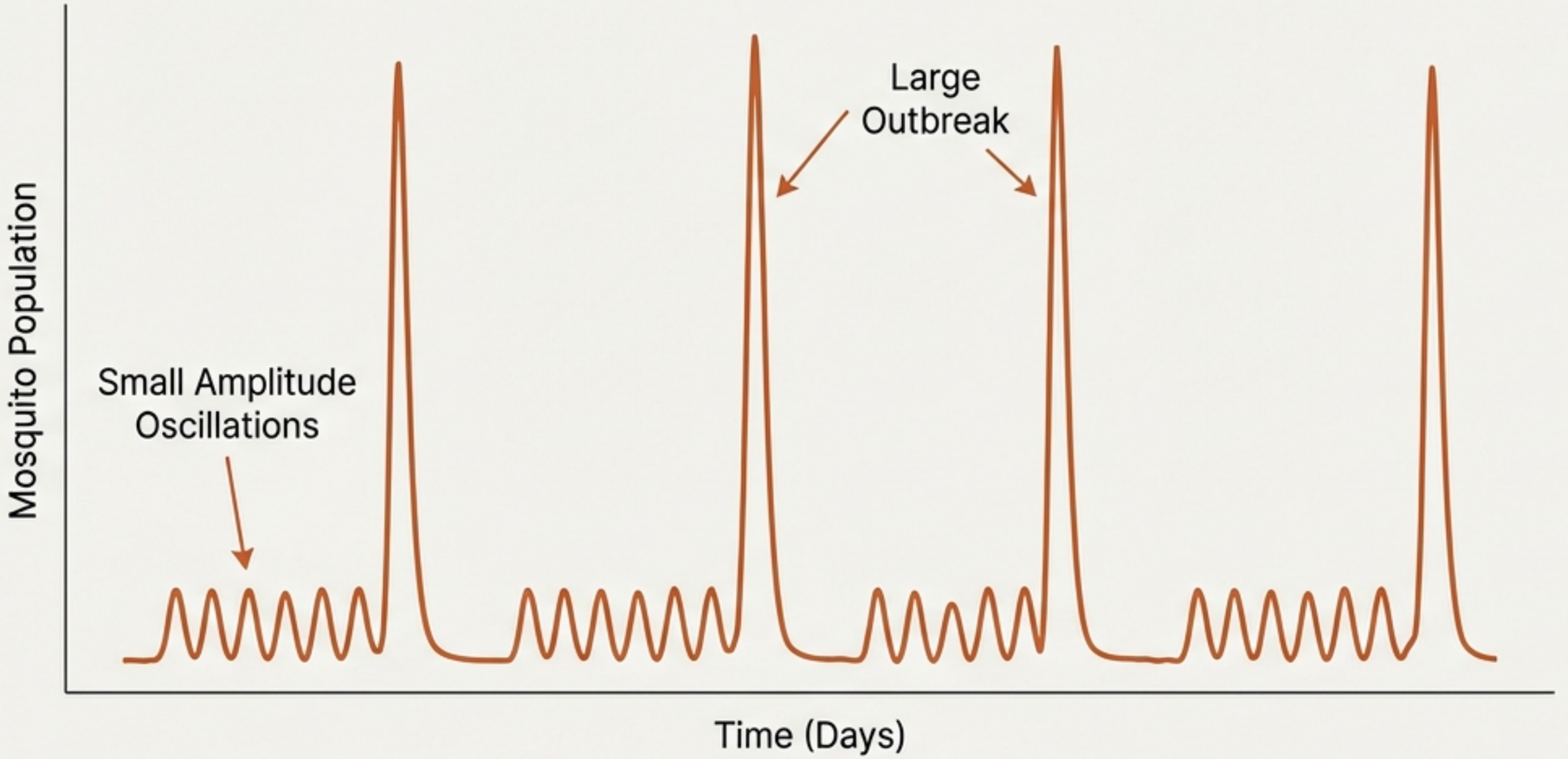
Hot (> 30°C)
→ Stable Epidemic.

Intermediate (20°–25°C)
→ Oscillations.

Insight: The thermometer drives the outbreak. Intermediate temperatures create the most complex, oscillatory dynamics.

