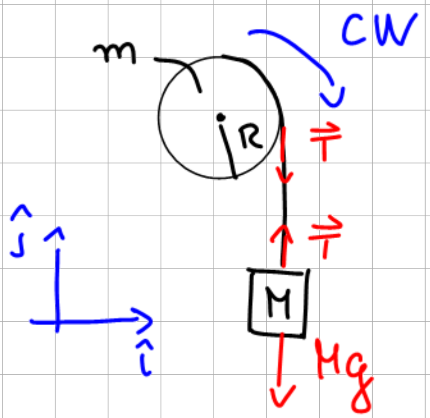
Rotational dynamics

fundamental equations to describe

	dynamics	equilibrium
translations	$m\vec{a} = \Sigma \vec{F}$	$\Sigma \vec{F} = 0$
rotations (about fixed axis)	$I\vec{\alpha} = \Sigma \vec{\tau}$ *)	$\Sigma \vec{\tau} = 0$

example of rotational dynamics: pulling a pulley



$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times \vec{T} = -RT\hat{k} = I\vec{\alpha}$$

$$\text{so} \quad \alpha_z = -\frac{RT}{\underbrace{I}_{=\frac{m}{2}R^2 \text{ for disk (table 8.2)}}} = -\frac{2T}{mR}$$

Newton-2 for crate: $F_y = -Mg + T = Ma_y$
$\Leftrightarrow \quad T = M(a_y + g)$

insert in α_z equation (to eliminate tension):

$$\text{so} \quad \alpha_z = -\frac{2M}{mR}(a_y + g)$$

This is a relation between α_z and a_y. Another relation between these two (yet unknown) quantities is

$$a_y = a_t = \alpha_z R$$

(a_y = tangential acceleration of pulley if rope doesn't become slack)

$$\Leftrightarrow \alpha_z = \frac{a_y}{R}$$

$$= -\frac{2M}{mR}(a_y + g)$$

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*) One can derive this equation from Newton-2 for any object with moment of inertia I.

$$\Leftrightarrow \quad a_y + \frac{2M}{m} a_y = -\frac{2M}{m} g$$

$$\Leftrightarrow \quad a_y \left(\frac{m+2M}{m} \right) = -\frac{2M}{m} g$$

$$\Leftrightarrow \quad a_y = -\frac{2Mg}{m+2M} = -\frac{Mg}{M+\frac{m}{2}} \underset{m \ll M}{\approx} -g$$

(simple free fall if $m \ll M$)

$$\text{CO} \quad \alpha_z = \frac{a_y}{R} = -\frac{Mg}{R(M+\frac{m}{2})} \underset{m \ll M}{\approx} -\frac{g}{R}$$

$$T = M(a_y + g) = Mg\left(1 - \frac{2M}{m+2M}\right) = \frac{Mg}{\frac{2M}{m}+1} \underset{m \ll M}{\approx} \frac{Mg}{\frac{2M}{m}} = \frac{mg}{2}$$

Both a_y and α_z are constant. Hence, we can infer (angular) velocity and (angular) displacement using

$$\omega_z(t) = \underset{\overset{\shortparallel}{0}}{\omega_0} + \alpha_z t \quad , \quad \theta(t) = \theta_0 + \frac{\alpha_z}{2} t^2 \underset{m \ll M}{\approx} \theta_0 - \frac{gt^2}{2R}$$

$$v_y(t) = \underset{\overset{\shortparallel}{0}}{v_0} + a_y t \quad , \quad y(t) = y_0 + \frac{a_y}{2} t^2 \underset{m \ll M}{\approx} y_0 - \frac{gt^2}{2}$$