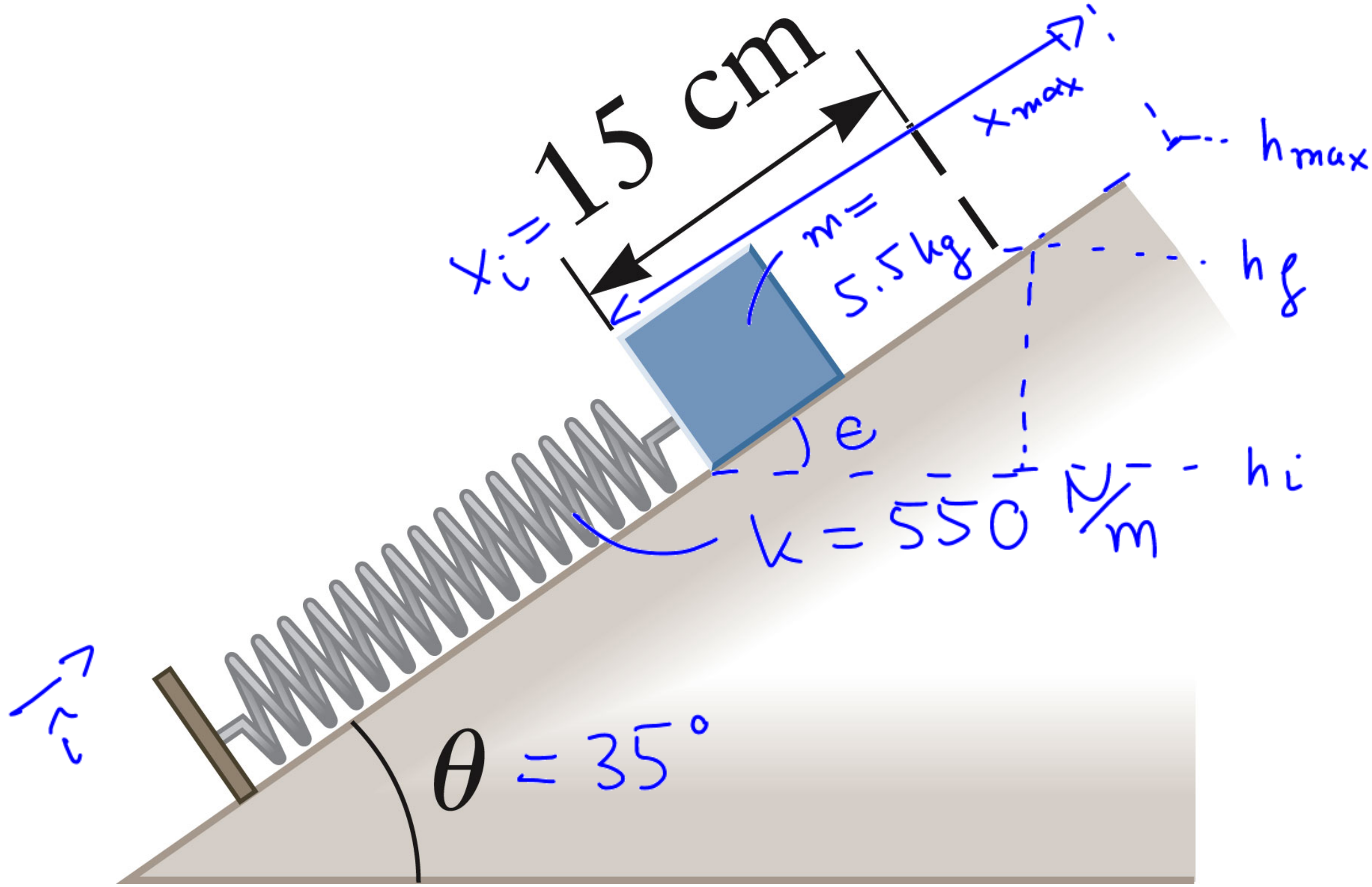
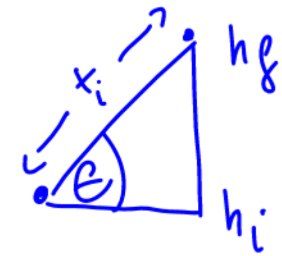
Tutorial Nov 20

