Towards a theory of thinking

Author(s)	Glatzeder, Britta ; Goel, Vinod ; Mèuller, Albrecht A. C. von
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Problem Solving

Michael Öllinger and Vinod Goel

There is no problem so big it can't be run away from.

– Charles Schultz

Abstract Problem solving and thinking are inseparably linked together. We propose that a theory of thinking has to consider and incorporate the notion of problem solving. In this chapter, we review the most important accounts of problem solving and hope to convince the reader that problem solving may provide an ideal framework for developing a theory of thinking.

We start with a broad summary on the Gestaltist perspective. The Gestaltists per se understood thinking as problem solving. They invented a large body of theoretical concepts and ingenious tasks that until now influence cognitive psychology in general and unexpectedly affects the development of the information processing account also. However, this influence becomes less and less explicit and is not appropriately recognized. We hope to stress this connection and bring it back to the readers' minds. Nevertheless, the Gestaltist approach has its weaknesses and methodological flaws, which will be dealt with in this chapter.

A large section is dedicated to the information processing account that still dominates the problem solving literature as a clear and proper account for describing and defining human problem solving. We elaborate on the differentiation between well and ill-defined problems and provide several foundations and models derived from this account. Nevertheless, the information processing account has its limits and we conclude with some extensions of the classical account and provide an integrative model for insight problem solving.

M. Öllinger (✉)
Parmenides Center for the Study of Thinking, Munich, Germany
e-mail: michael.oellinger@parmenides-foundation.org

V. Goel
Department of Psychology, York University, Toronto, Canada
Department of Psychology, University of Hull, UK
e-mail: vgoel@yorku.ca

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1 Introduction

In this chapter, we will focus on the study of thought processes through the study of problem-solving. Problem solving can be understood as the bridging of the gap between an initial state of affairs and a desired state where no predetermined operator or strategy is known to the individual. For example, consider the following task: $3 \times 4 = ?$ This task does not constitute a problem for most adults. They can automatically produce the result from memory. For a 7-year-old child, on the other hand, who is learning to multiply, it is a problem. The child has to consciously apply rules and procedures to bridge the gap between the initial problem state and a solution state. For a 2-year-old the situation is not recognizable as a problem because the child lacks the knowledge and semantics to understand what he/she is being asked to do.

In the following sections we will review psychological theories on problem solving, beginning with the work of the Gestaltist psychologists, and the subsequent development within the framework of information processing theory, and then point out two outstanding challenges that the information processing framework needs to confront and overcome.

2 The Gestaltist Perspective

At the beginning of the last century, the Gestaltist approach (Wertheimer 1912, 1925, 1959; Koffka 1935; Katona 1940; Duncker 1945; Köhler 1947) emerged as a countermovement to the dominant learning theory of Behaviorism. For the Gestaltist, thinking was not a reproductive recombination of learned associations but the meaningful effort to understand the fundamental nature and affordances of the given problem situation and the desired goal as a whole. They assumed that thinking obeyed similar basic principles (Gestalt laws) as perception. The Gestaltist idea was that, as in the flipping of the Necker Cube, there are also major transitions during the process of problem solving characterized by restructuring the given information in new and nonobvious ways. Restructuring reveals the fundamental structure of the problem. Problem solving was viewed as a process of transforming a disturbed Gestalt into a good Gestalt ("gute Gestalt"). It is a goal directed behavior that clears out existing barriers in the service of gaining a desired end (for an overview see Ash 1998; Öllinger and Knoblich 2009).

Between 1914 and 1917 Wolfgang Köhler investigated chimpanzees on Tenerife island. He addressed the question of whether chimps are able to solve problems in an intelligent way. He hoped to find evidence against the Behaviorist dictum that animals solve problems by pure trial and error (Thorndike 1911; Köhler 1921, 1925). He claimed that intelligent behavior can be observed when the obvious way to the goal is blocked by a barrier. That is, intelligence is used to elude existing barriers in new and unfamiliar situations. He created situations in which his apes had to solve problems. Sultan, the star pupil, was asked to get a banana that was out of reach. There were two sticks lying around in the compound. After a few minutes

Sultan purposefully joined the sticks together and successfully fished for the banana. For Köhler these findings provided evidence that some animals were able to solve problems not simply by blind and mindless trial and error attempts, but by insight into the affordances of the given situation.

Max Wertheimer the most famous and influential Gestaltist was particularly interested in the sudden moment of restructuring that accompanied insight in a given problem. Wertheimer contrasted *productive thinking* (Wertheimer 1959) with *reproductive thinking (Thorndike* 1911*)*. He was certain that productive thinking is superior to reproductive thinking, because it is characterized by gaining deep insight into the relations of the given problem constituents and their role in the given task, and the resulting solution. Wertheimer worked on a general psychological theory of problem solving that can be applied to various phenomena, ranging from low-level perceptual phenomena, to solving problems like crypt arithmetic, to explaining great scientific inventions, to problems in the social domain (Wertheimer 1959). Restructuring was the basic mechanism for resolving problems across a wide range of domains.

