PHYS 1410 : Solutions to the 1st test

Fall 2011

4) given : $x(t) = A \sin \omega t$

(a) SI units $[A] = m$ (since x is measured in meters)

$[\omega] = \frac{\text{rad}}{s}$ (since t is measured in seconds and ωt must be an angle)

(b) $v_x(t) = x'(t) = A \frac{d}{dt}(\sin \omega t)$

$= A\omega \cos \omega t$

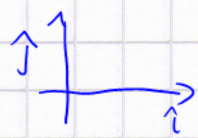
$a_x(t) = v_x'(t) = A\omega \frac{d}{dt}(\cos \omega t)$

$= -A\omega^2 \sin \omega t$

(c) Newton-2 $F_x = m a_x = -m\omega^2 A \sin \omega t$

$= -m\omega^2 x(t)$

$\Rightarrow \quad |F_x| \propto |x|$

5) Free fall on planet F 

(a) Newton-2 : $F_y = m a_y$ with $F_y = -mg$

$\Rightarrow \quad a_y = -g$

$v_y(t) = v_0 + a_y t = v_0 - gt$

proof : $v_y(0) = v_0$ ✓

$v_y'(t) = -g = a_y$ ✓

$y(t) = y_0 + v_0 t + \frac{1}{2} a_y t^2 = v_0 t - \frac{1}{2} g t^2$

proof: $y(0) = 0$ ✓ (this is why $y_0 = 0$)

$y'(t) = v_0 - gt = v_y(t)$ ✓

(b) If ball reaches max height at $t = 2s$ it reaches ground at $t = 4s$ (symmetry of up and down parts of motion).

(c) We know (in SI units) $t_{max} = 2 = \frac{v_0}{g}$ *) (*)

and $y(3) = 10 = 3v_0 - 4.5g$ (**)

solve (*) for v_0 and insert in (**):

$$10 = 3 \cdot 2g - 4.5g = 1.5g$$

$$\Leftrightarrow \quad g = 6.67 \; m/s^2$$

$$v_0 = 2g = 13.3 \; m/s$$

(d) $$\Delta x = v_{0,x} \, t_{ground} = \frac{2 v_{0,x} v_{0,y}}{g}$$

with $v_{0,x} = v_0 \cos\theta$, $v_{0,y} = v_0 \sin\theta$

obtain $\Delta x = \frac{2v_0^2}{g} \sin\theta \cos\theta = \ldots = 17.1 \, m$

plugging everything in

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*) follows from $v_y(t_{max}) \overset{!}{=} 0 = v_0 - g t_{max}$

e) Use $\Delta x = \frac{2v_0^2}{g} \sin\theta \cos\theta$

$$= \frac{v_0^2}{g} \sin 2\theta$$

solve for $\theta = \frac{1}{2} \sin^{-1}\left(\frac{g \cdot \Delta x}{v_0^2}\right) = 35°$