1. Motivation
2. Gauge fields on a D-Wave quantum annealer (York University)
3. Adding quarks and using IBM quantum computers (Waterloo and York)
From bits to qubits

Classical computers use bits. One bit is either $|0\rangle$ or $|1\rangle$.

Quantum computers use qubits. One qubit is a superposition of $|0\rangle$ and $|1\rangle$.

$$\theta \phi = |0\rangle = \cos\left(\frac{\theta}{2}\right) |0\rangle + e^{i\phi} \sin\left(\frac{\theta}{2}\right) |1\rangle$$

Multiple bits act independently. Multiple qubits can be entangled, so measuring one affects the others.

NOTE: Qubits can be in a superposition of all classically allowed states.
Where to get qubits

In principle, any two-state quantum system can be used for each qubit. The trick is to control the entangled state while avoiding environmental noise.

Our work uses qubits from D-Wave (Vancouver) and IBM (New York). In both cases, the qubits are superconducting circuits called transmons.

Two ways to use qubits

D-Wave, 5760 qubits, no gates

IBM, 7 qubits, universal gate set
Would qubits be helpful for your computational physics?

I don’t know. Quantum computing is new. Methods are still being developed.

Let me tell you about the situation in my field of research. Perhaps it will give you some ideas for your own context.

We use lattice gauge theory to study quarks and gluons. Quarks are spin-$\frac{1}{2}$ particles just like qubits, but gluons are not. Note that we need quantum fields, not just quantum mechanics.

We use a 4D lattice in Euclidean spacetime.
Lattice gauge theory is very successful without qubits

http://flag.unibe.ch/2019/Quark%20masses
What qubits might do for lattice gauge theory

Quantum computers offer an efficient Hamiltonian-based approach that might... allow us to avoid Euclidean time, thus moving from statics to dynamics. allow us to include a chemical potential, thus reaching nuclear densities.

*Lattice QCD at non-zero density* would be valuable for heavy-ion collisions, the early Universe and neutron-star structure. In practice, simulations at finite $\mu$ suffer from a “sign problem” and are at a rudimentary stage.

— paraphrased from Particle Data Group, Review of Lattice QCD
The state of the art


$SU(2)$ non-Abelian gauge field theory in one dimension on digital quantum computers
What a D-Wave quantum annealer calculates


SU(2) lattice gauge theory on a quantum annealer

The hardware performs its annealing by initializing the system into the ground state of a simple Hamiltonian and then moving quasi-adiabatically to a requested Ising Hamiltonian of this form:

\[
H(q) = \sum_{i=1}^{N} h_{i}q_{i} + \sum_{i=1}^{N} \sum_{j=i+1}^{N} J_{ij}q_{i}q_{j}
\]

Each \( q_{i} \) is either 0 or 1.
The user can choose any real-values coefficients \( h_{i} \) and \( J_{ij} \).
No gates required!

Is there an Ising Hamiltonian that describes SU(2) gauge theory?
The Hamiltonian for SU(2) gauge theory

We work in the basis where colour-electric terms are on-diagonal and colour-magnetic terms are off-diagonal.

\[ \hat{H} = \frac{g^2}{2} \left( \sum_{i=\text{links}} \hat{E}_i^2 - 2x \sum_{i=\text{plaquettes}} \hat{\Box}_i \right) \quad \text{with} \quad x \equiv \frac{2}{g^4} \]

As you might expect, the total colour-electric energy is a sum over links, and the total colour-magnetic energy is a sum over plaquettes.

There is only one parameter: the gauge coupling \( g \) (or more conveniently \( x \)).
Our lattices

SU(2) colour uses the algebra you know from angular momentum, $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$

two-plaquette lattice:

\[
\begin{array}{c|c}
\hline
j_B & j_D \\
\hline
j_E & j_F \\
\hline
j_A & j_C \\
\hline
\end{array}
\]

four-plaquette lattice:

six-plaquette lattice:
Constructing the Hamiltonian

\[
\langle \psi | \hat{E}_i^2 | \psi \rangle = j_i (j_i + 1) \\
\langle \psi_{\text{final}} | \hat{\square}_1 | \psi_{\text{initial}} \rangle = (-1)^{j_A+J_E+j_I} \sqrt{(2j_I + 1)(2J_E + 1)} \left\{ \begin{array}{ccc}
  j_A & j_E & j_I \\
  \frac{1}{2} & J_I & J_E \\
\end{array} \right\} \\
(-1)^{j_C+J_E+j_J} \sqrt{(2j_E + 1)(2J_J + 1)} \left\{ \begin{array}{ccc}
  j_C & j_E & j_J \\
  \frac{1}{2} & J_J & J_E \\
\end{array} \right\} \\
(-1)^{j_D+J_F+j_J} \sqrt{(2j_J + 1)(2J_F + 1)} \left\{ \begin{array}{ccc}
  j_D & j_F & j_J \\
  \frac{1}{2} & J_J & J_F \\
\end{array} \right\} \\
(-1)^{j_B+J_F+j_I} \sqrt{(2j_F + 1)(2J_I + 1)} \left\{ \begin{array}{ccc}
  j_B & j_F & j_I \\
  \frac{1}{2} & J_I & J_F \\
\end{array} \right\}
\]
For the two-plaquette lattice with $j_{\text{max}} = \frac{1}{2}$:

$$H = \frac{g^2}{2} \begin{pmatrix} 0 & -2x & -2x & 0 \\ -2x & 3 & 0 & -\frac{x}{2} \\ -2x & 0 & 3 & -\frac{x}{2} \\ 0 & -\frac{x}{2} & -\frac{x}{2} & 3 \end{pmatrix}$$

Our Hamiltonian matrices have these sizes:

<table>
<thead>
<tr>
<th>$N_{\text{plaq}}$</th>
<th>$j_{\text{max}}$</th>
<th>Size of $H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1/2</td>
<td>$4 \times 4$</td>
</tr>
<tr>
<td>4</td>
<td>1/2</td>
<td>$16 \times 16$</td>
</tr>
<tr>
<td>6</td>
<td>1/2</td>
<td>$64 \times 64$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>$27 \times 27$</td>
</tr>
<tr>
<td>2</td>
<td>3/2</td>
<td>$95 \times 95$</td>
</tr>
</tbody>
</table>
The quantum annealer eigensolver (QAE)

Recall the variational method: \( E_0 \leq \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} \).