The Gestaltists demonstrated what they meant by "restructuring" in a series of elegant examples (Wertheimer 1925, 1959). Figure 1a depicts a typical example. The task is to determine the area of the isosceles triangle, given the length of the side s and the angle at the apex (90°). At first glance people might try to determine the two segments g and h in order to apply the triangle formula $\frac{1}{2} g \times h$ for the area. This is a laborious approach. However, rotating the triangle reveals that the triangle can be understood as one half of a square with the diagonal g and the side with the length s (Fig. 1b). Now, the area can be determined by the simple formula $s \times s / 2$.

Restructuring the given situation requires a problem solver to overcome the reproductive tendency to compute the triangle area in the usual way and see it in a new way as part of a larger good Gestalt.

Another problem that Wertheimer analyzed was the famous enumeration problem solved by the young Gauss. The task was to add as quickly as possible the sum

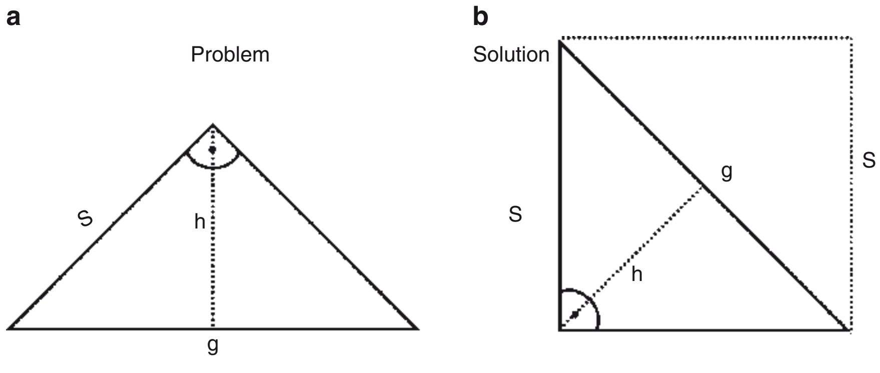

Fig. 1 Wertheimer's Triangle problem (**a**). The task was to determine the area of the given triangle. Insightful solution of the problem (**b**)

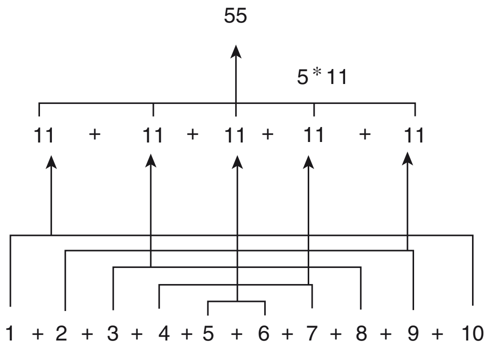

Fig. 2 Gauss' Enumeration Problem. Determine the sum of the given series. The trick is to re-cluster the problem elements and multiply the number of clusters. In this case the value of a cluster is 11 and there are 5 of those clusters

of 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10. Anecdotally, it is said that almost immediately Gauss proclaimed "Here it is!", and he showed his disbelieving teacher the correct solution (see Fig. 2). The reproductive solution requires stupid blind successive addition of consecutive numbers. However, Gauss found a productive solution in terms of a general principle of arithmetic progression ($(n + 1) \times (n/2)$; n = length of a series). Of course, the longer the series the more effective is the productive approach. The reader is invited to try both approaches on a series from 1 to 100.

Probably the most important Gestaltist work on problem solving was reported by Karl Duncker in his Monograph: On Problem-Solving (Duncker 1945). Duncker extended the basic principle of restructuring by a general framework that views problem solving as a stepwise process situated in a problem space which people navigate by means of strategies or heuristics. Duncker anticipated some concepts that later became the fundamentals of Newell and Simon's Problem Space Hypothesis (Newell and Simon 1972).

Duncker also introduced a number of classical problems, like the radiation problem and the candle problem, into the literature. The radiation problem asked the following question:

> Given a human being with an inoperable stomach tumor, and rays which destroy organic tissue at sufficient intensity, by what procedure can one free him of the tumor by these rays and at the same time avoid destroying the healthy tissue which surrounds it? (Duncker 1945, p. 1–2).

This has proved to be a fairly difficult problem. The solution is to use more than one laser of weak intensity and arrange them in a way that their rays exactly meet right in the heart of the tumor. The superimposed radiation destroys the tumor and does no harm to the surrounding tissue. In Duncker's studies, participants were asked to "think aloud" or verbalize thoughts and ideas as they are attended to, while solving the problem. This technique has become an important methodical instrument, within information processing theory, for mapping intermittent steps in thinking processes onto cognitive models (Schooler and Engstler-Schooler 1990; Ericsson and Simon 1993; Schooler et al. 1993, Goel and Pirolli 1992).

Duncker analyzed the thinking-aloud protocols and systematically developed graphs, such as in Fig. 3.

