Magnetic Force: applications and further understanding

1) $\vec{F}_M = q\,\vec{v} \times \vec{B}$ implies that this force "bends" the particle trajectory, but does not change the speed.

Since $|\vec{v}(t)| = v_0 = \text{const}$, the KE $= \frac{1}{2} m v_0^2 = \text{const}$

$\Rightarrow$ The magnetic force does no work on the particle!

2) Given a uniform $\vec{B}$ field, and a charged particle entering this region with $\vec{v}$: what trajectory can we expect?

force acting at right angle to $\vec{v}$?

$\longrightarrow$ remember uniform circular motion:

$\vec{B}$ into plane:

$$\vec{r}(t) = R\,(\hat{\imath}\cos\omega t + \hat{\jmath}\sin\omega t)$$

$$\vec{v}(t) = R\omega\,(-\hat{\imath}\sin\omega t + \hat{\jmath}\cos\omega t)$$

$$\vec{a}(t) = R\omega^2(-\hat{\imath}\cos\omega t - \hat{\jmath}\sin\omega t)$$

$$= -\omega^2 \vec{r}(t)$$

show: $\vec{v} \cdot \vec{a} = 0 \quad \therefore \vec{a} \perp \vec{v}$

$$\vec{F}_M = q\,\vec{v} \times \vec{B}$$

$$= q\,(v_0\,\hat{\imath}) \times (-B_0\,\hat{k}) = -q\,v_0 B_0\,\underbrace{(\hat{\imath} \times \hat{k})}_{-\hat{\jmath}} = +q v_0 B_0\,\hat{\jmath}$$

2nd step: more complicated: $\vec{v} = v_x\,\hat{\imath} + v_y\,\hat{\jmath}$

$\therefore \vec{F}_M = F_x\,\hat{\imath} + F_y\,\hat{\jmath}$

However, circular motion is simple to analyze: $\dfrac{m v_0^2}{R} = |\vec{F}_M| = |q v_0 B_0| = |q|\, v_0 B_0$

For $\vec{v}_0 \perp \vec{B}$ (no velocity component along $\vec{B}$):

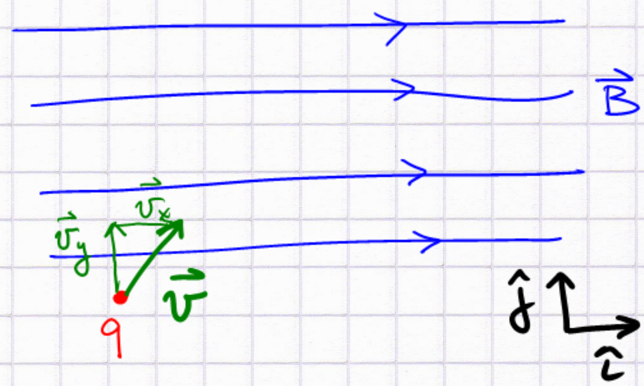
gyration with radius $R = \dfrac{m v_0^2}{|q| v_0 B} = \dfrac{m v_0}{|q| B}$

application: mass spectrometry $\rightarrow$ isotope separation

[figure out what molecules are contained in a sample of air by: 1) ionizing ($q=1$); 2) accelerating to known v_0 by a dialed-up $\vec{E}$ field (via voltage); 3) sending through known (dialed up) $\vec{B}$ field, and selecting mass according to trajectory radius (a slit after $90°$ turn in a uniform $\vec{B}$)]

Helical Trajectory

Now suppose the charge enters the $\vec{B}$ field region with some velocity component parallel to the field ($v_x \longrightarrow$)



Circular motion occurs in the y-z plane based on $q(\vec{v}_y \times \vec{B}) \rightarrow \vec{v}_y$ changes into $(\vec{v}_y, \vec{v}_z) \rightarrow$ gyration as before

but a translation without acceleration (uniform motion) in the $\hat{i}$ direction is superimposed

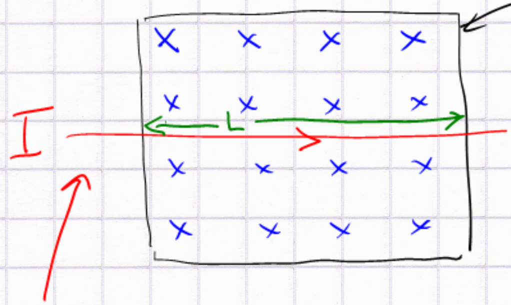
$\rightarrow$ helical (= spiralling) motion

Applications: galaxy, sun, earth through their $\vec{B}$ fields capture incoming charged cosmic particles which spiral to the poles and back $\rightarrow$ aurora (Northern Lights) + protection for us!