Problem 6.92



(a) use $(TE)_i = (TE)_f$

$$(TE)_i = \frac{m}{2}\underbrace{v_i^2}_{=0} + \frac{k}{2}x_i^2 + mgh_i$$



$$(TE)_f = \frac{m}{2}v_f^2 + mgh_f$$

$$\sin\theta = \frac{h_f - h_i}{x_i}$$

$$\Leftrightarrow h_f = x_i \sin\theta + h_i$$

$$(TE)_i = \frac{k}{2}x_i^2 + \cancel{mgh_i}$$

$$= (TE)_f = \frac{m}{2}v_f^2 + mgx_i\sin\theta + \cancel{mgh_i}$$

$$\Leftrightarrow \quad v_f = \sqrt{\frac{k}{m}x_i^2 - 2gx_i\sin\theta} = 0.75\,\text{m/s}$$

(b) Maximum distance x_{max}

energy conservation (again): $(TE)_f = (TE)_{max}$

$$\frac{m}{2} v_f^2 + mgh_f = mgh_{max}$$

$$h_{max} = x_{max}\sin\theta + h_i$$

$$h_f = x_i \sin\theta + h_i$$

$$\Leftrightarrow \quad \frac{v_f^2}{2} + g x_i \sin\theta = g x_{max} \sin\theta$$

$$\Leftrightarrow \quad x_{max} = \frac{v_f^2}{2g\sin\theta} + x_i = 20\ \text{cm}$$

Additional problem for Nov. 20

Consider an elastic head-on (i.e. one-dimensional) collision that involves two objects of mass m_1 and m_2. The latter mass is initially at rest.

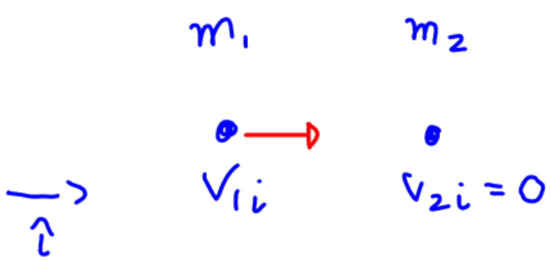
1. Starting from momentum and energy conservation derive equations for the final velocities of both objects after the collision.

2. What happens if $m_1 = m_2$?

3. Calculate the final velocities for $m_1 = 2.5$ kg, $m_2 = 4.2$ kg, and $v_1 = 12$ m/s.

4. Assume now that both objects stick together after the collision. Derive a formula for the final velocity and calculate it.

5. Calculate the change in total kinetic energy ΔKE for the sticky collision.



(1) momentum conservation

$$m_1 v_{1i} + \underbrace{m_2 v_{2i}}_{0} = m_1 v_{1f} + m_2 v_{2f} \qquad (1)$$

kinetic energy conservation

$$\frac{m_1}{2} v_{1i}^2 = \frac{m_1}{2} v_{1f}^2 + \frac{m_2}{2} v_{2f}^2 \qquad (2)$$

now combine (1) and (2) to find v_{1f}, v_{2f} :

(1)

$$\Leftrightarrow \quad v_{1f} = v_{1i} - \frac{m_2}{m_1} v_{2f} \quad (3)$$

insert in (2): $m_1 v_{1i}^2 = m_1 \left(v_{1i} - \frac{m_2}{m_1} v_{2f} \right)^2 + m_2 v_{2f}^2$

$$\Leftrightarrow \quad \cancel{v_{1i}^2} = \cancel{v_{1i}^2} - 2 \frac{m_2}{m_1} v_{1i} v_{2f} + \left(\frac{m_2}{m_1} \right)^2 v_{2f}^2 + \frac{m_2}{m_1} v_{2f}^2$$

$$\Leftrightarrow \quad 2 \frac{m_2}{m_1} v_{1i} \cancel{v_{2f}} = v_{2f}^{\cancel{2}} \left(\frac{m_2^2 + m_1 m_2}{m_1^2} \right)$$

$$\Leftrightarrow \quad \boxed{v_{2f} = 2 v_{1i} \frac{m_1}{m_1 + m_2}}$$

insert in (3) $\boxed{v_{1f}} = v_{1i} - \frac{m_2}{\cancel{m_1}} \frac{\cancel{m_1}}{(m_1 + m_2)} 2 v_{1i} = \boxed{\frac{m_1 - m_2}{m_1 + m_2} v_{1i}}$