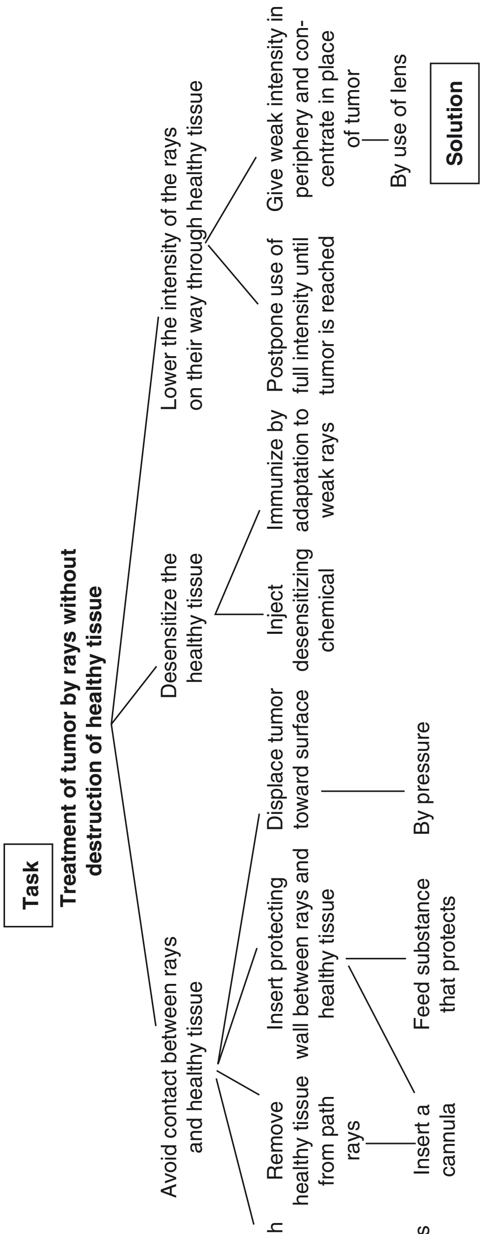

Fig. 3 Incorrect solution attempts and the correct solution for Duncker's radiation problem

This graph classifies different kinds of solution attempts for the radiation problem according to their "functional value". The functional value of a solution "is exactly what is called the sense, the principle or the point of the solution" (Duncker 1945, p. 4). For the radiation problem Duncker identified three basic methods participants used to solve the problem, but eventually only the "Lower the intensity of rays" path led to a correct solution.

The graph illustrates a hierarchical problem solving structure. The top of the graph represents the initial state and constraints given in the task, followed by potential solutions for meeting task requirements. The process of traversing the graph is accompanied by crosschecking for the suitability of steps and their relationship to the solution. Duncker found out that people search in both directions, that is, either from the initial state to the goal state or from a potential goal state back to the initial state (Holyoak 1995). This kind of task analysis is fairly similar to the more recent concepts of problem space and search within information processing theory (Newell and Simon 1972) that still dominates the current problem solving research (see below).

The Gestaltists were also interested in how prior knowledge influences the solution of problems. They assumed that prior knowledge can hamper productive thinking. In his famous Candle Problem, Duncker asked participants to find a way to create a ledge on the wall to place a candle. Matches, a candle, and a box of thumbtacks were placed on a table top in front of subjects. The solution is to remove the thumbtacks from the box, and use the box as a ledge and fasten it with the tacks onto the wall, and rest the candle on the box/ledge. The problem is quite difficult. Only a small percentage of people find the correct solution. Duncker explained the problem by assuming that problem solvers *fixate* on the container function of the box. That is, the experience with boxes as containers to put things in was detrimental to seeing it in another way. In a second experiment he placed the thumbtacks either inside the box or beside the box, thus emphasizing or deemphasizing the container function of the box. He found that in the case where the thumbtacks were placed beside the box (deemphasizing its container function) the probability of using the box as a platform was increased.

Further experimental studies provided support for Duncker's assumptions (Maier 1931; Birch and Rabinowitz 1951). Luchins (1942) showed that functional fixedness did not only appear when using objects in an uncommon way. He demonstrated also that the repeated application of the same solution procedure can result in a *mental set* that prevents people from applying alternative and more efficient solution strategies.

Luchins examined mental set by using water jug problems (Luchins 1942; Luchins and Luchins 1959; Luchins and Luchins 1994; Lovett and Anderson 1996; Öllinger et al. 2008). For example, given three jugs A, B, and C, with volumes of 21, 127, and 3 units, respectively, the goal is to end with an amount of 100 units. The solution is to pour water into B (127), then use the water in B to fill C twice, leaving 121 units in B. Now, pour 21 units from B into A.

Luchins created a set of problems that could be solved by the same solution procedure (B – 2 × C – A). After participants learned the solution they were confronted with a test problem that either could be solved with the previously learned solution procedure or with a simpler procedure. For example, given the volumes 23, 49, and 3 in jugs A, B, and C, with the goal of attaining 20 units, the simple alternative is to pour once water from A into C, and 20 units are left in A. Luchins' experiments showed that participants who had learned a solution procedure based on previous problems, continued to use the same procedure, even though a simpler procedure would suffice. A control group that only solved the test problems applied the easier procedure.

Functional fixedness and mental set became the key concepts of the Gestaltists to explain why productive thinking is often so difficult and why blind and stupid drill in school is detrimental for creative and insightful problem solving. Although the Gestaltists provided a number of valuable ideas and concepts (Novick and Bassok 2005) their theoretical and empirical contributions, and language were sometimes vague, unclear, phenomenological, and hard to formalize. In the next section we will introduce the information processing theory and the Problem Space Hypothesis (Newell and Simon 1972), which formalizes and builds upon a number of the ideas and insights of the Gestalt psychologists.

