PHYS 1410 6.0: PHYSICAL SCIENCE (Fall 2012)

– sample solutions –

Class test #2 **Nov 28, 1:30 p.m. – 2:20 p.m. = 50 min**

NAME: STUDENT NR:

Formulae at the end and calculator = only aids; total = 18 points (16 points = 100 %)

Note on units: You don't have to write out the units in intermediate steps as long as you are working consistently in the SI system. Give your final answers in SI units (where applicable).

Note on format: to earn full marks you have to support your results by explicit calculations and/or convincing arguments, i.e, just providing final equations and plugging in numbers does not yield full marks.

1. (2 points) A car ($m = 1100$ kg) is travelling at an initial speed of 18 m/s. It then slows to a stop over a distance of 85 m due to a force from the brakes.
   (a) (1 point) Use the work-energy theorem to calculate the work done on the car while slowing down from the initial speed to the stop.
   (b) (1 point) Calculate the acceleration of the car (assuming that it is constant).

(a) $W = \Delta KE = 0 - \frac{m}{2} v_i^2 = -180{,}000 \text{ J}$ (1)

(b) $W = F_{ave} \Delta x = m a \Delta x$

$\Leftrightarrow \quad a = \frac{W}{m \Delta x} = -1.9 \text{ m/s}^2$ (1)

2. (9 points) Consider a rock of mass $m = 2.5$ kg and a string of length $l = 1.3$ m as shown in the figure. The string is fastened to a hinge that allows it to swing completely around in a vertical circle. The rock starts at the lowest point on this circle and is given an initial speed v_i. Air drag can be neglected in this problem, but gravity is acting on the rock.

   (a) (1 point) Provide an equation for the total energy of the rock at the lowest point of the circle.

   (b) (1 point) Provide an equation for the total energy of the rock at the highest point of the circle.

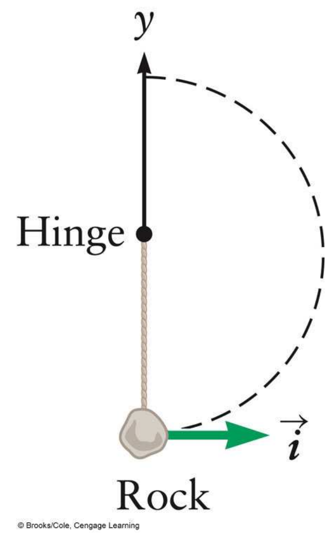
   (c) (1 point) Show that the minimum speed v_f at the highest point of the circle that ensures that the rock travels completely around the circle without the string becoming slack at the top is given by
   $$v_f = \sqrt{gl}\,.$$

   (d) (2 points) Calculate the smallest value of v_i that will allow the rock to travel completely around the circle without the string becoming slack at the top.

   (e) (1 point) Calculate the total work done on the rock while moving from the lowest to the highest point of the circle.

   (f) (1 point) The total work done on the rock is due to
   - ✓ gravity
   - ○ tension force
   - ○ both gravity and tension
   - ○ none of the above.

   (g) (2 bonus points) Calculate the tension in the string at the highest point and at the lowest point of the circle.



(a) $(TE)_{bot} = \frac{m}{2} v_i^2 + mg h_i$ (1)

(b) $(TE)_{top} = \frac{m}{2} v_f^2 + mg h_f$ with $h_f = h_i + 2l$ (1)

(c) Condition: $m a_c = \frac{m v_f^2}{l} \stackrel{!}{=} mg \iff v_f = \sqrt{gl}$ (1)

(d) Use $(TE)_{bot} = (TE)_{top}$ (0.5)

$\iff v_i^2 + 2gh_i = v_f^2 + 2g(h_i + 2\ell)$ (0.5)

$\iff v_i^2 = v_f^2 + 4g\ell \overset{(c)}{=} g\ell + 4g\ell = 5g\ell$ (0.5)

$\implies v_i = \sqrt{5g\ell} = 8.0 \text{ m/s}$ (0.5)

(e) $W = \Delta KE = \frac{m}{2}(v_f^2 - v_i^2) = \frac{m}{2}(g\ell - 5g\ell) = -2mg\ell$ (1)

$= -64 J$

(alternatively: $W = PE_i - PE_f = mgh_i - mg(h_i - 2\ell) = -2mg\ell$)

(f) Work is due to gravity (1)

(g) top: $\frac{mv_f^2}{\ell} = T_{top} + mg \iff T_{top} = m\left(\frac{\overbrace{v_f^2}^{=g\ell}}{\ell} - g\right) = 0$ (1)

bottom: $\frac{mv_i^2}{\ell} = T_{bot} - mg \iff T_{bot} = m\left(g + \frac{\overbrace{v_i^2}^{=5g\ell}}{\ell}\right)$ (1)

$= 6mg = 150 \text{ N}$

3. (7 points) Consider uniform circular motion.