(2) special case $m_1 = m_2$: $\boxed{v_{1f} = 0, \quad v_{2f} = v_{1i}}$

- another special case: $m_1 << m_2$ such that $m_1 \pm m_2 \approx \pm m_2$

$$\Leftrightarrow \quad v_{1f} = -v_{1i} \qquad \text{(reflection)}$$

$$v_{2f} << v_{1f}$$

(3) $v_{1f} = -3.0\ m/s$, $v_{2f} = 9.0\ m/s$

(4) Sticky collision: $m_1 v_{1i} + \underbrace{m_2 v_{2i}}_{=0} = (m_1 + m_2) v_f$

$$\Leftrightarrow \quad v_f = \frac{m_1 v_{1i}}{m_1 + m_2} = 4.5\ m/s$$

(5) $KE_i = \frac{m_1}{2} v_{1i}^2$, $KE_f = \frac{m_1 + m_2}{2} v_f^2$

$$\Leftrightarrow \quad \Delta KE = KE_f - KE_i = \frac{1}{2}\left[(m_1 + m_2) v_f^2 - m_1 v_{1i}^2\right]$$

$$= -110\ J$$

Problem 7.20

Completely inelastic = sticky collision of masses m and 3m; combined system at rest after collision

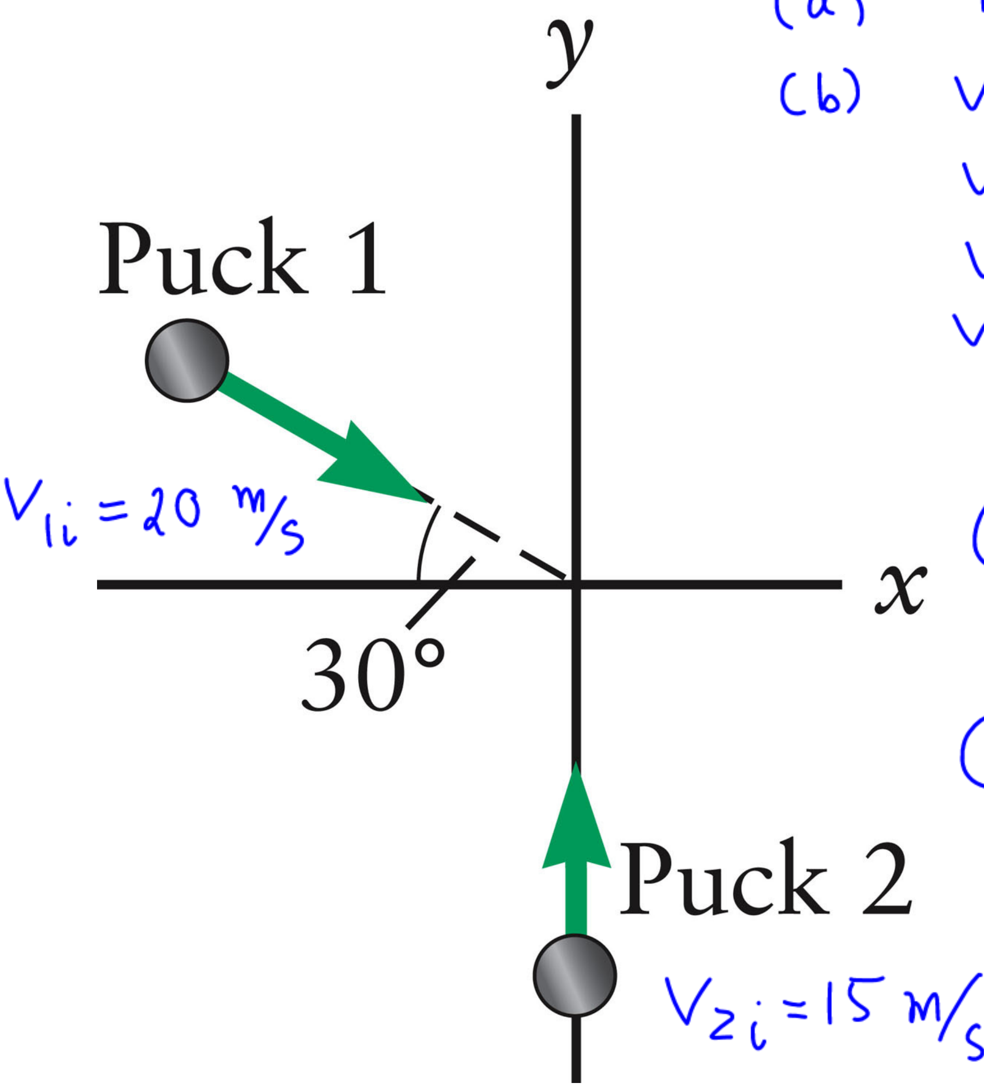
(1D) $$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

