PHYS 1410 6.0: PHYSICAL SCIENCE (Fall 2012)

– sample solutions –

Class test #1 **Oct 24, 1:30 p.m. – 2:20 p.m. = 50 min**

NAME: STUDENT NR:

Formulae at the end and calculators = only aid; total = 22 points (20 points = 100 %)
Note on units: You don't have to write out the units in intermediate steps as long as you are working consistently in the SI system. Give your final answers in SI units (where appropriate).
Note on format: to earn full marks you have to support your results by explicit calculations and/or convincing argument, i.e, just providing final equations and plugging in numbers does not yield full marks.

1. (2 points) A book lies on a table. Identify two action-reaction pairs and the associated forces in this set-up.

   book (0.5) – table (normal (0.5) forces)

   book (0.5) – Earth (gravity (0.5))

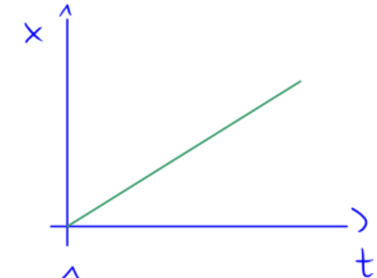
2. (2 points) Alice throws a beach ball straight up into the air. Taking air drag into account, what is true about the magnitude of the acceleration a of the ball on its way up after it has left Alice's hands?

   ✓ $a > g$
   ○ $a = g$ (1)
   ○ $a < g$
   ○ $a = 0$

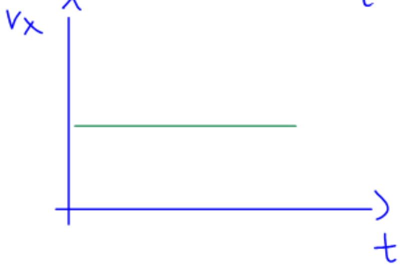
   What is true on the way down once the ball has reached its terminal velocity?

   ○ $a > g$
   ○ $a = g$ (1)
   ○ $a < g$
   ✓ $a = 0$

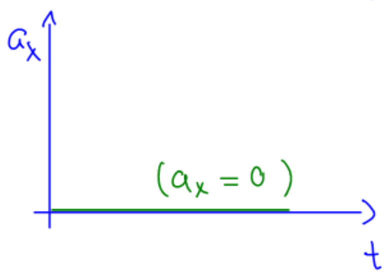
3. (3 points) Provide sketches of the $x(t)$, $v_x(t)$, and $a_x(t)$ graphs (where $x(t)$ characterizes the horizontal motion) of a ball thrown at the launch angle of 30°. Consider the time interval from the moment, when the ball leaves the hand of the person to the moment, when it hits the ground. Air drag can be neglected in this problem.



(1)

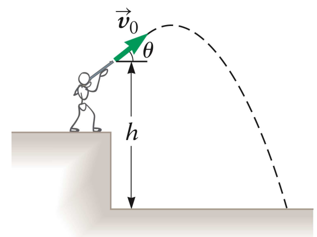


(1)



(1)

4. (9 points) A bullet of mass m is fired from a rifle with speed v_0 at an angle θ with respect to the horizontal axis as shown in the figure. Air drag can be neglected in this problem.



(a) (1 point) Provide a free-body diagram for the bullet.

(b) (3 points) Starting from Newton's second law derive equations for the velocity vector $\vec{v}(t) = v_x(t)\hat{\imath} + v_y(t)\hat{\jmath}$ and for the trajectory (position vector) $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$.

(c) (1 point) Show that the following equation holds for the time $t_{\rm gr}$, when the bullet hits the ground:

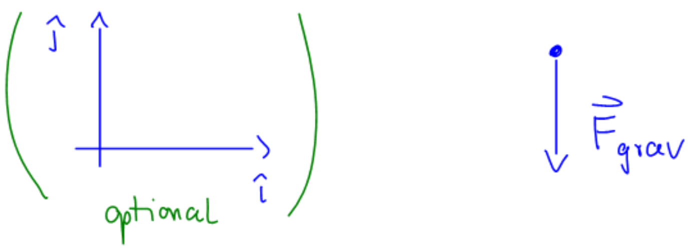
$$t_{\rm gr}^2 - \frac{2v_0 \sin\theta}{g} t_{\rm gr} - \frac{2h}{g} = 0\,. \qquad (1)$$

(d) (3 points) Using your results of part (b) and Eq. (1) find an expression for the speed v_f of the bullet right before it strikes the ground.