Complexity Ladder: Simple Delays → Environmental Forcing → Multi-species Interaction

Mixed-Mode Oscillations: Chaos in the Middle



Math Translator

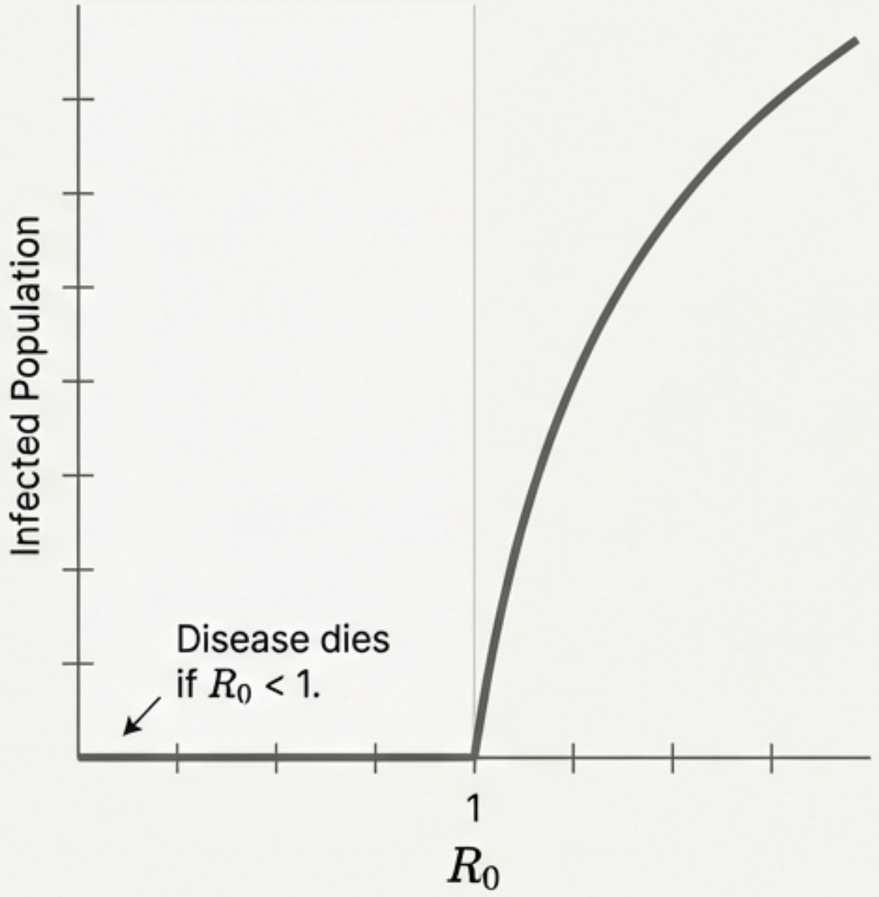
Mixed-Mode Oscillation = A rhythm that stutters.

Driven by intermediate temperatures, the system alternates between minor background cases and sudden, explosive outbreaks.

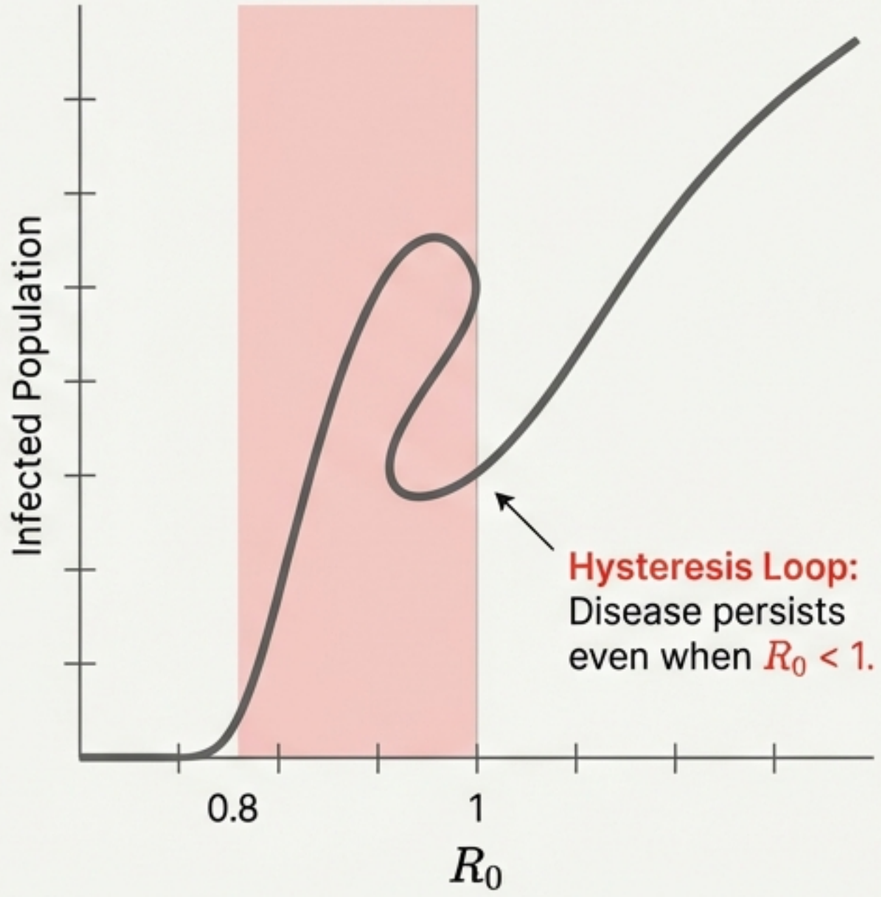
Reproducing the mixed-mode oscillation pattern found in West Nile Virus models.