(a) (1 point) Given the position vector

$$\vec{r}(t) = R[\cos(\omega t)\hat{\imath} - \sin(\omega t)\hat{\jmath}].$$

calculate the velocity vector $\vec{v}(t)$.

(b) (1.5 points) Show that $\vec{r}(t)$ and $\vec{v}(t)$ are perpendicular at all times.

(c) (4.5 points) Now assume that a rock of mass m is tied to a string of length R and hurled around such that it follows the circular trajectory given above.

- Make a sketch of the situation indicating the velocity vector at $t = 0$.
- Does the rock move clockwise or counterclockwise?
- Provide an expression for the net force the rock experiences in terms of m, R, T (with T being the period).
- How does the net force change if the period T doubles?
- Can the tension in the string alone provide the net force? Give reasons for your answer.

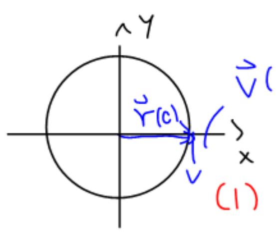
(a) $\vec{v}(t) = \frac{d\vec{r}}{dt} = R(-\omega \sin\omega t\ \hat{\imath} - \omega \cos\omega t\ \hat{\jmath})$ (1)

$= -R\omega(\sin\omega t\ \hat{\imath} + \cos\omega t\ \hat{\jmath})$

(b) $\vec{r} \perp \vec{v} \Leftrightarrow \vec{r}\cdot\vec{v} = 0$ (0.5)

$\vec{r}\cdot\vec{v} \overset{(0.5)}{=} xv_x + yv_y = -R^2\omega\left(\cos\omega t \sin\omega t - \sin\omega t\cos\omega t\right) = 0$ (0.5) ✓

(c)



$\vec{v}(c) = -R\omega\hat{\jmath}$

motion is clockwise. (0.5)

- $F_{net} = ma_c = m\omega^2 R = m\left(\frac{2\pi}{T}\right)^2 R = \frac{4\pi^2 mR}{T^2}$ (1)
- If T doubles, $F_{net} \to \tilde{F}_{net} = \frac{F_{net}}{4}$. (1)
- It can (and does) if there is no other force acting on the rock. If gravity is present, tension alone <u>cannot</u> provide F_{net} (1)

FORMULAE

$$\frac{d}{dt}t^n = nt^{n-1}\ ; \frac{d}{dt}\sin t = \cos t\ ; \frac{d}{dt}\cos t = -\sin t\ ; \frac{d}{dt}\exp t = \exp t$$

$$\frac{d}{dt}\left(f(t)+g(t)\right) = \frac{df}{dt}+\frac{dg}{dt}\ ; \qquad \frac{d}{dt}(\alpha f(t)) = \alpha\frac{df}{dt} \quad \text{for any } \alpha \in \Re$$

$$\text{product rule}:\ \frac{d}{dt}(fg) = \frac{df}{dt}g + f\frac{dg}{dt}\ ; \qquad \text{chain rule}:\ \frac{d}{dt}[f(x(t))] = \frac{df}{dx}\frac{dx}{dt}$$

$\frac{d}{dt}\vec{r}(t) = \vec{v}(t)$; $\frac{d}{dt}\vec{v}(t) = \vec{a}(t)$; $m\vec{a} = \vec{F}_{\text{net}}$; $F_{\text{grav}} = mg$; on Earth: $g = 9.8$ m/s^2
1D motion with $a =$const: $v = v_0 + at, x = x_0 + v_0 t + \frac{a}{2}t^2, v^2 = v_0^2 + 2a(x - x_0)$
on inclined plane with tilt angle θ: $F_{||} = mg\sin\theta, F_{\perp} = mg\cos\theta$
$F_k = \mu_k N$; $F_s \leq \mu_s N$; $\mu_k < \mu_s$; $F_{\text{drag}} = \frac{1}{2}\rho A v^2$
circular motion: $\vec{r}(t) = R\cos\theta(t)\hat{\imath} + R\sin\theta(t)\hat{\jmath}$; $\vec{a}(t) = \vec{a}_c(t) + \vec{a}_t(t)$ with $\vec{a}_c \cdot \vec{a}_t = 0$; $a_c = \omega^2 R$; $a_t = \alpha R$; $\alpha = \dot{\omega} = \ddot{\theta}$; $v = \omega R$, $\omega = 2\pi/T$;
$KE = \frac{m}{2}v^2$, $W = \vec{F}\cdot\vec{r} = F\Delta r\cos\theta$ (for constant forces); $W = \int_{x_i}^{x_f} F(x)dx$ (for 1D problems); $W = \Delta KE$; conservative systems: $W = -\Delta PE$; in 1D: $PE \equiv V(x)$ characterized by: $F(x) = -\frac{dV}{dx}$