(e) (1 point) Imagine the same experiment is done on the Moon. Would v_f be
- ○ larger
- ✓ smaller
- ○ unchanged

compared to v_f on Earth?

(a)



(1)

(b) Newton-2: $m\vec{a} = \vec{F}_{net} = -mg\hat{\jmath}$

i.e. $a_x \overset{(0.5)}{=} 0$, $a_y \overset{(0.5)}{=} -g$

⇒ $v_x = v_{0,x} \overset{(0.5)}{=} v_0 \cos\theta$, $v_y(t) = v_0 \sin\theta - gt$ (0.5)

$\left(\vec{v}(t) = v_0\cos\theta\,\hat{\imath} + (v_0\sin\theta - gt)\hat{\jmath}\right)$ (optional)

⇒ $x(t) = v_0 t\cos\theta + \overbrace{x_0}^{=0}$ (0.5), $y(t) = \overbrace{y_0}^{=h} + v_0 t \sin\theta - \frac{g}{2}t^2$ (0.5)

$\left(\vec{r}(t) = v_0 t\cos\theta\,\hat{\imath} + (h + v_0 t\sin\theta - \frac{g}{2}t^2)\hat{\jmath}\right)$ (optional)

c) Condition for hitting the ground: $y(t_{gr}) = 0$ (0.5)

$$\Leftrightarrow \quad 0 = h + v_0 t_{gr} \sin\theta - \frac{g}{2} t_{gr}^2$$

$$\Leftrightarrow \quad t_{gr}^2 - \frac{2 v_0 \sin\theta}{g} t_r - \frac{2h}{g} = 0 \quad (0.5)$$

d)

$$v_g = \left[v_x^2(t_{gr}) + v_y^2(t_{gr}) \right]^{1/2} \quad (0.5)$$

$$= \left[v_0^2 \cos^2\theta + (v_0 \sin\theta - g t_{gr})^2 \right]^{1/2} \quad (0.5)$$

$$= \left[v_0^2 \cos^2\theta + v_0^2 \sin^2\theta - 2 v_0 g t_{gr} \sin\theta + g^2 t_{gr}^2 \right]^{1/2}$$

$$= \left[v_0^2 - 2 v_0 g t_{gr} \sin\theta + g^2 t_{gr}^2 \right]^{1/2} \quad (1)$$

use (c): $t_{gr}^2 = \frac{2 v_0 \sin\theta}{g} t_{gr} + \frac{2h}{g}$

$$v_g = \left[v_0^2 - \cancel{2 v_0 g t_{gr} \sin\theta} + \frac{2 v_0 g^2 \sin\theta}{g} t_{gr} + 2gh \right]^{1/2}$$

$$= \left[v_0^2 + 2gh \right]^{1/2} \quad (1)$$

(e): v_g would be smaller on the Moon, since $g_{Moon} < g_{Earth}$; hence $\sqrt{v_0^2 + 2 g_{Moon} h} < \sqrt{v_0^2 + 2 g_{Earth} h}$

(optional)

5. (4+2 points) A car of mass $m = 1200$ kg is driving at constant speed $v_0 = 15$ m/s on a level street. Suddenly, the street becomes icy. The driver applies the brakes. The wheels lock and the car skids. The frictional coefficients between the tires of the car and the icy street are $\mu_k = 0.38$ and $\mu_s = 0.42$.

(a) (2 points) Starting from the equations for one-dimensional motion at constant acceleration derive an equation for the distance Δx the car is skidding until it comes to a stop. Calculate Δx for the values of m, v_0, μ_k, μ_s given above[1].