The Danger of Backward Bifurcation

Standard Forward Bifurcation (Safe)



Backward Bifurcation (Dangerous)



Math Translator

Backward Bifurcation
= A “sticky” outbreak.

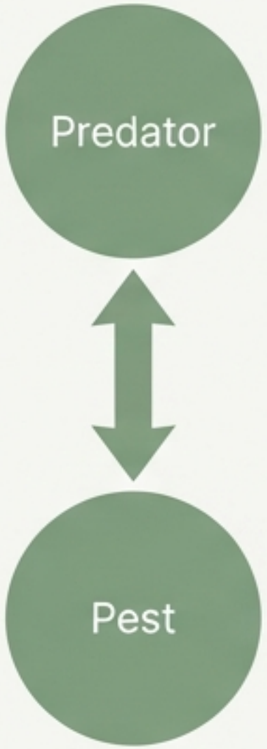
Lowering the reproduction number slightly is not enough.

The system **traps** the disease in a stable state, requiring drastic effort to crash the population.

Guardians of the Plantation: Mites vs. Leafhoppers

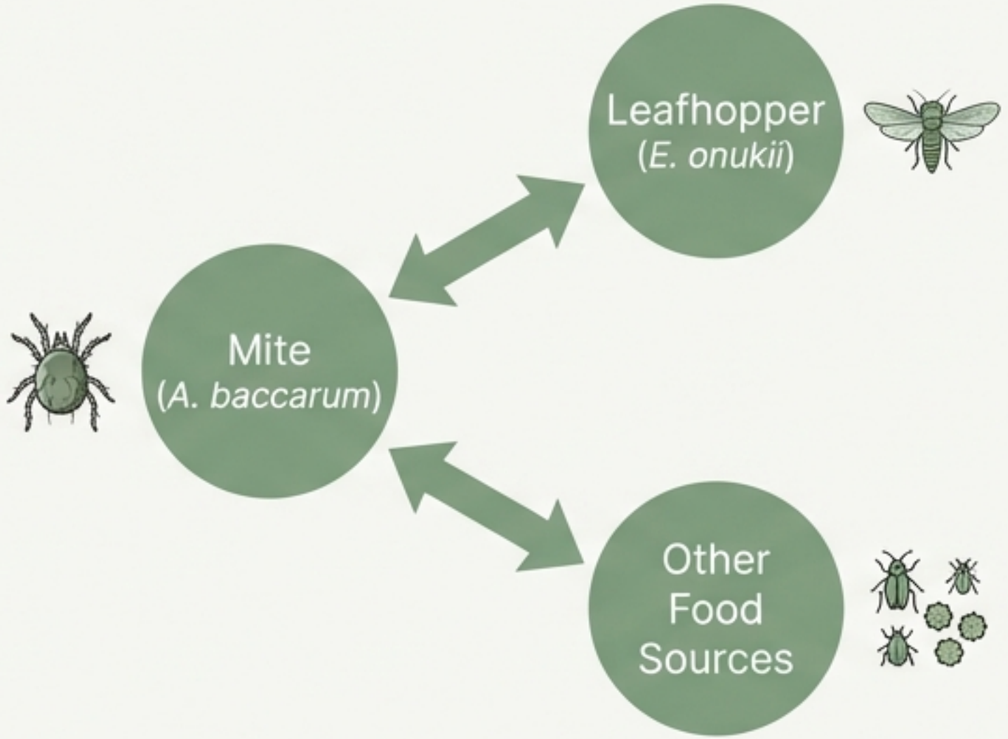
Yuan, Chen, You, Zhu (2021)