3 Information Processing Theory and the Problem Space Hypothesis

After the Gestaltist phase there was a short moratorium in problem solving research. In 1957 Newell and Simon presented their General Problem Solver (GPS) at the Dartmouth Conference (Newell et al. 1958). This meeting ushered in a new framework, accompanied by great optimism, that sooner or later thinking processes may be described and understood as computational processes (Ernst and Newell 1969). In the following sections we will review some of the key ideas and contributions of this research program, and then point out some limitations and challenges that it faces.

Information processing theory accounts of human problem-solving appeal to three main notions (see Fig. 4): (a) an information processing system, (b) the task

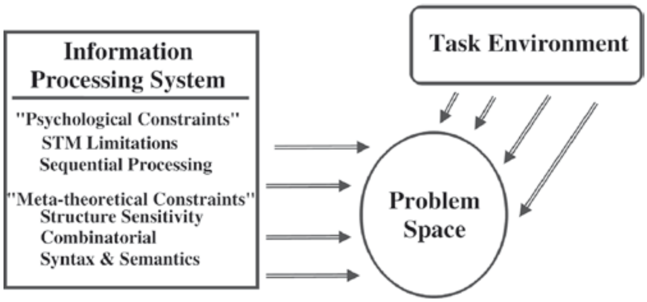

Fig. 4 The problem space is a computational construct shaped by the constraints imposed by the structures of the information processing system and the task environment. Reproduced from Goel (1995)

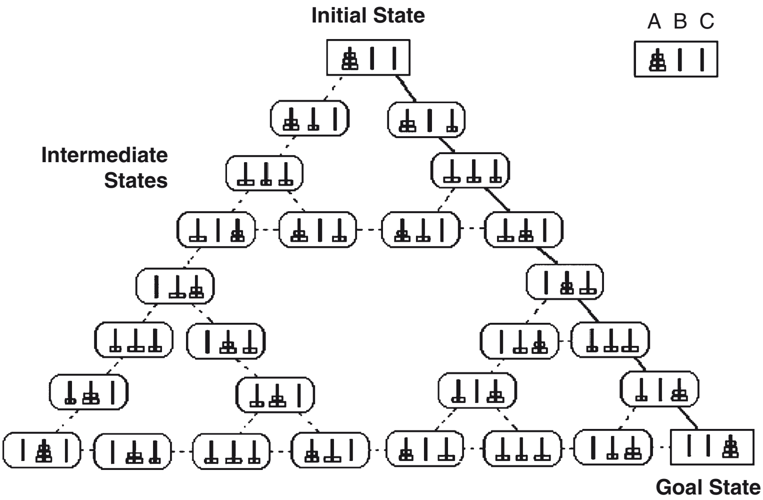

Fig. 5 The Tower of Hanoi puzzle consists of three pegs and several disks of varying size. Given a start state, in which the disks are stacked on one or more pegs, the task is to reach a goal state in which the disks are stacked in descending order on a specified peg. There are three constraints on the transformation of the start state into the goal state. (1) Only one disk may be moved at a time. (2) Any disk not being currently moved must remain on the pegs. (3) A larger disk may not be placed on a smaller disk. This is an example with three disks. The initial state, goal state and intermediate states are indicated. The optimal solution is the right-most sequence of consecutive moves

environment, and (c) the problem space. An Information Processing System (IPS) is a physical symbol manipulation system with memory stores (short term, long term, external), a processor, sensory receptors, and motor effecters. It brings to bear two sets of constraints. The psychological constraints consist of temporal and spatial limitations on working memory and sequential processing. The meta-theoretical constraints require that the information processing system be a computational system with combinatorial syntax and semantics, and structure sensitivity to process (see Fodor and Pylyshyn 1988). Task environments consist of (a) the goal, (b) the problem, and (c) other relevant external factors (Newell and Simon 1972). The problem space is a computational space shaped by the interaction of the constraints inherent in the information processing system and the task environment. It is defined by a state space, operators, evaluation functions, and search strategies.

We can illustrate each of the components of the problem space by reference to the well known Tower of Hanoi problem, depicted in Fig. 5. The state space consists of an initial state, a goal state, and intermediate states. The initial state (in Fig. 5) consists of three disks stacked on the left most peg of the puzzle. The goal state consists of the three disks stacked on the rightmost peg of this example. The initial state is transformed into the goal state by the application of a series of operators/transformation functions, resulting in intermediate states. In the case of the Tower of Hanoi the operator might consist of moving a disk from one peg to another peg, respecting the constraints that only one disk may be moved at a time, that any disk not being moved must remain on the peg, and that a larger disk may not be placed on a smaller disk. The application of operators to the initial state generates intermediate states.

Evaluation functions determine whether the solution path is moving one closer to or further away from the goal state. These functions may be used to select between different paths through the state space. For example, in the above Tower of Hanoi problem, one can move the small disk onto the middle peg or onto the rightmost peg. Most subjects will reason that if they move the disk to the rightmost peg, it will conflict with the goal state, because the largest peg needs to be placed on the rightmost peg first. Therefore, they will typically move the smallest disk to the middle peg. This actually turns out to be a mistake because it leads to some backward moves or the stacking of the disks on the middle (incorrect) peg.

