Complete description of motion (in 2d)

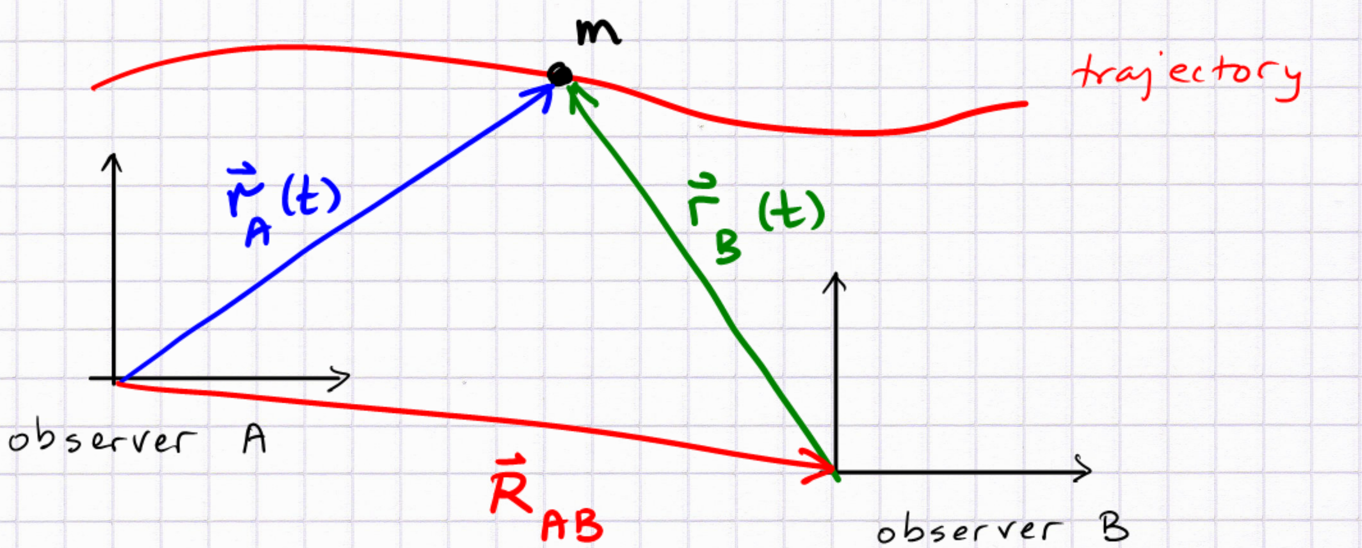
A particle trajectory is described by

$$\vec{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$$

$x(t)$, $y(t)$ are real functions of the real variable t = time

It depends on a choice of origin,

but the motion does not → we are free to choose some other reference frame



$$\vec{r}_A(t) = \vec{R}_{AB} + \vec{r}_B(t)$$

← motion as seen by B

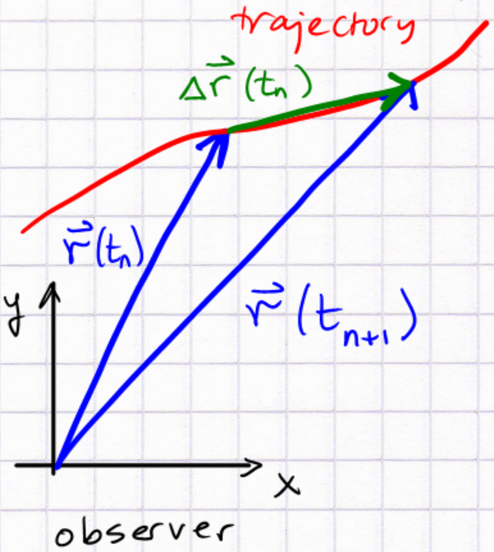
motion as seen by A

(fixed) position of observer B as seen by A

In Galilean mechanics (before special relativity, 1905) time is universal (the same for A, B, and m)

- Fundamental principle of physics: A and B should come up with the same laws; we must know how their respective observations are related.

Displacement vector



algebraic description

$t_n = n\,\Delta t$

two adjacent snapshots

$\Delta\vec{r}(t_n)$ = position @ t_{n+1} as seen relative to position at t_n

$$\Delta\vec{r}(t_n) = \vec{r}(t_{n+1}) - \vec{r}(t_n)$$

= change in the position vector over interval Δt

Define $$\vec{v}_{avg}(t_n) \equiv \frac{\Delta\vec{r}(t_n)}{\Delta t}$$

This implies: $$\vec{r}(t_{n+1}) = \vec{r}(t_n) + \Delta t\,\vec{v}_{avg}(t_n)$$

It means: given $\vec{r}(t_n)$ and $\vec{v}_{avg}(t_n)$, we can predict $\vec{r}(t_{n+1})$

This is used in physics simulation codes, such as in "real-physics-engine" based gaming programs

Computer programming languages working with floating-point numbers can only do arithmetic (+, *, −, ÷, ^)

Define the instantaneous velocity:

$$\vec{v}(t) = \lim_{\Delta t\to 0} \frac{\Delta\vec{r}(t)}{\Delta t} \equiv \frac{d\vec{r}}{dt}$$ "calculus-based"

Defining the derivative of a vector with respect to a parameter (time) works, since:

$$\vec{r}\,(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$$

$$\therefore \quad \vec{v}\,(t) = \underbrace{\frac{dx}{dt}}\,\hat{\imath} + \underbrace{\frac{dy}{dt}}\,\hat{\jmath} = \underbrace{\underset{v_x}{\overset{\shortparallel}{x'(t)}}}\,\hat{\imath} + \underbrace{\underset{v_y}{\overset{\shortparallel}{y'(t)}}}\,\hat{\jmath}$$

Known from single-variable calculus

$v_x = \frac{dx}{dt} = x'(t)$ represents slope in $x(t)$ graph

$\hat{\imath}, \hat{\jmath}$ are constant vectors (unaffected, $\frac{d}{dt}\hat{\imath} \equiv 0$)

Now we take this one step further:

Instantaneous acceleration vector:

$$\vec{a}\,(t) \equiv \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{dv_x}{dt}\,\hat{\imath} + \frac{dv_y}{dt}\,\hat{\jmath}$$

$$= a_x\,\hat{\imath} + a_y\,\hat{\jmath}$$

For finite Δt one has the arithmetic-based propagation:

$$\vec{v}\,(t_{n+1}) = \vec{v}\,(t_n) + \Delta t\,\vec{a}_{avg}\,(t_n)$$

Significance of $\Delta t \to 0$: Look at motion through recordings with increased frame rates (higher $f = \frac{1}{\Delta t}$).

For sufficiently high f (small Δt) the acceleration vectors and velocity vectors in neighboring frames are practically the same.