The Specialist Model



Linked Fate.
If Pest dies, Predator dies.

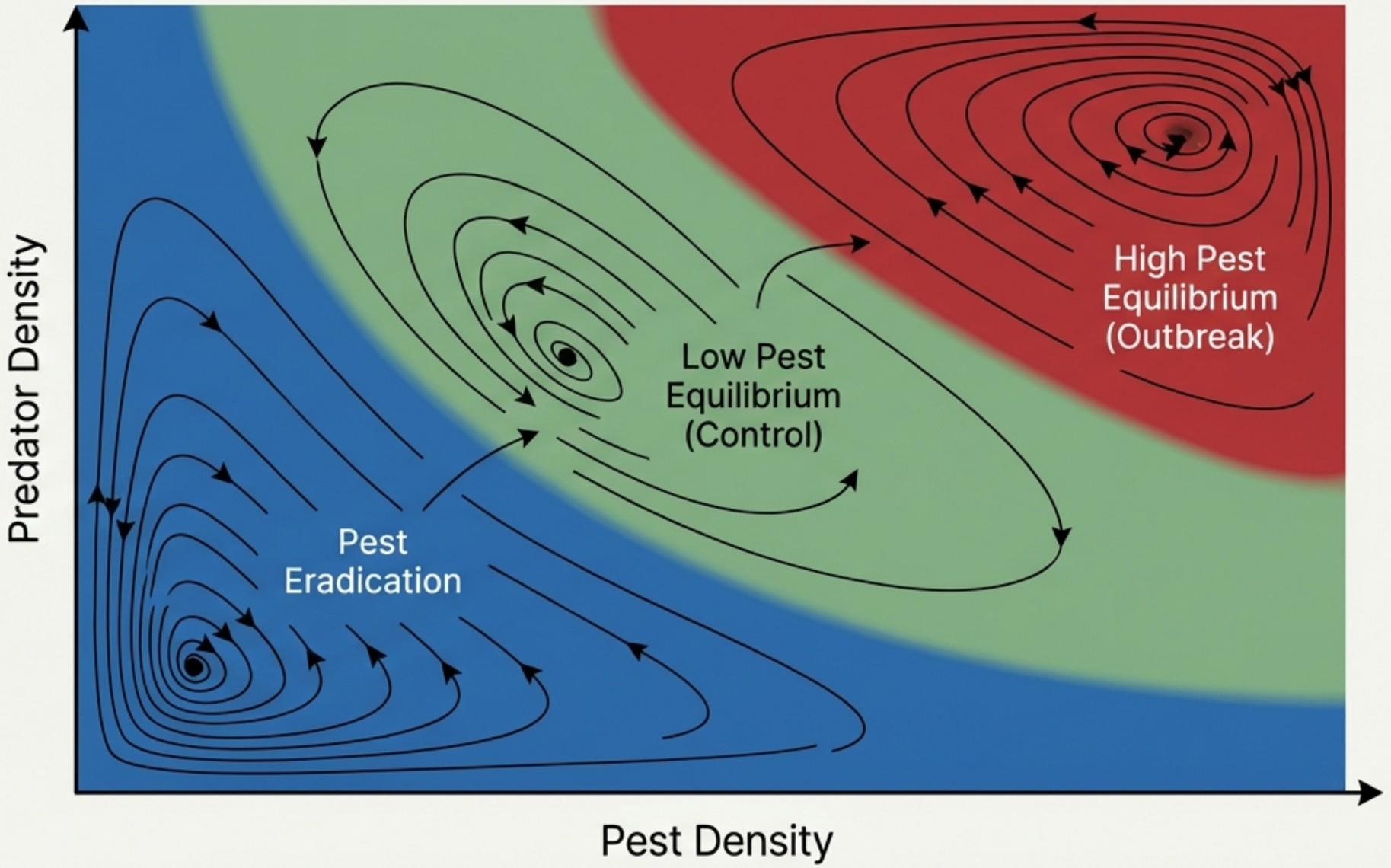
The Generalist Model (Tea Plantation)



Buffered Fate. Mite survives on other food, maintaining pressure on the pest.

