Electromagnetic Induction

Magnetic Flux

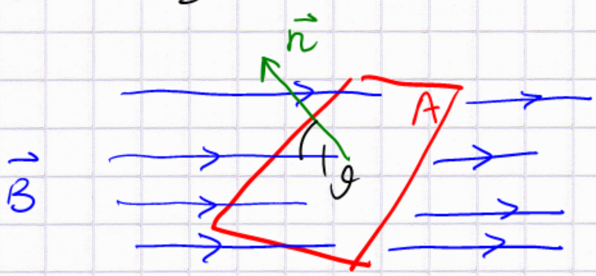
→ apply the same ideas as in the $\vec{E}$ field case when Gauss' law was introduced

A "window frame" is used to analyze how the $\vec{B}$ field flows through some area

$$\Phi_B = B A \cos\vartheta$$

A = area, B = field strength

ϑ = angle between $\vec{B}$ and normal to the area



Φ_B unit: $T m^2 \equiv Wb$ (Weber)

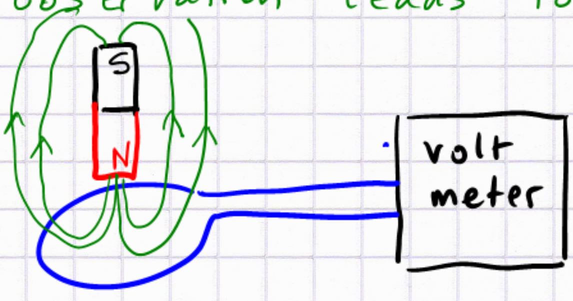
Faraday's law:

$$EMF = \mathcal{E} = -\frac{\Delta \Phi_B}{\Delta t}$$

1) $|\mathcal{E}| = \left|\frac{\Delta \Phi_B}{\Delta t}\right|$

2) significance of "−" (Lenz rule)

Which observation leads to this law?



wire loop (multi-turn works better)

stationary magnet: nothing happens

approaching magnet: generate a voltage

moving-away magnet: generate voltage of opposite sign

reverse magnet orientation → voltage sign flips

Use diff. calculus to understand what can cause a time-varying flux:

$$\frac{d}{dt}\,\phi_B = \frac{d}{dt}\left(B\ A\ \cos\vartheta\right) = \frac{dB}{dt}\cdot A\cos\vartheta$$ B field varies

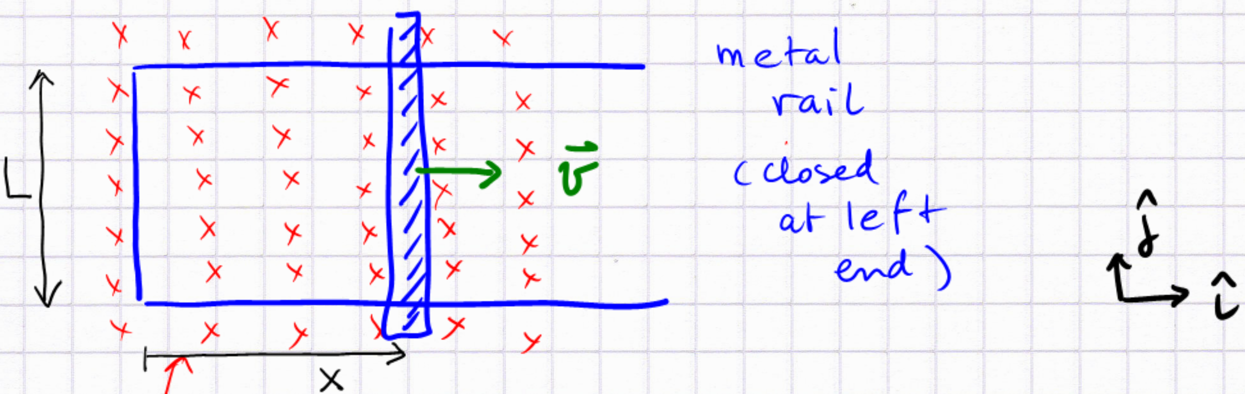
$$+\ B\,\frac{dA}{dt}\cos\vartheta$$ area changes

$$+\ B\,A\,\frac{d}{dt}(\cos\vartheta)$$ orientation $\vec{B}$ vs $\vec{n}$ changes

Three possibilities for flux change!

let's understand the $\frac{dA}{dt}$ → area change case first:

metal bar, moving to the right, $v_x > 0$



uniform $\vec{B}$ field into the plane, from permanent magnets or from electro-magnets. $\vec{B} = -B_0\,\hat{k}$

Q: what happens to the conduction electrons in the bar?

A: since these are $q = -e$ charged objects, moving through a $\vec{B}$ field, $\vec{v}\times\vec{B}$ points up, $\sim\hat{j}$

$\therefore$ $q\,\vec{v}\times\vec{B}$ points along $-\hat{j}$

Electrons are pushed towards the bottom of the bar

$\therefore$ top of bar $= +$ charged, bottom $= -$ charged

Can we understand the EMF from first principles, and derive the Faraday result?

F_M separates charge, but this causes a build-up of an electric field which will counteract further charge separation. Equilibrium, or steady state occurs when both forces balance:

$$F_M = q v B = F_E = qE$$

$$\therefore \quad E = vB$$

For a bar of length L the EMF produced at the ends: $\Delta V = EL = vBL$

Now look at Faraday:

The flux change $\frac{d}{dt}\Phi_B$ is caused by an increase in the "window" permeated by $\vec{B}$ as the bar moves.

$$\frac{d}{dt}\Phi_B = BL\frac{dx}{dt} = BLv$$

What we derived from the balance of forces is consistent with the flux change argument!

So far, we just discussed the generation of EMF or ΔV.

Let us look at the current loop formed by

- the bar (is thick, $\underbrace{\text{negligible resistance}}_{R_{int} \text{ of real EMF generator}}$)

- the rails → assume they have some resistance R at given position x
  [or assume only the end connection is a thin wire → load, as in Fig 21.6]

Now we have a circuit: moving bar = EMF generator

Use Ohm's law to determine the current:

$$|R I| = \Delta V \quad (= \text{EMF}) = B L v$$

$$\therefore \quad I = \frac{BLv}{R}$$

Note: we induced a current ∴ a magnetic field $\vec{B}_{ind}$ is also generated!
(come back to this)

dissipated power?

$$P = \Delta V \cdot I = R I^2$$

$$= \frac{B^2 L^2 v^2}{R}$$

Electrical power is dissipated (turned into heat)

Q: Where does this power come from?

A: We can't drag the bar across for free: when a current flows through the bar (positive charge moves up $\sim \hat{j}$)
$q \vec{v}_q \times \vec{B}$ provides a force opposing the bar's velocity v
∴ an external force to the right is required for $v = \text{const}$!