

**Alexander Nenashev**

**Math 6122    Algebra II**

**Winter 2022    "I. Categories (basics)**

Categories: definitions and examples. Isomorphisms; automorphisms of an object.

Functors: definitions and examples. Commutative diagrams. Categorical constructions: objects over or under a given object, category of arrows. Initial and terminal objects.

Products and coproducts. Natural transformations of functors, natural equivalences of functors; equivalent categories.

II.A. Field theory (Sections 13.1, 2, 4, 5, 6)

Characteristic. Field extensions: degree, finite/infinite extensions, algebraic and transcendental elements. Getting a root for a polynomial; minimal polynomial for an algebraic element. Splitting field of a polynomial (existence and uniqueness). Algebraic closures. Separable polynomials and separable extensions. Cyclotomic fields.

II.B. Galois theory (Sections 14.1-6)

Automorphisms of a field extension. Roots of polynomials under automorphisms.

Subgroups of  $\text{Aut}(K)$  versus subfields of  $K$ . Automorphisms of a splitting field, Galois extensions and Galois groups. Characters of a group with values in a field.

Characterization of Galois extensions (normal and separable). Conjugate elements and conjugate fields. The Fundamental Theorem of Galois Theory; examples. Galois theory of finite fields. Simple extensions, cyclotomic extensions, Abelian extensions. Galois groups of polynomials.

III. Homological algebra (basics)

Complexes (of modules) and their (co)homology modules. Possibility of extension to Abelian categories. Chain maps of complexes and induced maps in (co)homology; functoriality. The fundamental theorem of homological algebra (the snake lemma) and the 5-Lemma. Examples: homology groups in topology; (co)homology of groups.

IV. More of module theory (Section 10.5)

Injective, projective and flat modules and related exact sequences. Exact functors.

V. Introduction to commutative rings and algebraic geometry (Sections 15.1-3)(time permitting)

Affine algebraic sets versus ideals. Radicals of ideals. The role of prime ideals. Integral extensions of rings and Hilbert's zero locus theorem."

**Huaiping Zhu**

**Math 6340    Ordinary Differential Equations**

**Winter 2022    "Topics   General properties of Differential Equations**

Existence, uniqueness, dependence on initial data and parameters, extensibility.

- Linear Systems and Stability General theory of linear systems, Periodic coefficients and Floquet Theory, Stability of linear and nonlinear systems.

- Nonlinear Systems: Local Theory Linearization, Invariant manifolds, Hartman-Grobman Theorem, Normal form theory.

- Nonlinear Systems: Global Theory Limit sets and attractors, periodic sets and limit cycles; Poincare map, Poincare-Bendixson Theory, Lotka-Volterra system, Liénard systems. .

- Bifurcation theory of nonlinear systems: Saddle-node bifurcation, transcritical bifurcation, pitchfork bifurcation, Hopf bifurcation, homoclinic-bifurcations, co-dimension 2 and 3 bifurcations.

- Nonlinear dynamics and applications in physics, finance and biology.

**Comprehensive Exam****Course information**

Title Probability Theory

Course number MATH 6605

Semester Winter

**Outline of topics to be covered**

Weak law of large number, Strong law of large numbers, Borel Cantelli Lemma, convergence in probability, almost sure convergence, weak convergence, characteristic functions, central limit theorem, conditional expectations, martingales

**Reading sources and textbooks**

Rosenthal - A first look at rigorous probability theory

**Evaluation method**

The comprehensive exam will coincide with the final examination in the course, which will take place in-person.

Passing threshold Satisfactory mastery of the course concepts

% of the comprehensive exam A grade of B in the exam (70%)

This form must be submitted electronically by email to the Director of the Graduate Program with signature of the faculty member.

Faculty	Signature	Date
Name Tom Salisbury		Nov 2, 2021

**Comprehensive Exam****Course information**

Title Generalized Linear Models

Course number 6622

Semester Winter, 2021-2022

**Outline of topics to be covered**

Likelihood theory and maximum likelihood estimation, Link functions, Newton-Raphson and EM algorithm, The Fisher-scoring method and the reiterated weighted least squared method, Binary logistic regression, Nominal and ordinal logistic regression, Poisson regression and log linear models, Generalized estimating equation, Application to continuous, binary/multinomial, and discrete counting data.

**Reading sources and textbooks**

Textbook: A. J. Dobson and A. G. Barnett (2008). An Introduction to Generalized Linear Models (3rd ed).  
References: Agresti (2013). Categorical Data Analysis (3rd ed); Faraway (2016). Extending the Linear Model with R: Generalized Linear, Mixed Effects, and Nonparametric Regression Models; McCullagh and Nelder (1989). Generalized Linear Models (2nd ed); Venables and Ripley (2002). Modern Applied Statistics with S (4th ed).