Generalist predators introduce 'Tri-Stability'—creating a complex landscape of possible outcomes.

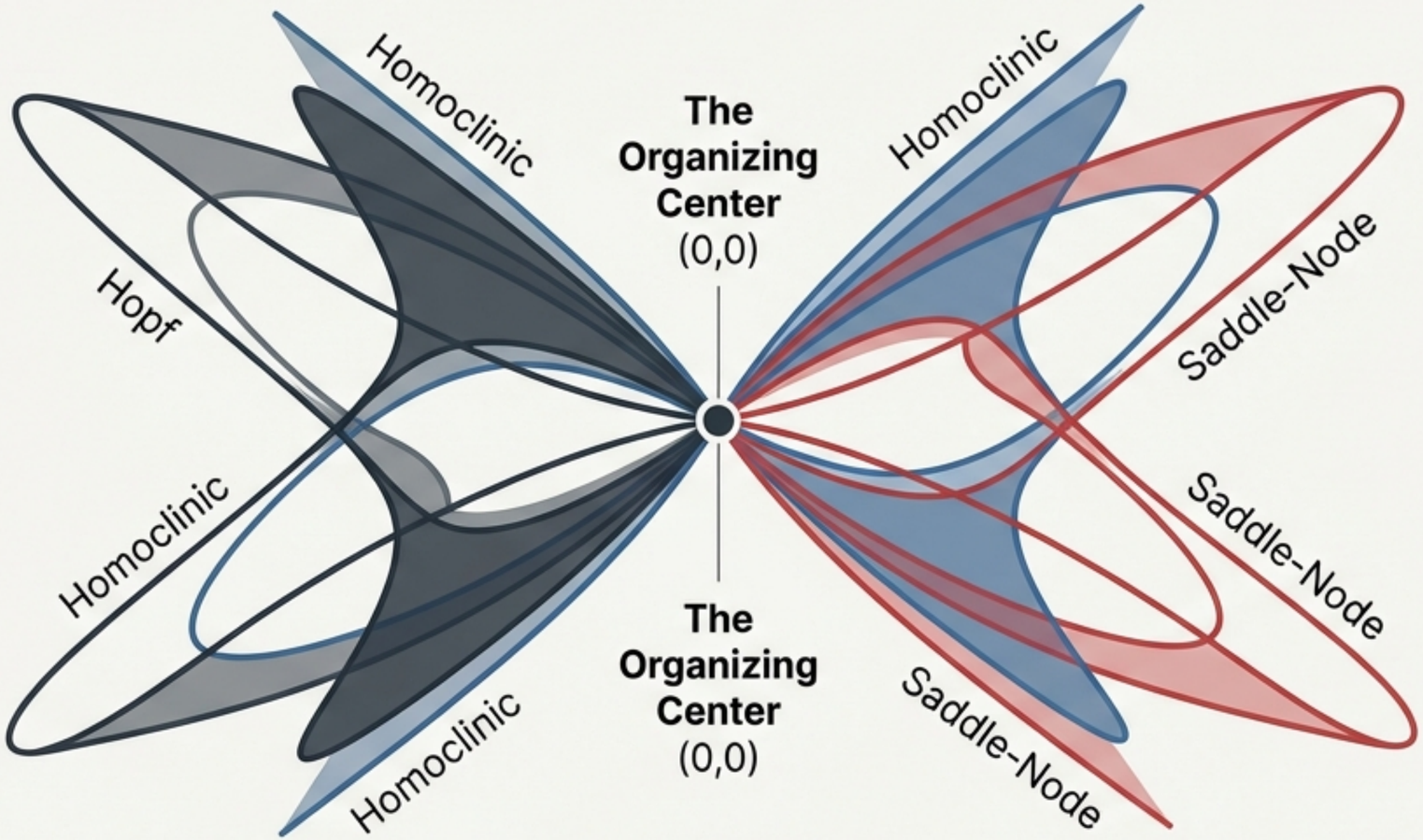
Tri-Stability: Three Possible Futures



Depending on the initial population numbers, the plantation will drift into one of these three states. Biological control requires pushing the system into the Green Zone.

