PHYS 3090: Homework 3 (due Friday Oct. 9)

1. Let $z = -2 - 2i$. Compute $\log(z)$, taking the principal branch with $0 \leq \arg(z) < 2\pi$. **(5 points)**

2. Express $z = \frac{2+i}{1-i}$ in the form $z = x + iy$. **(5 points)**

3. Show that for any complex numbers z_1 and z_2, the following are true:

- $|z_1 z_2| = |z_1|\,|z_2|$ **(5 points)**
- $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$ **(5 points)**
- $\arg(z_1/z_2) = \arg(z_1) - \arg(z_2)$ **(5 points)**

4. Find the solutions of the equation $z^4 - 16i = 0$. **(10 points)**

5. Find the solutions of the equation $z^4 - 6z^2 + 8 = 0$. **(10 points)**

6. Any complex function can be expressed as $f(z) = u(x, y) + i\,v(x, y)$, where u, v are purely real functions and $z = x + iy$. Find $u(x, y)$ and $v(x, y)$ for the following complex functions:

- $f(z) = e^{iz}/z$ **(5 points)**
- $f(z) = z\ln(z)$ **(5 points)**
- $f(z) = \cos(2z)$ **(5 points)**

7. Show explicitly that the following functions satisfy the Cauchy-Riemann relations and identify points z, if any, where $f(z)$ is not analytic.

- $f(z) = \tan(z)$ **(5 points)**
- $f(z) = e^{1/z}$ **(5 points)**