**Evaluation method**

The Ph.D. students should obtain at least 60/100 to pass the comprehensive exam.

Passing threshold 60

% of the comprehensive exam

This form must be submitted electronically by email to the Director of the Graduate Program with signature of the faculty member.

Faculty	Signature	Date
Name Wei Liu		October 5, 2021

## **Comprehensive Exam Winter 2022**

**Xin Gao**

**Math 6627    Practicum to Statistical Consulting**

**Winter 2022**

Procedure:

The comprehensive exam takes two components: theory exam and practical exam. For theory exam, the students take the final exam in the course.

The practical exam consists of a case-study project. The following is an outline of the process and expectations for the practical exam.

The student should help identify statistical aspects of the problem presented by the case study and formulate the problem statistically as precisely as possible at the preliminary state.

The student has about two weeks to perform the proposed statistical analysis. Based on the statistical analysis, the student summarizes the findings and write a summary report on the project.

The student is expected to give an informal presentation of the contents of the report following the manner in which an actual consultant would present and interpret statistical results to a client.

Evaluation is based on the performance on both final exam and on the practical exam.

The evaluation of the practical exam is based on :

- 1) Your ability to understand the research questions presented by the case study, including the ability to probe for relevant additional information,
- 2) Your attention to important aspects of data design for methods used in analysis and for the scope of conclusions.
- 3) Your creativity in crafting appropriate statistical approached
- 4) Your ability to communicate statistical finings effectively.

Preparations for the exam:

Before taking the exam you should have a broad background in statistics at a level corresponding to completion of a master's program in statistics.

## **MATH 6652 3.0 Numerical Solutions to Differential Equations: Syllabus for comprehensive exam, Winter 2021-22**

### **Evaluation:**

The final exam for Math 6652 is to serve as the comprehensive exam in numerical analysis. As per the course outline, the final exam is weighted as 60% of the course grade. The exam will have 6 questions; four questions will be evaluated out of 15 marks while two questions will be evaluated out of 20 marks (total 100 marks). A passing grade for the exam will be 60/100 marks. The exam will be written in person at the University and invigilated by the course instructor.

The main themes stressed in the exam problems are consistency, stability, and accuracy of numerical schemes applied to parabolic, elliptic and hyperbolic partial differential equations. One problem at the end of the exam is designed to ensure the students have learned basic skills in Matlab computing, reflecting the practical/lab component of the course.

### **Principal Text:**

K.W. Morton and D.F Mayers, Numerical Solution of PDEs, Cambridge University Press, 1994.

### **Other References:**

J.C. Strikwerda, Finite Difference Schemes and Partial Differential Equations, Second Ed., SIAM, Philadelphia, 2004.

C. Johnson, Numerical Solution of Partial Differential Equations by the Finite Element Method, Cambridge University Press, Cambridge, 1987.

## **COMPREHENSIVE EXAMINATION TOPICS:**

### **Section 1: Finite Difference Formulations**

- 1.1 Taylor series expansion
- 1.2 Mixed partial derivatives
- 1.3 Finite difference equations

### **Section 2: Parabolic Equations in One Space Dimension**

- 2.1 Explicit methods: FTCS, Richardson, and Dufort-Frankel
- 2.2 Implicit methods: Laasonen, Crank-Nicolson, beta (theta) formulation
- 2.3 Truncation error, consistency, and convergence

### **Section 3: Parabolic Equations in Two and Three Space Dimensions**

- 3.1 Standard finite difference methods
- 3.2 ADI method in two and three dimensions
- 3.3 Approximate factorization
- 3.4 Truncation error, consistency, and convergence
- 3.5 General problems

### **Section 4: Stability Analysis**

- 4.1 Discrete perturbation stability analysis
- 4.2 Fourier stability analysis
- 4.3 Stability of explicit and implicit methods
- 4.4 Multi-dimensional problems

### **Section 5: Linear Second Order Elliptic Equations in One and Two Space Dimensions**

- 5.1 Explicit iterative methods: Jacobi, point Gauss-Seidel, point successive over-relaxation.
- 5.2 Implicit iterative methods: Line Gauss-Seidel, Line successive over-relaxation, ADI method.
- 5.3 Stability, error analysis, and consistency.

### **Section 6: Hyperbolic Equations in One Space Dimension**

- 6.1 Explicit methods for linear equations: FTFS, FTCS, first upwind, Lax, Leapfrog, Lax-Wendroff.
- 6.2 Implicit methods for linear equations: BTCS, Crank-Nicolson.
- 6.3 Multi-step methods for linear equations: Richtmyer, Lax-Wendroff, MacCormack.
- 6.4 Explicit methods for non-linear equations: Lax, Lax-Wendroff, MacCormack.
- 6.5 Implicit methods for non-linear equations: Beam and Warming.
- 6.6 Linear damping.
- 6.7 Flux corrected transport.