The application of operators is guided by control strategies. The built-in strategies for searching this problem space include such content free universal methods as Means-Ends Analysis, Breath-First Search, Depth-First Search, etc. Means-ends analysis provides an important and elegant example of how a human like goal-directed behavior can be implemented in a computational system (Newell 1990; Anderson and Lebiere 1998). The application of the means-ends analysis follows three consecutive steps. First, the algorithm determines the distance between the current state and the goal state. Second, it tests whether there is an available operator that reduces the distance. If this is not the case a subgoal is created and pushed into a goal stack. Next, the procedure jumps back to the first step. The means-ends analysis creates subgoals until an operator is available that can be applied, third step. Now, the next stored subgoal is processed. The algorithm terminates when all subgoals are executed.

The strategy can be illustrated by application to the Tower of Hanoi puzzle in Fig. 5. The problem requires the large disk to be transferred to peg C by applying the move-the-largest-disk operator. This is only possible if the medium disk is moved (move-medium-disk operator; subgoal). To do so the smallest disk has to be cleared out of the way (move-small-disk operator, sub-sub-goal). This operator is available and the smallest disk can be moved to peg C. Now, it is possible to go back to the previous subgoal and apply the no longer blocked move-medium-disk operator to peg B. The small disk at peg C blocks the application of the move-large-disk operator therefore the move-small-disk operator is again applied and moves the smallest disk to peg B. Finally, the large disk is moved to peg C etc.

However the universal applicability of these formal methods comes at the cost of enormous computational resources. Given that the cognitive agent is a time and memory bound serial processor it would often not be able to respond in real time, if it had to rely on formal, context independent processes. So the first line of defense for such a system is the deployment of task-specific knowledge to circumvent formal search procedures.

A heuristic strategy for solving the Tower of Hanoi problem might be the following (Simon 1975): (1) on odd numbered moves, move the smallest disk; (2) on even numbered moves, move the next smallest exposed disk; (3) if the total number of disks is odd, move the smallest disk from the source peg to the target peg, to the other peg, to the source peg, etc. The strategy requires the concepts of odd-even moves and cycling a disk through the pegs in a particular order. The perceptual tests are quite simple, consisting of location of smallest and next smallest exposed disks and differentiating between target source and other pegs. The strategy also makes few computational demands, requiring only the retention of move parity in short-term memory and is therefore very easy to implement. However, it is an "unreasoned" heuristic strategy. It just happens to work for this problem. Gigerenzer and colleagues have undertaken extensive investigations into the nature and role of heuristics in human problem solving (Gigerenzer and Hug 1992; Gigerenzer and Todd, 2001a, 2001b).

The great achievement of the Information Processing Theory was the emphasis on detailed task analyzes, and the clear computational characterization of the problem space. It has resulted in new insights into the nature of certain types of problem-solving. However, 40 years after the onset of the program, the scope of the framework seems limited to a narrow range of problems. It is proving difficult, perhaps impossible, to encompass certain critical types of real-world problem solving within this framework. We've reviewed to search problem types below: Ill-structured problems and insight problems.

4 Challenge: Well-Structured Versus Ill-Structured Problems

The ill-structured/well-structured distinction originates with Reitman (1964). Reitman classified problems based on the distribution of information within the three components (start state, goal state, and the transformation function) of a problem vector. Problems where the information content of each of the vector components is absent or incomplete are said to be ill-structured. To the extent that the information is completely specified, the problem is well-structured.

A mundane example of an ill-structured problem is provided by the task of planning a meal for a guest. The start state is the current state of affairs. While some of the salient facts are apparent, it is not clear that all the relevant aspects can be immediately specified or determined (e.g., how hungry will they be?; how much time and effort do I want to expend?; etc.). The goal state, while clear in the broadest sense (i.e., have a successful meal), cannot be fully articulated (e.g., how much do I care about impressing the guest?; should there be 3 or 4 courses?; would salmon be appropriate?; would they prefer a barbecue or an indoor meal?; etc.). And finally, the transformation function is also incompletely specified (e.g., should I have the meal catered, prepare it myself, or ask everyone to bring a dish?; if I prepare it, should I use fresh or frozen salmon? etc.).

Well-structured problems on the other hand, are characterized by the presence of information in each of the components of the problem vector. The Tower of Hanoi (Fig. 5) provides a relevant example. The start state is completely specified (e.g., the disks are stacked in descending order on peg A). There is a clearly defined test for the goal state (e.g., stack the disks in descending order on peg C). The transformation function is restricted to moving disks within the following constraints: (1) Only one disk may be moved at a time. (2) Any disk not being currently moved must remain on a peg. (3) A larger disk may not be placed on a smaller disk.

Reitman's original characterization has been extended along a number of dimensions by (Goel 1995). One very important, but little noted, difference has to do with the nature of the constraints in the two cases. In the Tower of Hanoi, as in all puzzles and games, the constraints are logical or constitutive of the task. That is, if one violates a constraint or rule, one is simply not playing the game. For example, if I place a bigger disk on a smaller disk I am simply not doing the Tower of Hanoi task.

However, the constraints we encounter in real-world situations are of a very different character. Some of these constraints are nomological; many of them are social, political, economic, cultural, etc. We will encompass the latter category under the predicate "intentional". In fact one can view social, cultural, religious norms (e.g., Thou shalt not commit adultery! Thou shalt not lie!) as attempts to provide structure to our lives. However, as part of the educational processes, most of us quickly learn that these constraints are not definitional or constitutive of the task. On the contrary, they are negotiable/breakable, depending on circumstances (e.g., maybe it is ok if I don't get caught).