(b) (2 points) Now assume that the car is skidding up a hill that makes the angle $\beta = 15°$ with the horizontal. Calculate the distance the car is skidding until it comes to a stop in this case.

(c) (2 bonus points) Now assume that the car is skidding down a hill that makes the angle α with the horizontal. What is the smallest value for α, at which the car *does not* come to a stop?

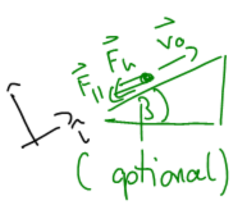
(a) use $v^2 = v_0^2 + 2a(x - x_0)$ for $v = 0$ (coming to a stop)

$\Rightarrow 0 \overset{(0.5)}{=} v_0^2 + 2a\Delta x \Leftrightarrow \boxed{\Delta x = -\frac{v_0^2}{2a}}$ (0.5) $(*)$

constant acceleration in this example: $a = -\frac{F_k}{m} = -\mu_k g$ (0.5)

$\overset{(*)}{\Rightarrow} \Delta x = \frac{v_0^2}{2\mu_k g} = 30\text{ m}$ (0.5)

(b) constant acceleration in this example: $F_\perp = mg\cos\beta$

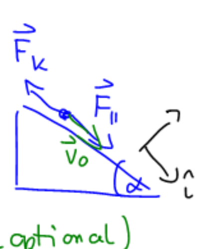


$a \overset{(0.5)}{=} -\frac{F_{\parallel}}{m} - \frac{F_k}{m} = -g\sin\beta - \frac{\mu_k N}{m}$

$= -g(\sin\beta + \mu_k\cos\beta)$ (1)

$\overset{(*)}{\Rightarrow} \Delta x = \frac{v_0^2}{2g(\sin\beta + \mu_k\cos\beta)} = 18\text{ m}$ (0.5)

(c)



constant acceleration in this example:

$a = \frac{F_{\parallel}}{m} - \frac{F_k}{m} = g\sin\alpha - \frac{\mu_k N}{m} = g(\sin\alpha - \mu_k\cos\alpha)$ (1)

car doesn't come to a stop if $a \geq 0$ (1)

$a = 0 \Leftrightarrow \sin\alpha - \mu_k\cos\alpha = 0 \Leftrightarrow \tan^{-1}\mu_k = \alpha = 20.8° \approx 21°$

[1]Note that not necessarily all values are needed to do this calculation.

FORMULAE

$$\frac{d}{dt}t^n = nt^{n-1}\ ;\ \frac{d}{dt}\sin t = \cos t\ ;\ \frac{d}{dt}\cos t = -\sin t\ ;\ \frac{d}{dt}\exp t = \exp t$$

$$\frac{d}{dt}\left(f(t) + g(t)\right) = \frac{df}{dt} + \frac{dg}{dt}\ ; \qquad \frac{d}{dt}(\alpha f(t)) = \alpha\frac{df}{dt} \quad \text{for any } \alpha \in \Re$$

$$\text{product rule}: \ \frac{d}{dt}(fg) = \frac{df}{dt}g + f\frac{dg}{dt}\ ; \qquad \text{chain rule}: \ \frac{d}{dt}[f(x(t))] = \frac{df}{dx}\frac{dx}{dt}$$

$\frac{d}{dt}\vec{r}(t) = \vec{v}(t)$; $\frac{d}{dt}\vec{v}(t) = \vec{a}(t)$; $m\vec{a} = \vec{F}_{\text{net}}$; $F_{\text{grav}} = mg$; on Earth: $g = 9.8$ m/s^2
1D motion with a =const: $v = v_0 + at, x = x_0 + v_0t + \frac{a}{2}t^2, v^2 = v_0^2 + 2a(x - x_0)$
on inclined plane with tilt angle θ: $F_{||} = mg\sin\theta, F_{\perp} = +mg\cos\theta$
$F_k = \mu_k N$; $F_s \leq \mu_s N$; $\mu_k < \mu_s$; $F_{\text{drag}} = \frac{1}{2}\rho A v^2$