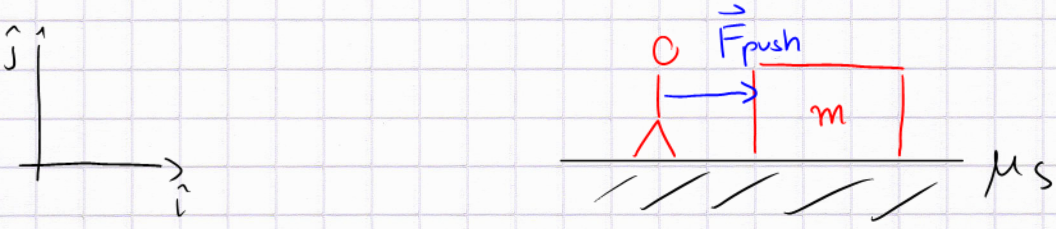
Tutorial, Oct 9

Challenge problem # 2

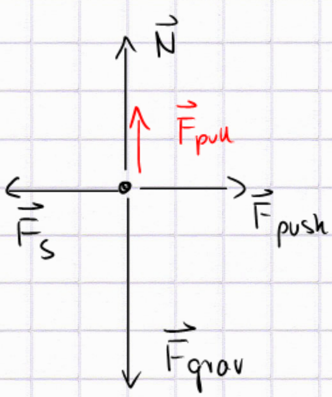


① $F_s^{max} = \mu_s N = \mu_s mg = 127\ N$

② Crate doesn't move $(F_s^{max} > F_{push})$

③ $F_s = F_{push} = 110\ N$ (such that $F_{net,x} = 0$)

④ FBD



equilibrium conditions:

$\hat{j}$: $F_{pull} + N = F_{grav}$

$\Leftrightarrow N = mg - F_{pull}$

$\hat{i}$: $F_s^{max} = \mu_s N = \mu_s(mg - F_{pull}) \overset{!}{=} F_{push}$

$\Leftrightarrow F_{pull} = mg - \frac{F_{push}}{\mu_s} = 46 N$

→ least vertical pull needs to be larger than 46 N.

⑤ $F_{pull}^{horizontal} = \mu_s mg - F_{push} = 17\ N$ → least horizontal pull needs to be larger than 17 N.

extra: is there an optimum angle for pulling the crate?

start from $\vec{F}_{pull} = F_{pull,x}\,\hat{i} + F_{pull,y}\,\hat{j}$

equilibrium conditions
$\hat{j}$: $N = mg - F_{pull,y} = mg - F_{pull} \sin\theta$
$\hat{i}$: $F_s^{max} = F_{pull,x} + F_{push} = F_{pull}\cos\theta + F_{push}$

$= \mu_s N = \mu_s(mg - F_{pull}\sin\theta)$

$$\Leftrightarrow \quad F_{pull}(\cos\theta + \mu_s \sin\theta) = \mu_s mg - F_{push}$$

$$\Leftrightarrow \quad F_{pull} = \frac{\mu_s mg - F_{push}}{\cos\theta + \mu_s \sin\theta}$$

this equation tells us how the minimum F_{pull} varies as a function of θ

Now, we need to find the minimum of $F_{pull}(\theta)$:

$$\Leftrightarrow \quad F'_{pull}(\theta) = \frac{-(\mu_s mg - F_{push})(-\sin\theta + \mu_s \cos\theta)}{(\cos\theta + \mu_s \sin\theta)^2} \overset{!}{=} 0$$

$$\Leftrightarrow \quad -\sin\theta + \mu_s \cos\theta = 0 \quad \Leftrightarrow \quad \theta = \tan^{-1}\mu_s$$

indeed. $F_{pull}(\theta = 20.3°) = 15.9\ N \approx 16\ N$

(3.45) Remember: rolling motion $\leftrightarrow$ static friction

$$F_s^{max} = \mu_s N = \mu_s \frac{m_{car}\, g}{4}$$ (consider one of the 4 tires)

Newton-2: $m_{car}\, a_{max} = 2 F_s^{max}$ (you have 2 drive tires)

$$= \frac{\mu_s m_{car}\, g}{2}$$

$$\Leftrightarrow \quad \mu_s = 2 \frac{a_{max}}{g} = 0.82$$

(3.46) (a) Car is skidding $\rightarrow$ kinetic friction

Newton-2: $m a_x = -\mu_k N = -\mu_k m g$ (constant acceleration)

$$\Leftrightarrow \quad v_x = v_0 - \mu_k g t$$

$$x = x_0 + v_0 t - \frac{\mu_k g}{2} t^2$$

coming to a stop: $v_x = 0 \Leftrightarrow t_{stop} = \frac{v_0}{\mu_k g}$

distance travelled: $\Delta x_{skid} = x(t_{stop}) - x_0$

$$= v_0 t_{stop} - \frac{\mu_k g}{2} t_{stop}^2$$

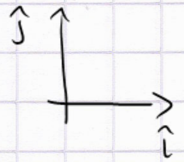
$$= \frac{v_0^2}{2 \mu_k g} = 54 \text{ m}$$

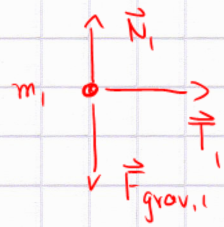
(b) Wheels not locked up $\rightarrow$ static friction

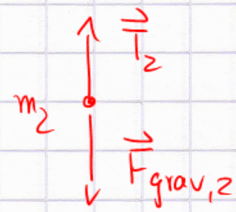
$$\Delta x_{no} = \frac{v_0^2}{2\mu_s g} = 48 \text{ m}$$

(c) $$\frac{\Delta x_{skid}}{\Delta x_{no}} = \frac{\mu_s}{\mu_k} = 1.13$$

4.44 (a) FBDs







note that $T_1 = T_2 = T$

(b) $\vec{N}_1 = m_1 g \hat{j}$ $\qquad$ $\vec{T}_2 = T\hat{j}$

$\vec{T}_1 = T\hat{i}$ $\qquad$ $\vec{F}_{grav,2} = -m_2 g \hat{j}$

$\vec{F}_{grav,1} = -m_1 g \hat{j}$

(c) $\hat{i}: \; m_1 a_x^{(1)} = T$ $\qquad$ $m_2 a_x^{(2)} = 0$

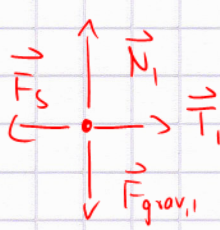
$\hat{j}: \; m_1 a_y^{(1)} = N - m_1 g = 0$ $\qquad$ $m_2 a_y^{(2)} = T - m_2 g$

(d) Note that $a_x^{(1)} = -a_y^{(2)} \iff \frac{T}{m_1} = -\frac{1}{m_2}(T - m_2 g)$

$$\iff T = \frac{m_1 m_2}{m_1 + m_2} g = 93\,N, \quad a = a_x^{(1)} = \frac{T}{m_1} = 2.1 \; m/s^2$$

4.12 Now include friction

FBD (crate 1)



$N_1 = m_1 g$ (still)

if equilibrium: $F_s = T = m_2 g \leq F_s^{max} = \mu_s N_1 = \mu_s m_1 g$

$\iff m_2 \leq \mu_s m_1$

→ fulfilled ⇒ static equilibrium situation!