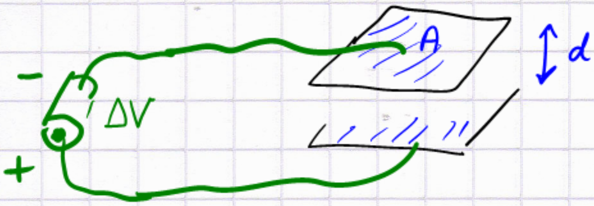
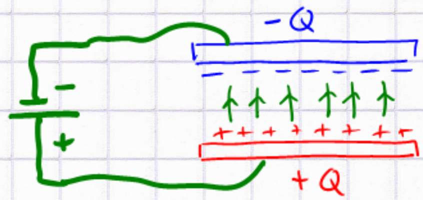
Capacitance



take 2 metal plates of area A each, separate by distance d, connect by wires to a battery of voltage ΔV

Charges will flow for a brief instant until the same potential difference is established across the plates as exists between the battery poles.

The + pole of the battery will suck in electrons, while the − pole will push out some



$\vec{E}$ field establishes itself

Question: what controls how much charge Q will be displaced?

A: we can figure that out from what we learned!

1) we know the strength of the field $\quad E = \dfrac{Q}{\varepsilon_0 A}$

$\left(\sigma = \dfrac{Q}{A}\right.$ = charge surface density$\left.\right)$

2) we learned what the potential difference is (across the gap): $\Delta V = Ed$

$$\therefore \quad \Delta V = \frac{Q}{\varepsilon_0 A} d$$

Recognize: 1) voltage $\Delta V \sim Q$

2) A, d = property of set-up

Define: $C \equiv \dfrac{\varepsilon_0 A}{d}$ = capacitance = plate set-up dependent

$$\boxed{\Delta V = \frac{Q}{C}}$$

unit for C:

farad $\equiv F = \frac{\text{Coulomb}}{\text{volt}}$

states: more charge is displaced if a "bigger" capacitor is used

bigger capacitor:

$$C \equiv \frac{\varepsilon_0 A}{d}$$

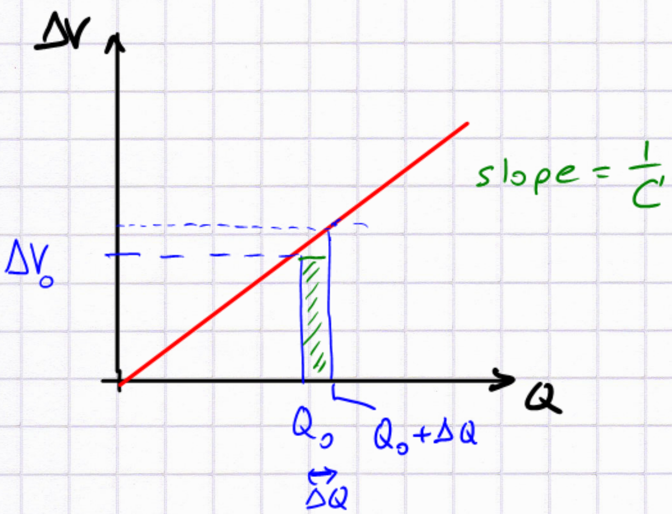
1) bigger area A
2) smaller distance d
3) different ε_0?

put something else than vacuum/air in between the plates (dielectric = polarizable medium)

Energy storage

We use the mechanical energy balance for a charged probe particle between the plates to understand the energetics behind the charge displacement [even though in electronics we never want this set-up of charges travelling between plates, that is reserved for vacuum tubes + physics experiments]

The voltage-charge relationship is linear: $\Delta V = \frac{1}{C} Q$



Suppose we are at the point $(Q_0, \Delta V_0)$.

How much energy is required to charge up the capacitor by an extra charge ΔQ?

