Tutorial, Oct 30

5.12 Centripetal acceleration

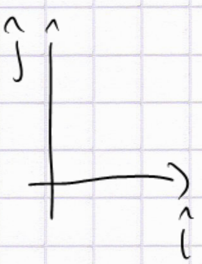
$a_c > 8g$: black out

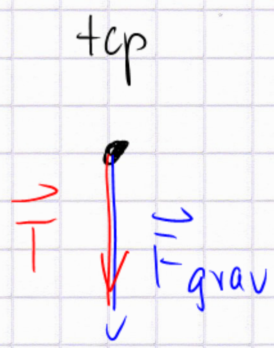
$V = 900\ m/s$

$$a_c = \frac{v^2}{R_{min}} \stackrel{!}{=} 8g$$

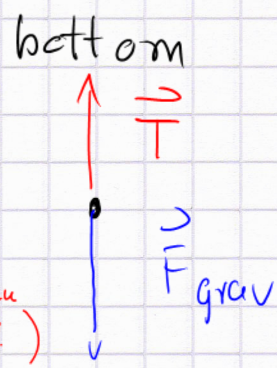
$$\Leftrightarrow \quad R_{min} = \frac{v^2}{8g} = 1.0 \times 10^4\ m$$

5.16 a) Free-body diagrams





(note that the $\vec{T}$ vector should actually be much longer than the $\vec{F}_{grav}$ vector!)



b) $$F_{net} = \frac{mv^2}{R}$$ with given values for m, v, R

$$= 270\ N$$

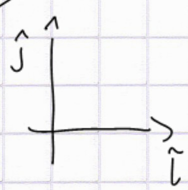
c) top $$F_{net} = \frac{mv^2}{R} = T + mg$$

$$\Leftrightarrow \quad T = \frac{mv^2}{R} - mg = 260\ N$$

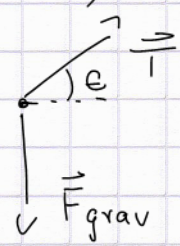
bottom $\quad F_{net} = \frac{mv^2}{R} = T - mg$ ②

$$\Leftrightarrow \quad T = \frac{mv^2}{R} + mg = 290 \text{ N}$$

5.24



FBD (side view)



$F_{net} = T\cos\theta$ (total force is directed toward centre of circle)

$$\hat{i}: \quad ma_c = \frac{mv^2}{R} = T\cos\theta \qquad (*)$$

$$\hat{j}: \quad 0 = T\sin\theta - mg \quad \Leftrightarrow \quad T = \frac{mg}{\sin\theta}$$

$$\overset{(*)}{\Leftrightarrow} \quad \frac{mv^2}{R} = mg\frac{\cos\theta}{\sin\theta} \quad \Leftrightarrow \quad \tan^{-1}\left(\frac{gR}{v^2}\right) = \theta = 8.4°$$

Extra problem

Start from $\vec{r}(t) = R\left(\cos\theta(t)\,\hat{i} + \sin\theta(t)\,\hat{j}\right)$

$$\Rightarrow \quad |\vec{r}(t)| = r = R$$

use short-hand notations $\dot{\theta} = \frac{d\theta}{dt}$, $\theta = \theta(t)$ etc.)

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t) = R\left(-\dot{\theta}\sin\theta\,\hat{i} + \dot{\theta}\cos\theta\,\hat{j}\right)$$

$$\Rightarrow \quad |\vec{v}(t)| = v = R|\dot{\theta}| = R|\omega|$$

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = R\left[\left(-\ddot{\theta}\sin\theta - \dot{\theta}^2\cos\theta\right)\hat{i} + \left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right)\hat{j}\right]$$

$$= \underbrace{-\dot{\theta}^2 R\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)}_{= \vec{a}_c(t)} + \underbrace{\ddot{\theta} R\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)}_{= \vec{a}_t(t)}$$

with $a_c = |\vec{a}_c(t)| = \dot{\theta}^2 R = \omega^2 R$

$$a_t = |\vec{a}_t(t)| = |\ddot{\theta}|R = |\dot{\omega}|R = |\alpha| R$$

angular acceleration

(4)

(1) $a_c = \dot{\theta}^2 R = \omega^2 R = \left(\frac{v}{R}\right)^2 R = \frac{v^2}{R}$ ✓

a_c changes with time if speed v changes with time

(2) $a_t = |\ddot{\theta}| R = |\dot{\omega}| R = \left|\frac{d}{dt}\left(\frac{v}{R}\right)\right| R$

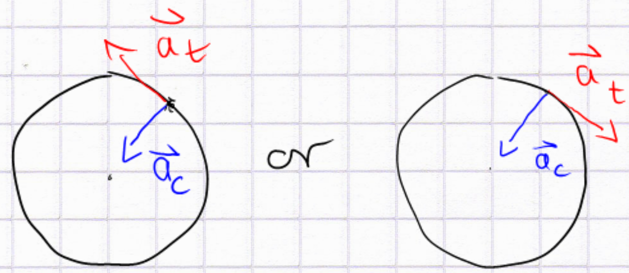
$= \frac{1}{\cancel{R}}\left|\frac{dv}{dt}\right|\cancel{R} = \left|\frac{dv}{dt}\right| = |\dot{v}|$

a_t changes with time if $\dot{v}$ does.

(3) Show $\vec{a}_c \cdot \vec{a}_t =$

$= -R^2 \dot{\theta}^2 \ddot{\theta}\left(-\sin\theta\cos\theta + \sin\theta\cos\theta\right)$

$= 0$



(4) Uniform circular motion: $v = \text{const}$

$\hookrightarrow \dot{v} = a_t = 0$