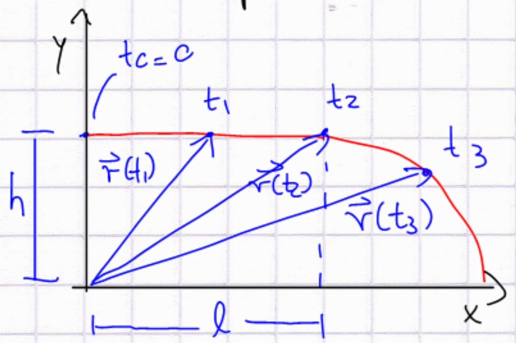
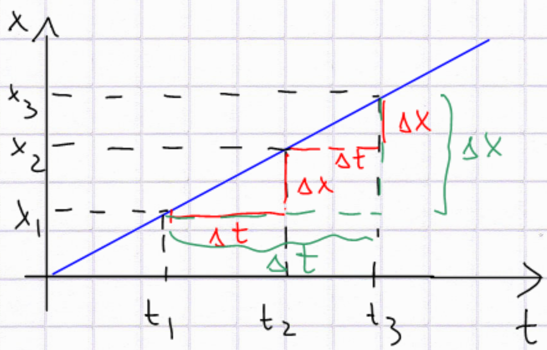
Motion := change of an object's position with time

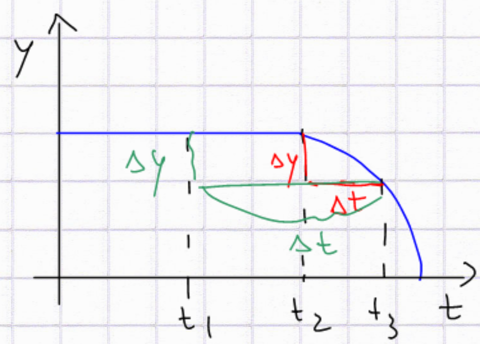
2D example: ball rolling on and falling off a table



motion is characterized by trajectory $\vec{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$

position – time graphs (ignoring friction + air drag)





- introduce velocity $\vec{v} = v_x\,\hat{\imath} + v_y\,\hat{\jmath}$ to characterize changes in object's position

$$v_x = \frac{\text{displacement}}{\text{time interval}} = \frac{\Delta x}{\Delta t}$$

slope of x – t graph

$$= \frac{x_2 - x_1}{t_2 - t_1} = \frac{x_3 - x_2}{t_3 - t_2}$$

$$= \frac{x_3 - x_1}{t_3 - t_1} = \text{const} > 0$$

$$v_y^{(2,1)} = \frac{y_2 - y_1}{t_2 - t_1} = 0$$

$$v_y^{(3,2)} = \frac{y_3 - y_2}{t_3 - t_2} < v_y^{(3,1)}$$

$$= \frac{y_3 - y_1}{t_3 - t_1} < 0$$

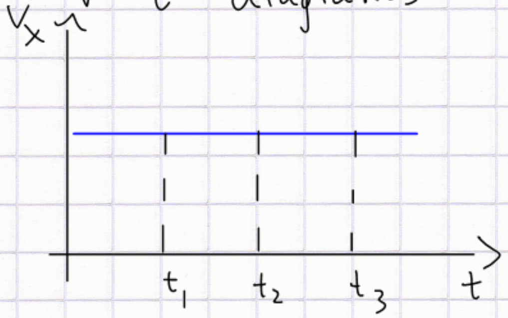
→ need instantaneous velocity for a complete characterization

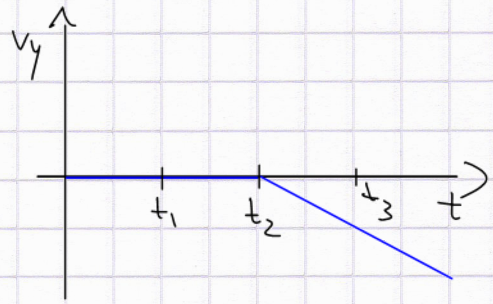
$$V_x(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \equiv \dot{x} \equiv x'(t)$$

$$V_y(t) = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} = \frac{dy}{dt} \equiv \dot{y} = y'(t)$$

different popular symbols for time-derivatives

v-t diagrams





instantaneous velocity vector:

$$\vec{V}(t) = V_x(t)\,\hat{i} + V_y(t)\,\hat{j} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$= \frac{d}{dt}\left(x(t)\,\hat{i} + y(t)\,\hat{j}\right) = \frac{d}{dt}\vec{r}(t) = \frac{d\vec{r}}{dt}$$

= time derivative of position vector

= slopes of position-time graphs at points t

- speed $\quad v(t) = |\vec{v}(t)| = \sqrt{v_x^2(t) + v_y^2(t)} \geq 0$
- average velocity (wrt given time interval) $\quad \vec{V}_{ave} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{dy}{dt}\hat{j}$

Summary: 2D motion is characterized by

- trajectory $\quad \vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$
- velocity $\quad \vec{v}(t) = \frac{d}{dt}\vec{r}(t)$
- acceleration $\quad \vec{a}(t) = \frac{d}{dt}\vec{v}(t) = \frac{d^2}{dt^2}\vec{r}(t)$ second derivative

i.e. $a_x(t) = \dot{v}_x = \ddot{x}$, $a_y(t) = \dot{v}_y = \ddot{y}$

study of motion: "kinematics"