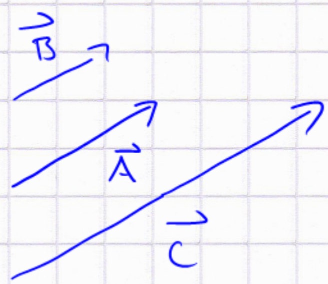
Math prep : vectors and trigonometry

Sept 7 and Sept 10

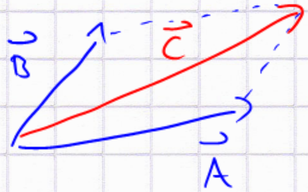
- vectors: = quantity with magnitude and direction (orientation)
  - can be represented by arrows (length corresponds to magnitude)
  - can be multiplied by scalars (to change length and possibly orientation)
  - can be added

ex 1:



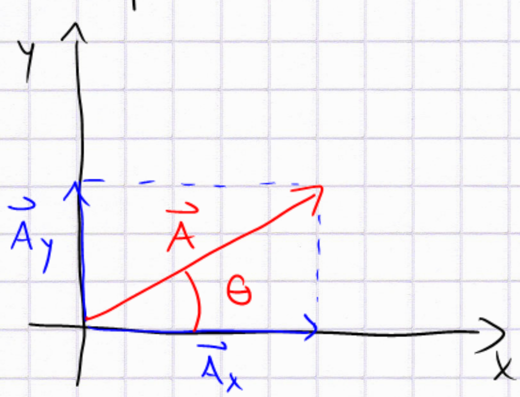
$$\left.\begin{aligned}\vec{B} &= 0.5\,\vec{A}\\ \vec{C} &= 2\,\vec{A}\end{aligned}\right\} \Rightarrow \vec{C} = 4\vec{B}$$

ex 2:



$$\vec{C} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

- Component vectors and vector components



$$\vec{A} = \underbrace{\vec{A}_x + \vec{A}_y}_{\text{"component vectors"}}$$

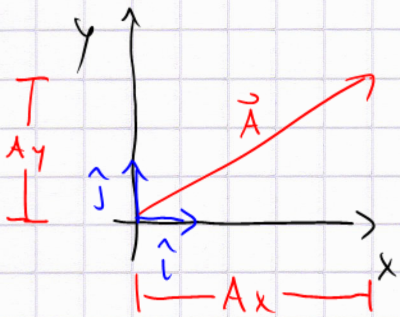
use trig relations to find relationships between $\vec{A}$ and $\vec{A}_x, \vec{A}_y$

- $\sin\theta = \dfrac{|\vec{A}_y|}{|\vec{A}|} = \dfrac{A_y}{A}$
- $\cos\theta = \dfrac{|\vec{A}_x|}{|\vec{A}|} = \dfrac{A_x}{A}$

- $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{A_y}{A_x}$ $\Rightarrow$ $\theta = \tan^{-1}(\tan\theta) = \tan^{-1}\dfrac{A_y}{A_x}$

- $A = \sqrt{A_x^2 + A_y^2}$ (Phythagoras)

- unit vectors $(|\hat{\imath}| = |\hat{\jmath}| = 1)$

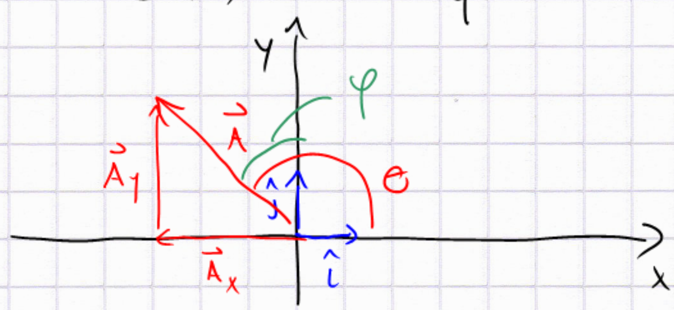


$$\Rightarrow \vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} = \overbrace{(A_x, A_y)}^{\text{"vector components"}}$$

$$(\vec{A}_x = A_x\hat{\imath}\,;\; \vec{A}_y = A_y\hat{\jmath})$$

note: so far, we have assumed that $\vec{A}$ be in the 1st quadrant.

consider $(\measuredangle)$ 2nd quadrant now



$$\vec{A} = \vec{A}_x + \vec{A}_y \quad \text{(still)}$$
$$= A_x\hat{\imath} + A_y\hat{\jmath}$$

$\Rightarrow$ $\vec{A}_x = A_x\hat{\imath}$ with $A_x = -|\vec{A}_x| < 0$

$\vec{A}_y = A_y\hat{\jmath}$ with $A_y = |\vec{A}_y| > 0$

consistency checks: $A\cos\theta = A\cos(90+\varphi) = -A\sin\varphi = A_x < 0$

$A\sin\theta = A\sin(90+\varphi) = A\cos\varphi = A_y > 0$

$\Rightarrow$ vector components can be negative, i.e., in general they are <u>not</u> the magnitudes of the corresponding component vectors!

- addition of vectors (do it algebraically now)

$$\vec{C} = \vec{A} + \vec{B} = A_x\hat{\imath} + A_y\hat{\jmath} + B_x\hat{\imath} + B_y\hat{\jmath}$$

$$= (A_x + B_x)\,\hat{i} + (A_y + B_y)\,\hat{j}$$

$$\equiv \quad C_x \quad \hat{i} \quad + \quad C_y \quad \hat{j}$$

example : $A_x = 2 \qquad B_x = 1$
$A_y = 1 \qquad B_y = 2$

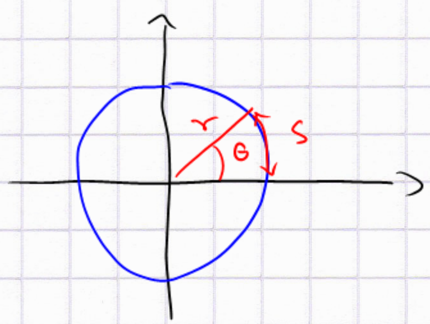
$\Rightarrow \quad \vec{C} = (2+1)\,\hat{i} + (1+2)\,\hat{j} = (3,3)$

$C = |\vec{C}| = \sqrt{C_x^2 + C_y^2} = \sqrt{3^2 + 3^2} = \sqrt{18} = 4.25$

$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1} 1 = 45°$

(graph it !)

Angles : radians + degrees



define angle $\theta := \frac{s}{r}$

< one complete trip : $s = 2\pi r$

$\Rightarrow \quad \theta = \frac{2\pi r}{r} = 2\pi$ (radians)

$= 360°$

$\longrightarrow \quad \theta\,[\text{radians}] = \theta\,[\text{degrees}] \times \frac{2\pi}{360}$

exercise : complete the table

θ [rad]	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$y = \sin\theta$									
$x = \cos\theta$									

and graph the functions $y(\theta) = \sin\theta$ and $x(\theta) = \cos\theta$!