It is also the case that in most ill-structured situations, there are no right or wrong answers, though there are certainly better and worse answers (Rittel and Webber 1974). In the above dinner example, if our dinner guest eats what we serve, did we reach the correct goal state? This seems like an odd question. There will always be better and worse possibilities than any given outcome.

In well-structured problems there are right and wrong answers, and clear ways of recognizing when they have been reached. So if I succeed in stacking my disks in descending order on peg 3 in the Tower of Hanoi task, that is the one and only possible correct answer.

All problems require registration and decomposition, or at least individuation. There are differences with respect to the lines of decomposition/individuation and the interconnectivity of components. Well-structured problems have a predetermined structure, which is either explicitly given with the problem, or is implied by the logical structure of the problem. (So, for example, on a standard interpretation of the game of chess, each player starts with 16 game pieces. One does not have the option of claiming that the conjunction of one of the "rooks" and "knights" constitutes a game piece.).

In ill-structured problems, on the other hand, lines of decomposition/individuation are determined by the subject, taking into consideration the physical structure of the world, social and cultural practices, and personal preference.

In terms of the interconnectivity of parts, one finds logical interconnections in well-structured problems (e.g., in cryptarithmetic there is always the possibility that any row will sum to greater than 9 and affect the next row). Thus the subject has no choice or selectivity in attending to interconnections. Interconnections in ill-structured problems are contingent and one has considerable latitude in determining which ones to attend to and which ones to ignore.

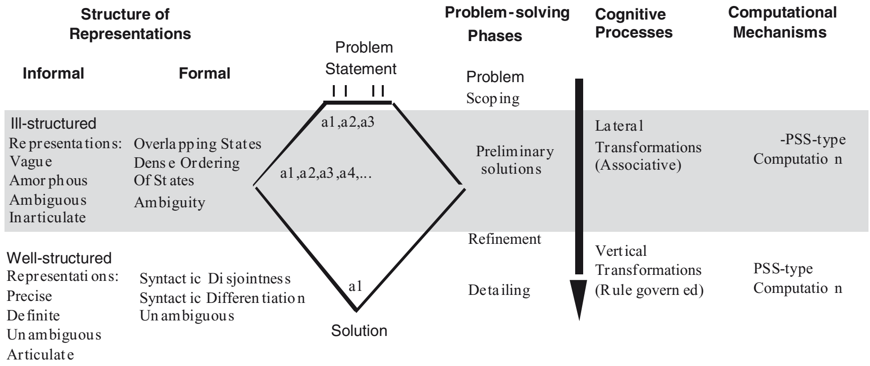

Fig. 6 Aspects of real-world problem-solving. Reproduced from Goel (1995)

Simon (1973) has famously argued that the distinction between ill-structured and well-structured problems is ill-conceived. The so-called "Ill-structured problems" are simply structured by adding information from our background knowledge and external sources and then one can specify the problem space and search for a solution. Subsequent research does not bear out this claim.

Goel (1995) views ill-structured problem-solving as typically involving the following four phases: *problem scoping*, *preliminary solutions*, *refinement*, and *detailing* of solutions. Each phase differs with respect to the type of information dealt with, the degree of commitment to generated ideas, the level of detail attended to, the number and types of transformations engaged in, the mental representations needed to support the different types of information and transformations, and the corresponding computational mechanism (Goel 1995). As one progresses from the preliminary phases to the detailing phases, the problem becomes more structured. This is depicted in Fig. 6.

Preliminary solution generation is a classical case of creative, ill-structured problem solving. It is a phase of "cognitive way-finding", a phase of concept construction, where a few kernel ideas are generated and explored through transformations. This generation and exploration of ideas/concepts is facilitated by the abstract nature of information being considered, a low degree of commitment to generated ideas, the coarseness of detail, and a large number of *lateral transformations*. A lateral transformation is one where movement is from one idea to a slightly different idea rather than a more detailed version of the same idea. Lateral transformations are necessary for the *widening* of the problem space and the exploration and development of kernel ideas. The rules underlying lateral transformations cannot be articulated (Goel 1995).

The *refinement* and *detailing* phases are more constrained and structured. They are phases where preconstructed concepts are manipulated. Commitments are made to a particular solution and propagated through the problem space. They are characterized by the concrete nature of information being considered, a high degree of commitment to generated ideas, attention to detail, and a large number of *vertical transformations*. A vertical transformation is one where movement is from one idea to a more detailed version of the same idea. It results in a *deepening* of the problem space. The rules underlying vertical transformations can often be articulated (Goel 1995).

Goel (1995) has argued that the ability to engage in lateral transformations is underwritten by a mechanism that supports ill-structured mental representations and computation. Ill-structured representations are imprecise, ambiguous, fluid, indeterminate, vague, etc. The ability to engage in vertical transformations is underwritten by a mechanism that supports well-structured mental representations and computation. Well-structured representations are precise, distinct, determinate, and unambiguous.