The Mathematical Heart of Chaos

Nilpotent Singularity of Codimension 3 & 4



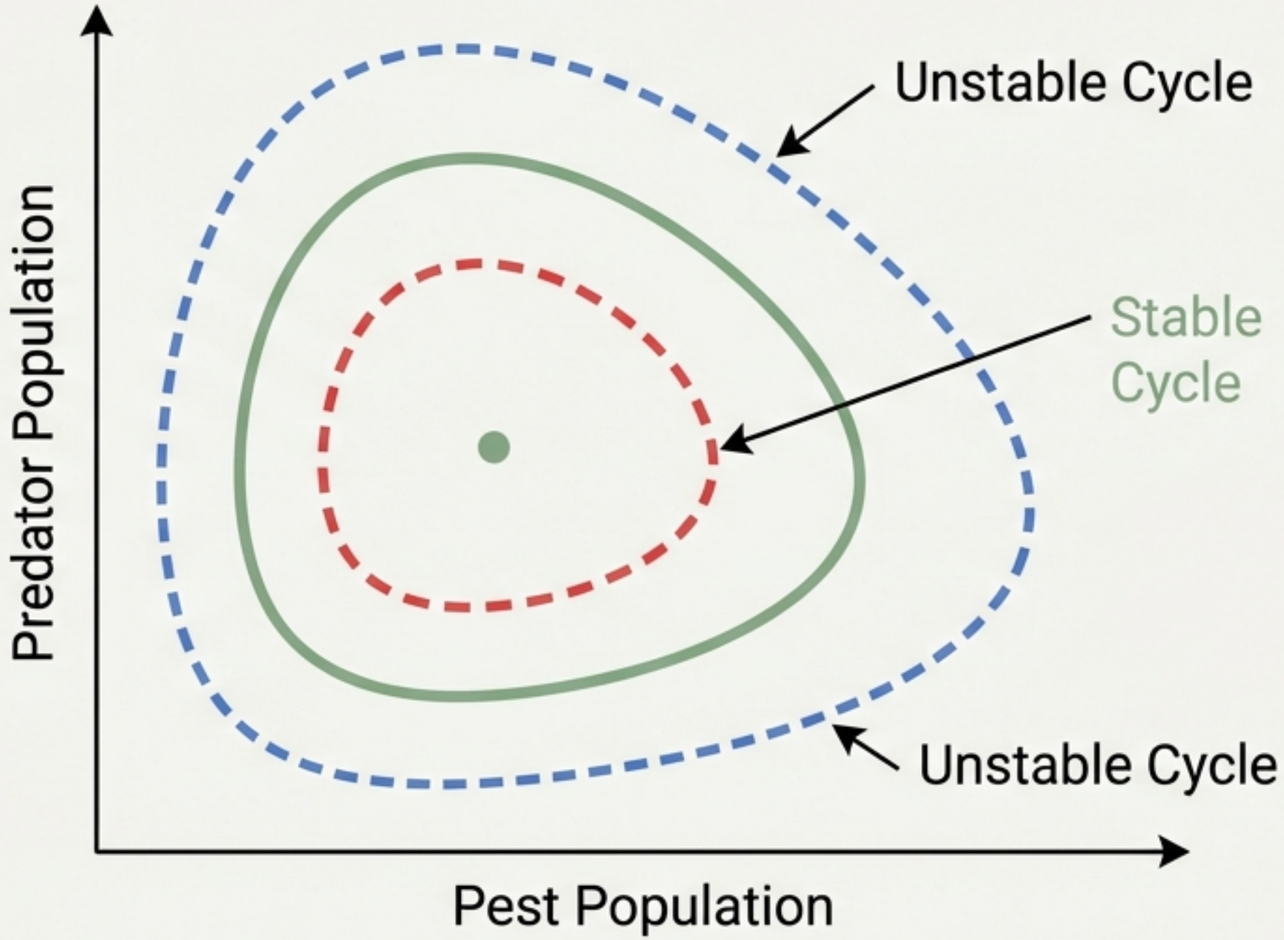
Math Translator

Nilpotent Singularity = The 'Big Bang' of the model.

