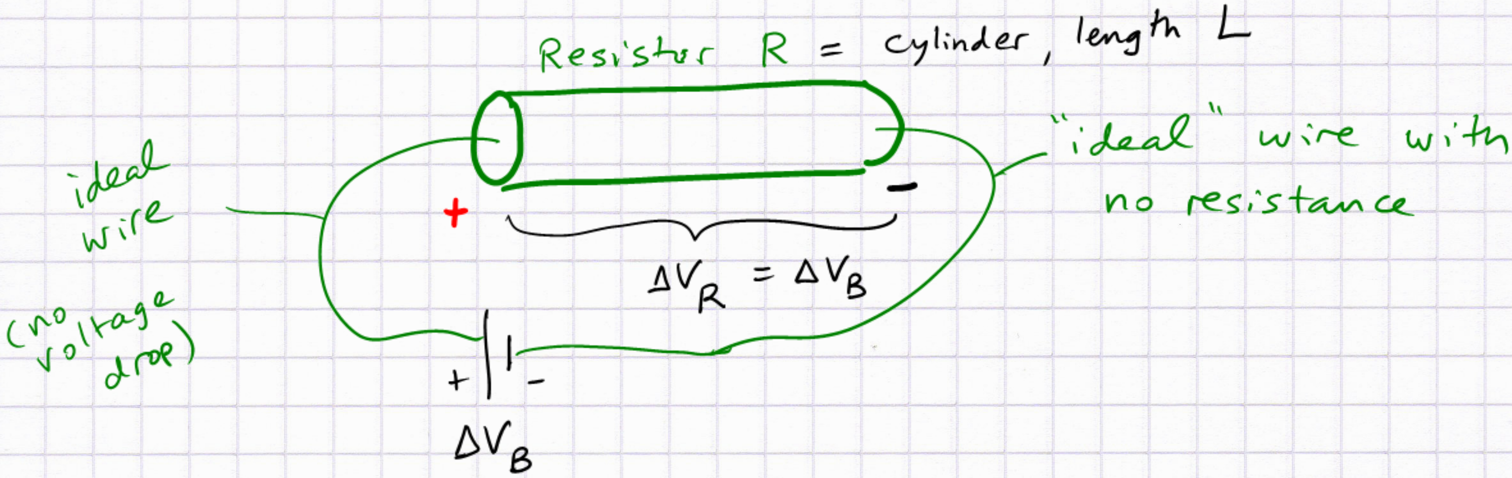
Current and Resistance

Suppose a charged capacitor (or a battery) tries to push charge through a wire. The wire is assumed to be of cylindrical shape and contains conduction electrons.



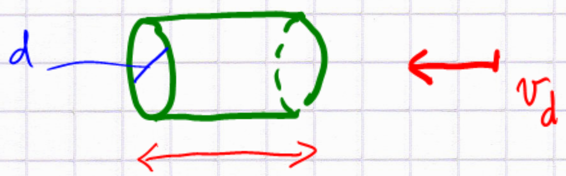
An electric field of strength $E = \frac{\Delta V_R}{L}$ would push positive charges left $\to$ right, i.e., it pushes conduction electrons right $\to$ left.

This $\vec{E}$ field continuously accelerates electrons, but they reach a constant terminal velocity rather quickly. The ions in the metal lattice deflect the electrons, remove energy (lattice vibrates $\to$ heats up) and thus provide a drag force.

$\Rightarrow$ A constant charge current through any cross sectional area of the wire

Quantitative : Electrical current : $i = \dfrac{\Delta q}{\Delta t}$

Consider a segment of the cylinder : cross section $= \pi \dfrac{d^2}{4}$
(d = diameter of wire)



$\left(\pi \dfrac{d^2}{4} \times v_d \Delta t\right) =$ volume

average distance travelled by a drifting e^- in time Δt
$= v_d \Delta t$

The volume contains the electrons that will cross the front surface in time Δt. Estimate how much charge that is.

We need the density of charge carriers involved :

$n =$ volume density of conduction electrons $=$ material property

Number of charge carriers $N = n \cdot V = n \left(\pi \dfrac{d^2}{4} \cdot v_d \Delta t\right)$

Amount of charge : $N(-e) = \Delta q = -ne \left(\pi \dfrac{d^2}{4} v_d \Delta t\right)$

Charge current: $I = \dfrac{\Delta q}{\Delta t} = -ne \left(\pi \dfrac{d^2}{4} v_d\right)$

How many amps $= I$ depends on
- drift velocity (faster = more current)
- cross sectional area $\pi \dfrac{d^2}{4}$ (more is better)
- conduction electron volume density

(Cu better than Al, ...)
but mostly similar for metals

metal	n ($\#/m^3 = \frac{1}{m^3}$)
Al	6.0×10^{28}
Cu	8.5×10^{28}
Fe	8.5×10^{28}
Au	5.9×10^{28}
Ag	5.8×10^{28}

