PHYS 3090: Homework 7 (due Friday Nov. 20)

Problem 1: Consider a quantum mechanical particle of mass m in the ground state of an infinite one-dimensional square well with walls at $x = 0$ and $x = L/2$.

- What is the normalized wavefunction $\psi_0(x)$ for this state?

Suppose at time $t = 0$, the right wall is moved suddenly from $x = L/2$ to $x = L$. The wavefunction $\Psi(x, t)$ now evolves according to the Schrodinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = i\hbar \frac{\partial \Psi}{\partial t}\,,$$

with the initial condition

$$\Psi(x, 0) = \begin{cases} \psi_0(x) & 0 < x < L/2 \\ 0 & L/2 < x < L \end{cases}\,, \qquad \dot{\Psi}(x, 0) = 0\,,$$

and boundary condition $\Psi(0, t) = \Psi(L, t) = 0$.

- Using Fourier series methods, compute $\Psi(x, t)$.
- Compute $\langle x(t) \rangle$ and show that the particle undergoes simple harmonic motion. What is the oscillation frequency?

Hint: As a simplifying approximation, you may keep only the first two Fourier modes.

Problem 2: Show that when a string of length L is plucked at a position L/k (where $k \geq 2$ is an integer), no Fourier modes of order $n = k, 2k, 3k, \ldots$ are excited.

(This effect is utilized in pianos to eliminate the 7th harmonic by having the hammer strike the piano string at a position $L/7$.)

Problem 3: Consider a periodic function $f(t)$, with period T, defined by

$$f(t) = \begin{cases} \frac{1}{\varepsilon} & 0 < t < \varepsilon \\ 0 & \varepsilon < t < T \end{cases}\,.$$

- Compute the Fourier coefficients a_n, b_n, c_n for this series.
- In the limit $\varepsilon \ll T$, show that high frequency Fourier modes ($n \gg T/\varepsilon$) are suppressed compared to low frequency modes ($n \ll T/\varepsilon$).