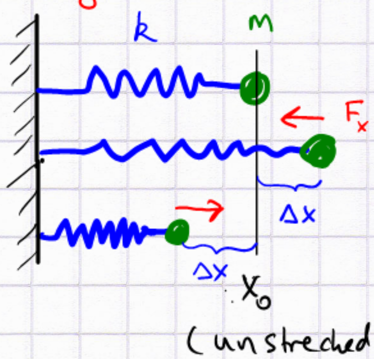
Potential Energy

The work-energy theorem allows to find the change in KE from the area under the F(x) curve (in 1d)

We do not wish to recalculate this for each problem

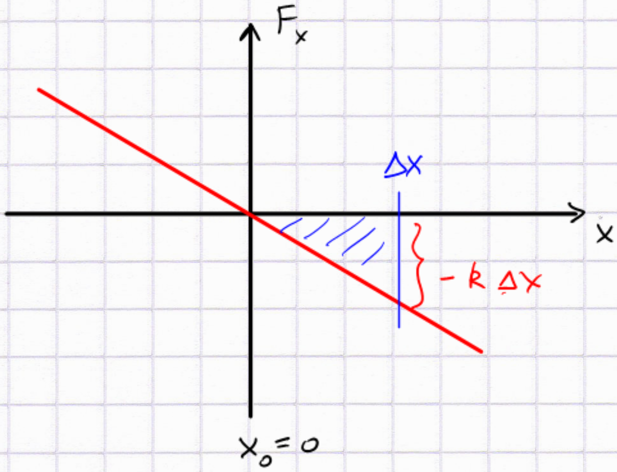
Example: compressing / stretching a spring

$$F_x = -k\,\Delta x = -k\,(x - x_0)$$



strength of force grows with displacement Δx

graphical representation:



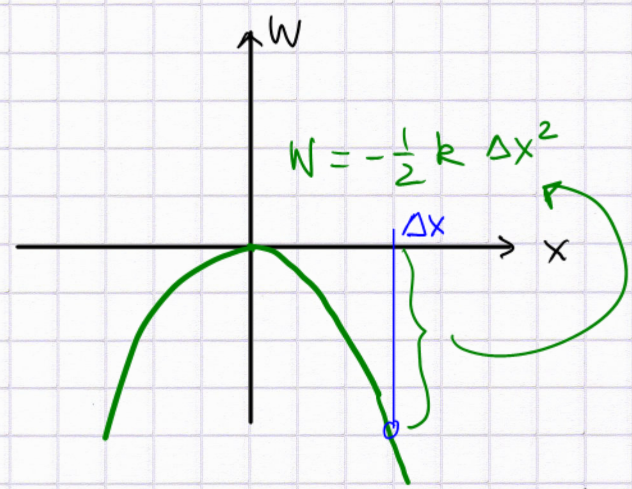
$$W = \frac{1}{2}\,\Delta x\,(-k\,\Delta x)$$

$$= -\frac{1}{2}\,k\,\Delta x^2$$

independent of the sign of Δx!

What does it mean?

to stretch or to compress a spring from equilibrium by $\pm\,\Delta x$: required work is $-\frac{1}{2}\,k\,\Delta x^2$



SI unit: Nm = Joule (= Ws)

from energy work theorem: if we have a spring compressed to Δx, with mass m attached, and let it go: $x_i = \Delta x$, $x_f = 0$,

$\frac{1}{2} m v_f^2 = KE_f = -W = +\frac{1}{2}\,k\,\Delta x^2$ why? reverse process to $\{v_i, x=0\} \rightarrow \{v=0, x=\Delta x\}$

Scenario 1: $x_i = \Delta x$, spring is loaded, $PE = -W = +\frac{1}{2}k\,\Delta x^2$, (potential energy)

m is at rest, $v_i = 0$, $KE = 0$, total $E = KE + PE = \frac{1}{2}k\Delta x^2$

release the catch: 2 observe at spring equilibrium, $\Delta x = 0$

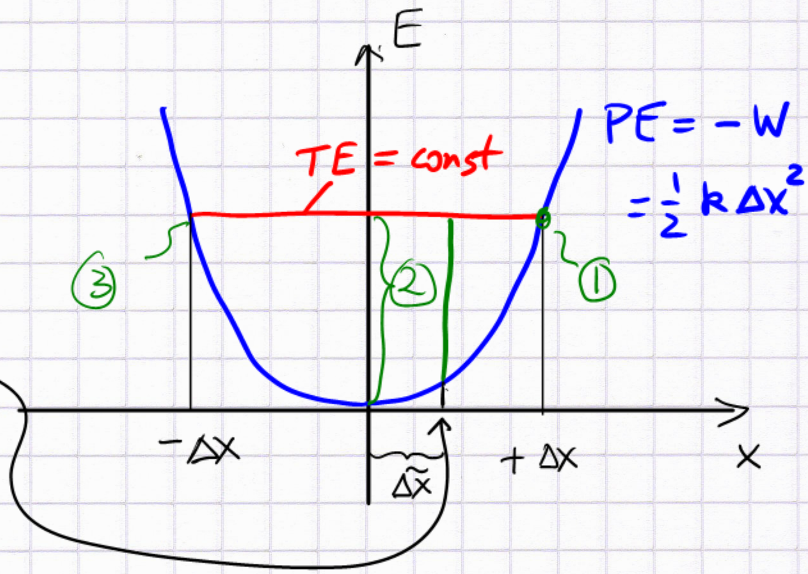
$v = v_f$, ($v_f < 0$!), $KE = \frac{1}{2}m v_f^2$, $PE = \frac{1}{2}k\Delta x^2 = 0$, $TE = KE = \frac{1}{2}m v_f^2$

let the motion continue, observe at $x = -\Delta x$, $v = 0$

3 $TE = PE = \frac{1}{2}k\,\Delta x^2$

Now go to a time in between: 4

$$\underbrace{TE}_{} = \underbrace{KE}_{\frac{1}{2}m\tilde{v}^2} + \underbrace{PE}_{\frac{1}{2}k\,\tilde{\Delta x}^2}$$



System w/o friction or drag:

- total mechanical energy $TE = KE + PE$ is conserved

  (oscillator rocks forever)

Friction (drag) convert mechanical energy into heat;

Mechanical energy recoverable from heat? only partially

Heat = fast-moving atoms/molecules (also molecule vibrations, rotations)

atoms bounce of a paddle ⟶ paddle moves → turbine
(but atoms continue with some motion)