Given a material (n is fixed)

Given a cross sectional area $\pi \frac{d^2}{4}$ = gauge of wire

What controls v_d ?

$$v_d \sim E = \frac{\Delta V}{L} \quad \therefore \quad I \sim \Delta V$$

or $I = \frac{\Delta V}{R}$ ← Ohm's law

R = resistance

$R \equiv \rho \frac{L}{A}$

ρ = material property
= resistivity $\sim \frac{1}{ne}$?

unit for R : $\frac{\text{volt}}{\text{ampere}}$ ≡ ohm → Ω (Greek long "O" = Omega)

⇒ unit for ρ = (ohm) × m .

To understand ρ: we need a conduction model

$$v_d = \frac{e\tau}{m} E$$

(dimensionally correct!)

τ = average time between collisions of e^- with lattice (depends on material)

m = mass of charge carrier

Two material properties, n and τ together determine ρ

material	ρ [Ωm]
Al	2.7×10^{-8}
Cu	1.7×10^{-8}
Fe	9.7×10^{-8}
Au	2.2×10^{-8}
Ag	1.6×10^{-8}
Pb	22×10^{-8}
nichrome	1.5×10^{-6}

← worse than Cu, but cheaper (but doesn't last, breaks)

nichrome ↳ heater wire

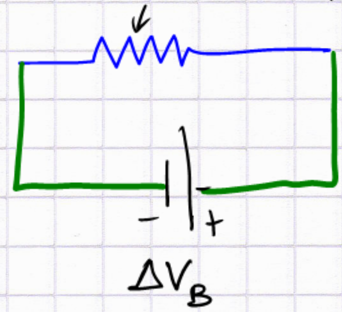
What is this table good for?

Given a material, given a gauge of wire ($A = \frac{\pi d^2}{4}$ cross sectional area), given the length of this cylindrical wire:

$R = \rho \frac{L}{A}$ → absolute resistance in Ω

Meaning of our circuit:

nichrome wire, small A, large L } big R

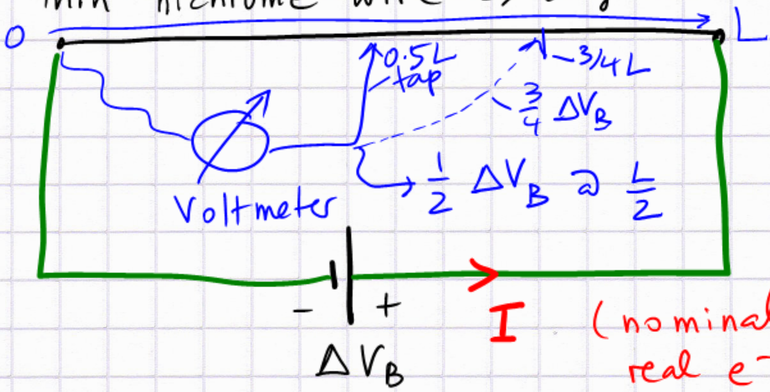


— copper wire, large A, short L $\Rightarrow$ negligible R = ideal wire $R \approx 0$

Ohm's law verification:

$$\Delta V_R = R\,I$$ voltage drop along resistor

thin nichrome wire $\Rightarrow$ large resistance; $R \sim L$ $\left(= \rho \frac{L}{A}\right)$

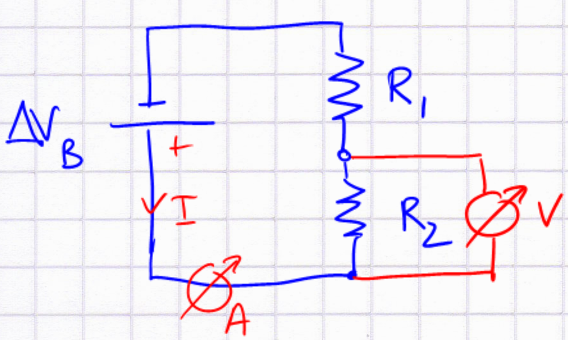


voltage reading is proportional to length at which tap of probe applies

(nominal current goes + → −)
real e^- current is − → +

There is no voltage drop along the ideal wire, as the resistance of an ideal wire $R \equiv 0$.

Voltage divider:



Voltage drop along $R_1 + R_2$: ΔV_B

current $I = \dfrac{\Delta V_B}{R_1 + R_2}$

$$\Delta V_2 = R_2 \cdot I = \frac{R_2}{R_1 + R_2}\,\Delta V_B$$

In principle, this allows us to step down a voltage and "dial up" voltages $0 \leq \Delta V_2 \leq \Delta V_B$. In practice, this works if the "loading" of ΔV_2 is minimal