Furthermore, there is a computational dissociation between these two mechanisms (Giunti 1997; Goel 1995). Laboratory problems emphasize well-structured mental representations while real-world problems require both ill- and well-structured mental representations. Ill-structured and well-structured representations differ with respect to modes of inference and computational mechanisms. It has also been suggested that *there is an anatomical dissociation corresponding to the computational dissociation* (Goel 2005). If this analysis is correct, it severely limits the scope and relevance of the Newell and Simon framework to our understanding of human problem solving.

5 Challenge: Insight Problem Solving

Insight problems reveal further limitations of the classical information processing theory. The enigma of insight problems is that they mostly have a fairly small problem space and sometimes can be solved by only one single move; however they can be extremely difficult. Insight problems present a challenge for classical Information Processing Theory.

5.1 Definition of Insight

There is no general agreement on the characterization of insight problems. There are phenomenological, task, and process definitions (Knoblich and Öllinger 2006; Öllinger and Knoblich 2009). Two theoretical accounts try to extend the application of information processing theory to insight problems. The first account (Kaplan and Simon 1990) assumes that insight problems are nothing special. A second "process definition" account states that insight problems are characterized by the requirement of a representational change – in Gestalt terms a moment of restructuring, often accompanied by a kind of Aha! experience (Bowden et al. 2005).

5.2 Nothing Special Account

Insight problems are difficult because of an ill-defined initial problem representation that integrates far too much information. A much too large problem space is established and therefore an exhaustive search is impossible. A further related point is that in most cases, problem solvers do not have the appropriate heuristics for constraining the search space – that is the transformation function sensu Reitman is not specified. Therefore, insight problems are so difficult because they require the problem solvers to find an appropriate heuristics within an ill-structured problem representation (see also Chronicle et al. 2004; MacGregor et al. 2001; Ormerod, MacGregor, and Chronicle 2002).

> … noticing invariants is a widely applicable rule of thumb for searching in ill-defined domains, [but] there can be no guarantee that those noticed will be the critical ones for the particular problem. Nevertheless, the constraints offered by the notice-invariant heuristic are a vast improvement over blind trial and error search. Kaplan and Simon (1990) p. 404

Kaplan and Simon (1990) investigated their assumptions with the mutilated checkerboard problem (Wickelgren 1974). The task is to determine whether the remaining 62 squares (Fig. 7) can be covered with 31 dominos, or to show that this is impossible. The task was extremely difficult, only a few persons were able to correctly solve it. The answer is, it is impossible to cover the mutilated board. The reason is that two white squares have been removed. And a domino can only cover to adjacent squares of different colors. The problem became significantly easier by providing a version that explicitly emphasizes this aspect (Fig. 7; Bread-and-Butter version).

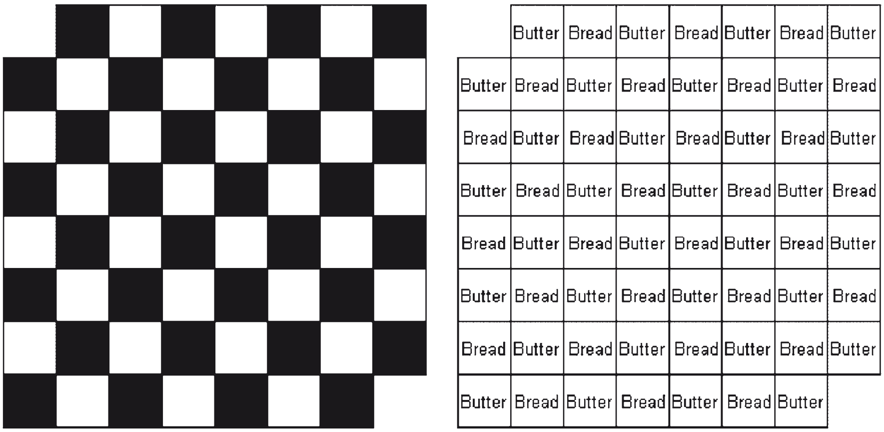

Fig. 7 Wickelgreen's mutilated checkerboard problem. On the left side the original problem, on the right side the Bread-and-Butter version that facilitates finding the solution

5.3 Representational Change

The second account (Ohlsson 1990; Knoblich et al. 1999) holds that people apply self-imposed and unconscious constraints on the problem representation. Such an over-constraint problem representation dramatically hampers the solution of the problem. The key mechanism for gaining insight is a representational change. A representational change modifies either the specification of the initial state (chunk decomposition), or the representation of the goal (constraint relaxation, see Knoblich et al. 1999; Öllinger et al. 2008).

During a successful problem solving process three different phases are crossed, namely before, within and after an *impasse*. Before an impasse, problem solving is driven by prior knowledge that suggests that the problem can be solved as usual. For insight problems those attempts normally lead into an impasse. An impasse is defined as a state of mind where problem solving attempts cease and the impression arises that the problem is unsolvable. A representational change either of the given problem situation or the goal representation is necessary. A new and more general representation is established. After an impasse the common strategies (e.g., means-ends analysis) are applied onto the new representation (Öllinger and Knoblich 2009). This assumption fits quite well to Wallas' (1926) four tier model of scientific problem solving. In the *preparation phase* people gather information and make first solution attempts. After a number of failed attempts the problem is put aside and other things come into focus. This phase is called *incubation*. After a while, *illumination* occurs. The key to the solution is found accompanied by insight and an Aha! experience. Finally, in the *verification* phase people verify whether the found solution works.