(momentum conservation)

$$\left.\begin{array}{l} m_1 = m \\ m_2 = 3m \\ v_f = 0 \end{array}\right\} \Rightarrow \quad \cancel{m} v_{1i} + 3\cancel{m} v_{2i} = 0$$

$$v_{1i} = -3 v_{2i} \Leftrightarrow \frac{v_{1i}}{v_{2i}} = -3$$

Problem 7.24: 2D sticky collision



(a) Total momentum is conserved

(b) $V_{1ix} = \overbrace{V_{1i}}^{20}\cos 30 = 17.3$ m/s

$V_{1iy} = -20\sin 30 = -10$ m/s

$V_{2ix} = 0$ m/s

$V_{2iy} = +15$ m/s

(c) + (e) use momentum conservation

($\hat{i}$) $m_1 V_{1ix} = (m_1 + m_2) V_{fx}$

$m_1 = m_2 = m$ $\Leftrightarrow$ $V_{fx} = \frac{V_{1ix}}{2} = 8.66$ m/s

($\hat{j}$) $mV_{1iy} + mV_{2iy} = 2mV_{fy}$

$\Leftrightarrow$ $V_{fy} = \frac{V_{1iy} + V_{2iy}}{2} = 2.5$ m/s

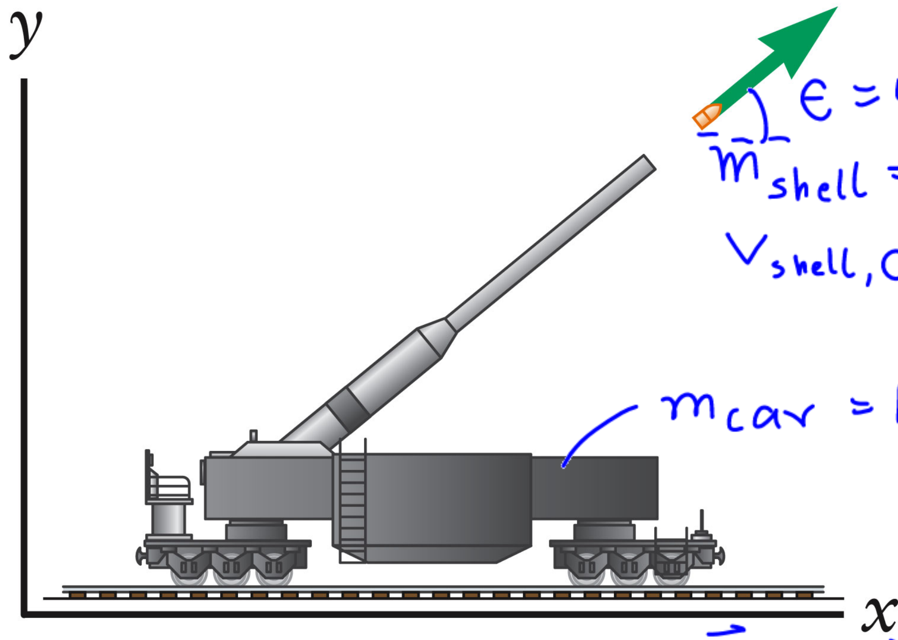
(d) It's an inelastic collision.

(f)
$$KE_i = \frac{m}{2} V_{1i}^2 + \frac{m}{2} V_{2i}^2$$

$$KE_f = \frac{2m}{2} V_f^2 = m \left(V_{fx}^2 + V_{fy}^2 \right)$$

$$\text{fraction} = \left| \frac{\Delta KE}{KE_i} \right| = \frac{KE_i - KE_f}{KE_i} = 0.74$$

Problem 7.34



$\theta = 40°$

$m_{shell} = 30\,kg$

$v_{shell,0} = 300\,m/s$

$m_{car} = 1500\,kg$

momentum conservation: $\vec{P}_i = \vec{p}_{car,i} + \vec{p}_{shell,i} = 0$

$$= \vec{P}_f = \vec{p}_{car,f} + \vec{p}_{shell,f}$$

$\hat{i}$: $0 = m_{car} v_{car,x} + m_{shell} v_{shell,x}$, $v_{shell,x} = v_{shell,0}\cos\theta$

$$v_{car,x} = -\frac{m_{shell}}{m_{car}} v_{shell,0}\cos\theta$$

$$= -4.6\,m/s$$