This is equivalent to move a probe charge ΔQ across the plates through a field $E = \frac{\Delta V}{d}$

Naively, we would say: this change in PE of the probe ΔQ is $\Delta Q \cdot \Delta V$ = rectangle area. This is correct for a small probe ΔQ, i.e., this is an energy change

We understand now the energy change associated with charging further a capacitor that sits at charge Q_0 (and potential ΔV_0) and which moves to $(Q_0 + \Delta Q)$ (and associated potential $\Delta V_0' = \frac{Q_0 + \Delta Q}{C}$)

The energy content itself (not just the change) is then given by adding up such rectangles incrementally

$\Rightarrow$ area under the curve (integral calculus, or geometry)

$$(PE)_{cap.} = \frac{1}{2} Q_0 \Delta V_0 \rightarrow \frac{1}{2} Q \Delta V$$

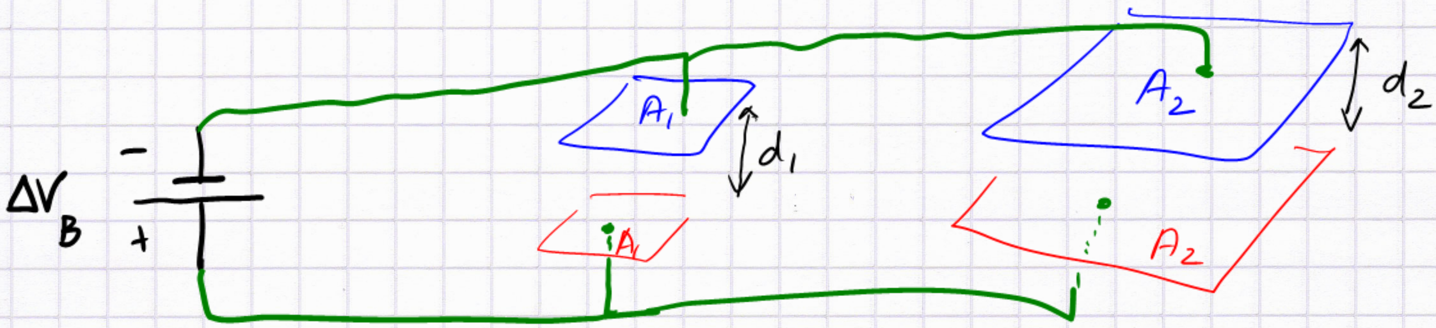
$$= \frac{1}{2} C \cdot \Delta V \cdot \Delta V = \frac{1}{2} C \Delta V^2$$

$$= \frac{1}{2} Q \cdot \frac{Q}{C} = \frac{1}{2} \frac{Q^2}{C}$$

Example 18.6 $\rightarrow$ practical calculation

Capacitors in series vs capacitors in parallel

1) parallel : Suppose you have two parallel-plate set-ups



$$C_1 = \frac{\varepsilon_0 A_1}{d_1} \qquad C_2 = \frac{\varepsilon_0 A_2}{d_2}$$

The same ΔV_B appears across the plates in either C_i:

A <u>different</u> amount of charge displacement!

$$Q_1 = C_1 \Delta V_B \qquad Q_2 = C_2 \Delta V_B$$

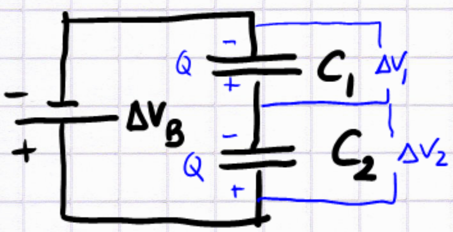
The combined system acts like <u>one</u> capacitor with bigger capacitance

$$Q_{tot} = Q_1 + Q_2$$

$$C_{eq} \Delta V_B = C_1 \Delta V_B + C_2 \Delta V_B$$

$$\therefore \quad C_{eq} = C_1 + C_2$$

The series case is tricky:



The same charge displacement $Q = Q_1 = Q_2$!

$$\Delta V_1 + \Delta V_2 = \Delta V_B$$

$$\frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_{eq}} \quad \therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\therefore \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

putting a big C_1 in series with a small C_2 $(C_1 >> C_2)$ results in a small equivalent capacitance.

physics: the small C_2 can't hold much charge (at given voltage drop)

→ limits the combined system

math: $C_{eq} \approx \frac{C_1 C_2}{C_1} \approx C_2$

why?

$C_1 + C_2 \approx C_1$ when $C_1 >> C_2$.

understand more about capacitor networks? → electronics course

We will study the charging process as a function of time in PHYS 1010.