# Research statement

**Jihyeon Jessie Yang**[1]

The diagram below describes the broad landscape of my research: three foundational fields in pink, seven areas of questions in green, four connections to other sciences in blue, and three arrows of the directions of my approach. My current research is to solve questions in Algebraic Geometry and Representation Theory in the approach of two theories from Combinatorics: Newton-Okounkov Body Theory and Tropical Geometry. These new (about 30 years old) branches have been proved to be powerful with their scopes growing very rapidly.

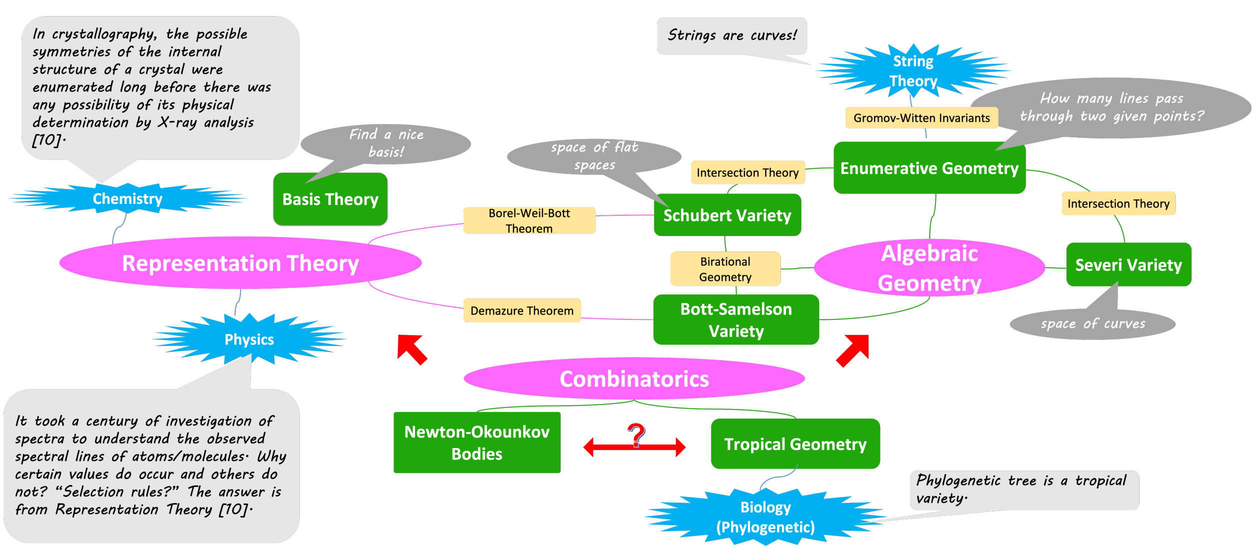
"What can we compute?": spaces arising from interesting questions are often too abstract to obtain information by computation or visualization. For a simple example, while we can visualize a real plane curve on a piece of paper, it is hard for a complex plane curve since it lives in a real 4-dimensional space. A key idea to resolve this obstacle is to construct a combinatorial object that is related to the original space. Going back to the example above, we have a tropical plane curve associated with a complex plane curve. A tropical plane curve is a planar graph so that we can use a computer for computations.

In general, given a geometric object $X$, each of Tropical Geometry and Newton-Okounkov Body Theory suggests a way to construct a polyhedral object that is closely related to $X$. Toric Geometry, the well-established branch in Algebraic Geometry, was the primary motivation of these two developments where we have an excellent dictionary between the combinatorics of polyhedra and the structures of Toric varieties. There are three important questions in these two theories:

- What are they?: Find an explicit description of the combinatorial object.
- How to use them?: Find applications.
- How are they related? : Find relations between these two theories.

In attempts to find answers to these three questions, I have been researching two fundamental examples from Algebraic Geometry: Schubert variety and Severi variety. Severi varieties parameterize plane curves with prescribed singularities. Their connection to String Theory through Enumerative Geometry regained geometers' attention. I have found a description of the Tropical Severi varieties [8], which uses a much simpler notion: subdivisions of polygons. This description has provided an intersection-theoretic understanding of certain multiplicities that were introduced in

[1]updated on January 2021



the celebrated Mikhalkin's Correspondence Theorem [7]. Also, I have found explicit criteria [9] when the Tropical Severi varieties fail to be subfans of Secondary fans, which are well-studied polyhedral fans [1]. This result reflected the fact that Tropical Severi varieties are not completely determined by combinatorial properties. On the other hand, Schubert varieties parameterize flat spaces such as lines and planes with given geometric conditions. Schubert varieties are in general not smooth and their birational models, Bott-Samelson varieties, are important tools in Geometric Representation Theory. With Megumi Harada [5], we have found examples of Newton-Okounkov Bodies of Bott-Samelson varieties , and as a byproduct, we have found several equivalent conditions which guarantee certain character formulae in Representation Theory to be purely positive ones [2]. Furthermore, as another application, we have suggested a way to resolve singularities of Schubert varieties [3].

Recently, with Christopher Manon [6], we have completed our task to answer to the third question (How are these two theories related?) for the case of Grassmannian of planes. This paper provides a concrete example to build a connection between these two theories, which in general can unify many different results in the literature. I am interested in developing this work further for the case of Severi varieties, which may provide some pieces of information that may be lost in the Tropical approach. Also, with Yael Karshon [4], we have been working to find bases of irreducible representations of reductive groups. These bases, Bott canonical bases, are constructed geometrically, motivated by an idea of Raoul Bott. This work brings up further questions. First, our construction relies on certain cohomology vanishing conditions. Also, it shows a close connection to the Newton-Okounkov Body Theory which may provide another interesting example of Newton-Okounkov Bodies of Bott-Samelson varieties. It is also an interesting question how the Bott canonical bases are related to many well-known bases in Representation Theory, whose study may find applications both in Representation Theory and Algebraic Geometry.

## References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