Momentum

Newton's 2nd law $\qquad m \dfrac{d^2\vec{r}}{dt^2} = \vec{F}(\vec{r}(t), \vec{v}(t))$

? when

can be written as

1) $\dfrac{d}{dt}\vec{p} = \vec{F}(\vec{r}(t))$ $\qquad \vec{F}\left(\vec{r}(t), \dfrac{\vec{p}(t)}{m}\right)$

(more general, e.g., with drag)

2) $\dfrac{d}{dt}\vec{r} = \dfrac{\vec{p}(t)}{m}$

$\vec{p} = m\vec{v}$ or $\vec{p}(t) = m\vec{v}(t)$ for a single point particle

this is a trivial re-write.

For objects with variable mass $m(t)$, e.g., rockets,

this form of Newton's law is correct: $\boxed{\dfrac{dp_x}{dt} = \dfrac{dm}{dt} v_x + m\dfrac{dv_x}{dt} = F_x}$

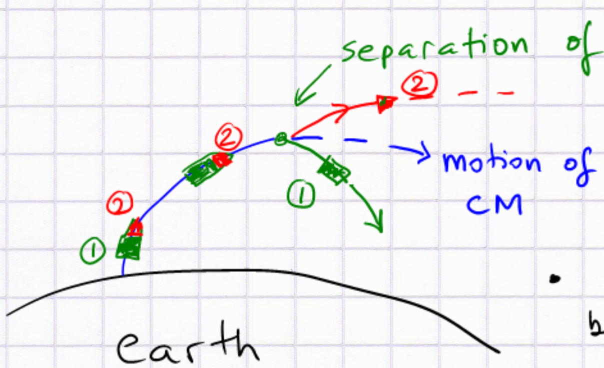
We also want to understand the modeling of extended objects by the particle model (entire mass is concentrated in C.M.)

center of mass
" " gravity

$$\vec{P}_{tot} = \sum_{i=1}^{N} \vec{p}_i = \sum_{i=1}^{N} m_i \vec{v}_i$$

Example: $N=2$ a) two colliding cars (Ex. 7.2 in text)

b) break-up of a rocket (2-stage)



(stage 1 = big motor + fuel tank
Stage 2 = capsule (payload) + small motor

(stage 1 could be booster rocket)

- A relative force between S1 and S2 produces separation (small explosion)
- CM → virtual continuation of complete S1 + S2 trajectory

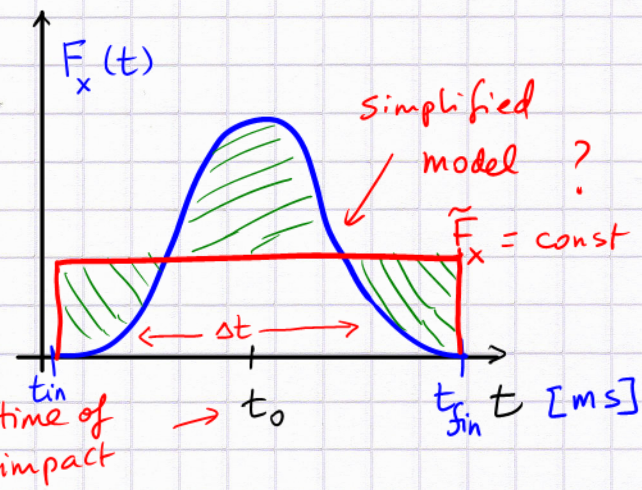
Impulse

Newton's 2nd law fully describes changes in motion due to the net force [based on free-body diagrams].

Often we don't have the details about short-term forces, such as:

- ball hits the ground → reverses velocity (momentum) vector
- tennis racquet hits suspended ball → sets it into horizontal motion
- hand flicks + tosses ball vertically into the air

... → impulsive forces



bouncing ball: free fall, $v_f = \sqrt{2gh}$,

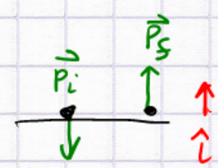
TE = KE before contact, then the KE is converted into ball compression (+ a little floor compression?)

KE → $PE_{\text{"spring"}}$ (KE = 0),

then deformed tennis ball decompresses + accelerates the ball in the opposite (upward) direction.

This bounce happens on the millisecond time scale

Macroscopic observation: $\vec{p}_f - \vec{p}_i = \Delta\vec{p} = 2\vec{p}_f$



$$\Delta p = \tilde{F}_x \, \Delta t = J_x \quad \text{impulse (x component)} \quad \text{[one dim.]}$$

$$\Delta\vec{p} = \vec{F}_{ave} \, \Delta t = \vec{J} \quad \text{impulse vector}$$

exactly: $J_x = \int_{t_{in}}^{t_{fin}} F_x \, dt$

Impulse - Momentum theorem:

The change in momentum = impulse = area under $\underbrace{F_x(t)}_{\text{force - time}}$ curve