It is a highly sensitive point where the system is perfectly balanced.

All complex behaviors (cycles, stability shifts) radiate from this single mathematical seed.

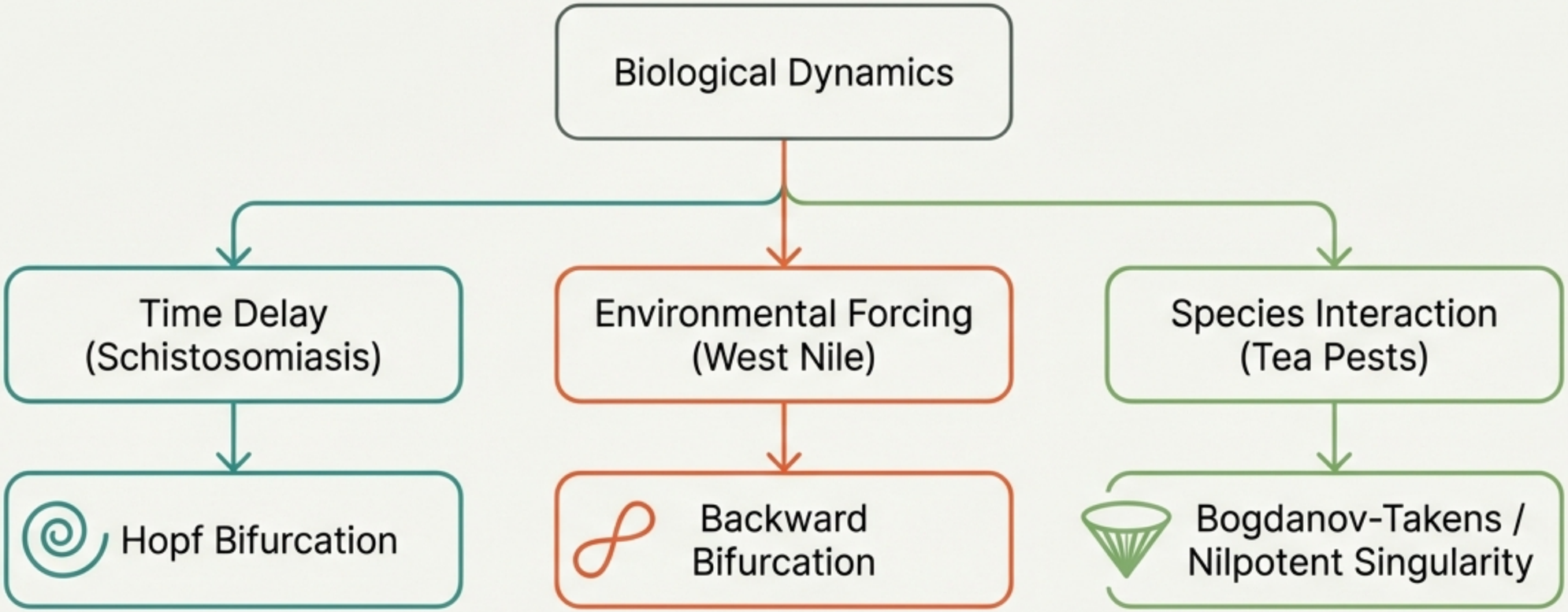
The Phenomenon of Three Limit Cycles



A rare biological rhythm.

The system can trap predator and pest in a stable oscillation (Green), but if pushed too far, it snaps to a different behavior. Period ~150 days.

The Grand Unification: Bifurcation Theory



Mathematics acts as the universal map, classifying the distinct “catastrophes” inherent in nature.

Strategies for Control

Schistosomiasis

Manage Initial Size

Due to Saddle-Node dynamics, infection must be kept below the unstable threshold to prevent endemic establishment.

West Nile Virus

Break the Hysteresis

Backward Bifurcation means marginal reduction fails. Massive vector reduction is required to push the system out of the “sticky” outbreak loop.

Tea Pests

Manipulate Carrying Capacity (K_2)

Increase predator capacity (e.g., cover crops) to force the system from High Pest Equilibrium to Low Pest Equilibrium.

Blueprints for a Complex World

Mathematics does not just describe nature’s complexity; it provides the blueprints for managing it.