Current-carrying wire inside a $\vec{B}$ field

Suppose we are given a strong homogeneous $\vec{B}$ field,

e.g., the region between two strong permanent magnet poles



S pole is behind the paper plane

N pole is in front

$\Rightarrow$ $\vec{B}$ is into the plane

a current-carrying wire is now placed inside the field.

suppose the current is weak, i.e., ignore the B field caused by the wire (according to Biot-Savart)!

We have $\Delta q > 0$ running left $\rightarrow$ right: $\vec{v} = v_0 \hat{\imath}$

A magnetic force: $\Delta \vec{F}_M = \Delta q \, \vec{v} \times \vec{B} = \Delta q \, v_0 B_0 (\hat{\imath} \times (-\hat{k}))$

$$= \Delta q \, v_0 B_0 \hat{\jmath}$$

acts on each segment containing Δq running at speed v_0.

Now express the total magnetic force on a wire of length L carrying current I:

$$I = \frac{\Delta q}{\Delta t} \quad ; \quad F_M = \Delta q \, v_0 B_0$$

(magnitude)

Q: what is v_0?

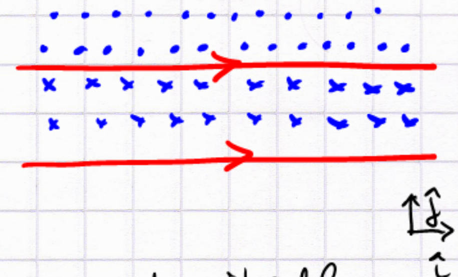
A: The amount of "running" charge, Δq, is contained in the total wire length immersed in the field, i.e., within L

Thus, $v_0 = \frac{L}{\Delta t}$, since $I = \frac{\Delta q}{\Delta t}$ corresponds to the entire "wire content" Δq moving to the right within Δt

$$F_M = (I \Delta t) \frac{L}{\Delta t} B_0 = I L B_0$$

assuming the wire is perpendicular to $\vec{B}$

Two current-carrying wires



Now we just consider two parallel wires carrying equal current I.

According to Biot-Savart the top wire surrounds itself with a $\vec{B}$ that is into the plane at the location of the bottom wire. $\vec{F}_{M \text{ on bottom wire}} = q\,(v_0\hat{\imath} \times (-B_0\hat{k}))$
$= q v_0 B_0 \hat{\jmath}$

On the other hand the bottom wire creates a $\vec{B}$ field at the location of the top wire that points out of the plane. Thus, $\vec{F}_{M \text{ on top wire}} = q\,(v_0\hat{\imath} \times (+B_0\hat{k})) = -q v_0 B_0 \hat{\jmath}$

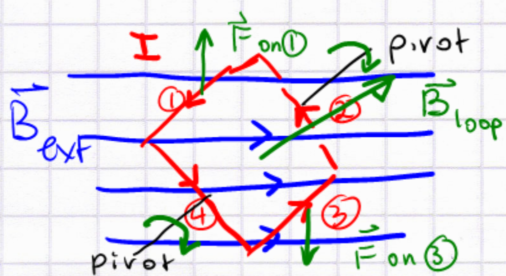
∴ Two parallel wires carrying currents in the same direction attract each other. Opposite current directions → repulsion

NOTE: we never consider the force on the wire from its own magnetic field! The B field from the top wire caused a current in the bottom wire to experience a push towards the top wire and vice versa

→ No self interactions.

Torque on a current loop:

Given a homogeneous $\vec{B}_{ext}$ (permanent magnets) and a rectangular current-carrying loop



Forces on segment ①, ③ straighten the loop such that its magnetic field $\vec{B}_{loop}$ (RH rule!) aligns with $\vec{B}_{ext}$ (see Fig. 20.23)