A prototypical example for the necessity of a change of an over-constraint goal representation is provided by Katona's triangle problem (Katona 1940). The task is to arrange six given matchsticks in a way that four equilateral triangles result. Most participants try to solve the problem in a 2D fashion (see Fig. 8a), but in 2D the problem is unsolvable. The solution requires overcoming the 2D constraint and searching for a solution in 3D.

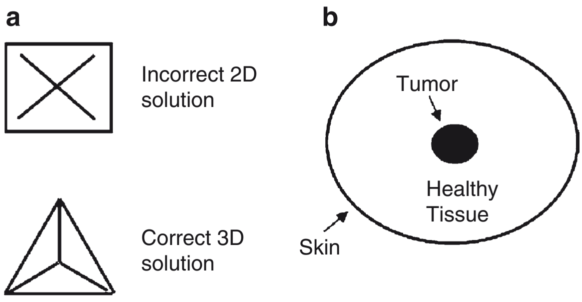

Fig. 8 (**a**) Solution and incorrect solution attempt for Katona's triangle problem. Note: The incorrect solution did indeed produce four equal triangles, but not equilateral triangles. (**b**) Duncker's radiation problem. Sketch used in an eye-movement study by Grant and Spivey (2003)

To investigate the dynamics and preconditions of representational change Grant and Spivey (2003) introduced a very elegant method. In two successive experiments they asked participants to solve Duncker's radiation problem (as introduced in the Gestalt section). Additionally, participants received a sketch (see Fig. 8b) of the problem. In the first experiment participants solved the problem and in parallel their eye movements were recorded. Analyzing the eye movement patterns revealed that solvers looked significantly longer at the skin than nonsolvers did.

The authors concluded that fixating at the tumor – the site where the desired effect should occur – constrains the problem representation and prevent a solution. If so, then, it should be possible to increase people's solution rate by guiding their attention towards the area around the skin. In the second experiment they introduced three conditions. In the first condition the surrounding skin slightly flickered. In the second condition the tumor flickered and finally, in the control condition participants got the static sketch. The result showed that guiding attention to the skin strongly increased the solution rate. This might provide evidence that some insight problems are difficult due to an over-constrained initial problem representation.

5.4 An Integrative Perspective

Currently, researchers are trying to develop more integrative models of insight problems that extend the classical Information Processing Account. There is a long standing and heated debate about the Nine-Dot problem (Maier 1930; Scheerer 1963; Weisberg and Alba 1981; MacGregor et al. 2001; Kershaw and Ohlsson 2004). The task is to connect nine dots with four connected straight lines without lifting the pen or retracing a line (see Fig. 9a). The problem proved extremely difficult. The solution rates usually lay between 0 and 10%.

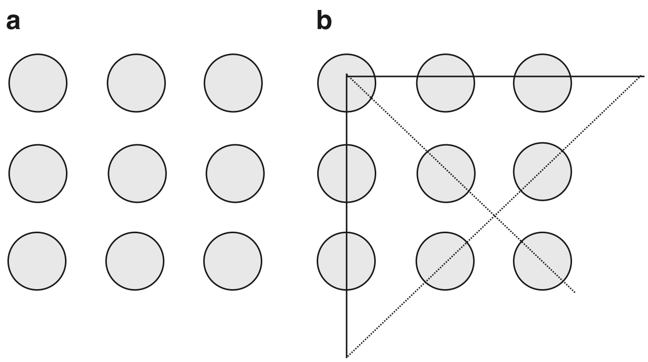

Fig. 9 Nine-Dot problem (**a**) with solution (**b**)

The Gestaltist's standard explanation was that fixation on the 3 × 3 dot matrix that forms a virtual square prevents moves outside the square's boundaries (Fig. 9b). The fixation is the result of perceptual Gestalt laws like figural integrity and figure ground perception (Maier 1931; Ohlsson 1990; Scheerer 1963). Weisberg and Alba (1981) questioned this interpretation. They deduced the following from the fixation assumption: if fixation is the main source of problem difficulty then providing a line that goes beyond the barriers of the virtual square should dramatically increase the solution rate. In their experiments they did exactly this manipulation and found no facilitation. This null-effect was often taken as evidence that Gestalt principles did not play a major role for the solution of the Nine-Dot problem.

However, Kershaw and Ohlsson (2004) demonstrated on the basis of several experimental manipulations that no single source is responsible for the poor solution rate, the interplay of at least three different factors are required. They identified perceptual (Gestalt laws), conceptual (over-constraint goal representation), and process factors (look-ahead, working memory demands) whose interplay impede the solution.

We conclude that understanding the dynamic interplay between problem and goal representation, contextual factors and the process factors, like working memory and executive functions might provide the key for a general understanding of human problem solving. This might help to get deeper insights in the involved processes and to complement the classical Information Processing Theory.

6 Closing Remarks

In this chapter we have provided a selective overview of psychological theories of human problem solving, starting with the Gestaltist perspective, and following its adoption, transformation, and development within information processing theory. We hope to have shown the thread linking these apparently disconnected fields and provided a coherent perspective on human problem solving. In addition, we have highlighted two issues, ill-structured problem-solving and insight problem-solving, that test the limits of the information processing theory approach and suggest that additional, or even alternative, concepts and frameworks are necessary if we are to enhance our understanding of human thinking processes.

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