Recall that D-Wave finds the minimum of \( H(q) = \sum_{i=1}^{N} h_i q_i + \sum_{i=1}^{N} \sum_{j=i+1}^{N} J_{ij} q_i q_j \).

If the \( |\psi\rangle \) vector has only 0 and 1 as entries, then those are basically the same!

\[
\begin{align*}
q & \rightarrow |\psi\rangle \\
h_i & \rightarrow \text{on-diagonals of } H \\
J_{ij} & \rightarrow \text{off-diagonals of } H
\end{align*}
\]

QAE handles a general vector (fixed-point representation) and the normalization. It uses one penalty term (called \( \lambda \)).


We built an adaptive QAE to use fewer qubits and solve larger Hamiltonians. Its only parameter is the \( \lambda \) from original QAE.
Ground state eigenvalue for two plaquettes and $j_{\text{max}} = \frac{1}{2}$

Data points are from QAE.

Curves are exact eigenvalues.

Raw data for $x = 0.5$ in the graph above.

1000 anneals were used.

Each anneal took 20 microseconds.
The importance of our adaptive algorithm

The original QAE has no adaptive step, so zoom=0.
Our AQAE is helpful on a classical simulator.
Our AQAE is vital for larger Hamiltonians on noisy quantum hardware.
Assessing the gauge truncation

\[ \langle 0 | H_E | 0 \rangle \]
\[ \langle 0 | H_{SU(2)} | 0 \rangle \]
\[ \langle 0 | H_{\square} | 0 \rangle \]

solid curves are \( j_{\text{max}} = \frac{1}{2} \)
dashed curves are \( j_{\text{max}} = 1 \)
dotted curves are \( j_{\text{max}} = \frac{3}{2} \)
Including quarks

Consider a one-dimensional lattice. It will have no colour-magnetic fields.

Put quarks and antiquarks on alternating sites ("staggered fermions").

We need two qubits per lattice site.
Absorbing the gauge fields

There are two physics parameters: the gauge coupling and the quark mass.

With open lattice boundaries, gauge field effects are long-range quark interactions.

\[ \hat{H} = x\tilde{m}\hat{H}_m + \hat{H}_{el} + x\hat{H}_{kin} \]

\[ \hat{H}_m = 2 \sum_{n=1}^{N-1} \left( \frac{(-1)^n}{2} \left( \hat{\sigma}^z_{2n-1} + \hat{\sigma}^z_{2n} \right) + 1 \right) \]

\[ \hat{H}_{kin} = - \sum_{n=1}^{N-1} \left( \hat{\sigma}^+_{2n-1} \hat{\sigma}^z_{2n} \hat{\sigma}^-_{2n+1} + \hat{\sigma}^+_{2n} \hat{\sigma}^z_{2n+1} \hat{\sigma}^-_{2n+2} + \text{h.c.} \right) \]

\[ \hat{H}_{el} = \frac{3}{8} \sum_{n=1}^{N-1} (N - n)(1 - \hat{\sigma}^z_{2n-1} \hat{\sigma}^z_{2n}) \]

\[ + \frac{1}{8} \sum_{n=1}^{N-2} \sum_{m=n+1}^{N-1} (N - m) \left( \hat{\sigma}^z_{2n-1} - \hat{\sigma}^z_{2n} \right) \left( \hat{\sigma}^z_{2m-1} - \hat{\sigma}^z_{2m} \right) \]

\[ + \sum_{n=1}^{N-2} \sum_{m=n+1}^{N-1} (N - m) \left( \hat{\sigma}^+_{2n-1} \hat{\sigma}^-_{2n} \hat{\sigma}^+_{2m} \hat{\sigma}^-_{2m-1} + \text{h.c.} \right) \]
Computing the meson mass

VQE preparation of the low-lying energy spectrum

- Meson (VQE)
- Vacuum (VQE)
- Exact meson energy
- Exact vacuum energy

N=2

Circuit for the variational estimation of the overlap

- \( \theta_1 \)
- \( \theta_2 \)
- \( \theta_1^\vee \)
- \( \theta_2^\vee \)
- \( \Theta_1 \)
- Control Y gate
- Parameterized Y gate
- X gate

Circuit for variational energies

- \( \theta_1 \)
Computing the baryon mass

a  VQE circuit to prepare baryon and vacuum states

b  VQE preparation of the baryon mass

\[ M_b \]

- Baryon mass (VQE)
- Exact baryon mass

SU(2) “proton”

SU(2) “quark”

\[ n = 4 \]
Computing the meson-to-baryon mass ratio

For continuum SU(2), the meson and baryon are exactly degenerate.

Our staggered lattice calculation is consistent with this continuum limit.
The future

What approach to lattice gauge theory will be most practical on larger lattices? For which observables can a quantum advantage be attained? Can we quantify the speed of quantum annealing versus classical computing?

What can quantum computers do for you?