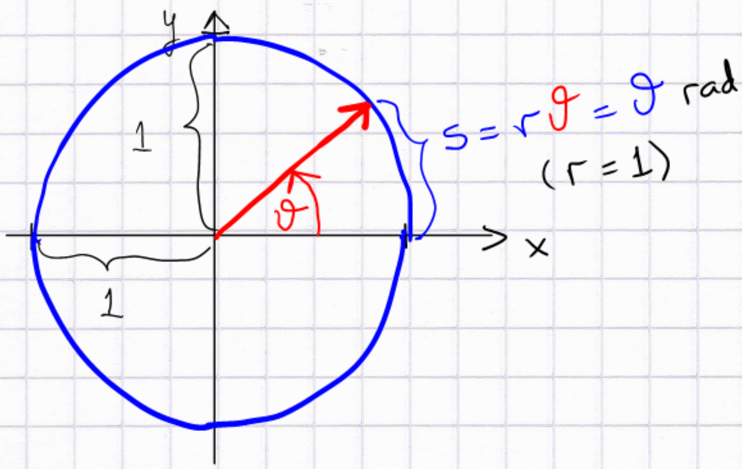
Trigonometry $\longrightarrow$ why in physics?

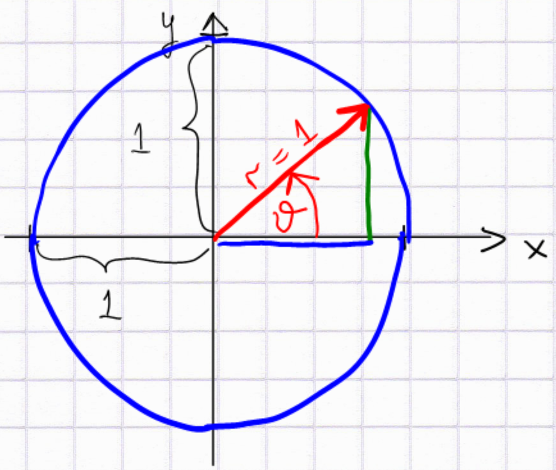
1) unit circle



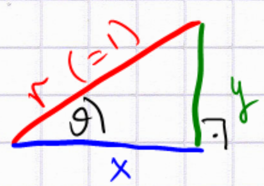
ϑ in degrees

$s = \vartheta$ in radians

2π rads $\hat{=}$ $360°$



$$\left.\begin{aligned} y &= r \sin\vartheta \\ x &= r \cos\vartheta \end{aligned}\right\} \text{why?}$$



$$\frac{x}{r} = \cos\vartheta$$

$$\frac{y}{r} = \sin\vartheta$$

$$\frac{\sin\vartheta}{\cos\vartheta} = \tan\vartheta = \frac{y/r}{x/r} = \frac{y}{x}$$

Suppose you know x, y and need ϑ $\longrightarrow$

$$\vartheta = \tan^{-1}\left(\frac{y}{x}\right) \quad \leftarrow \tan^{-1} = \arctan$$

has nothing to do with $\frac{1}{\tan x}$!

Math idea: inverse functions = undo a function

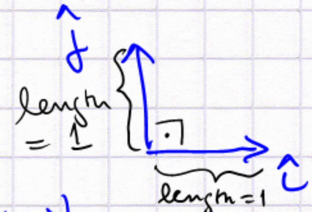
$$\tan^{-1}[\tan(x)] = x$$

back to physics!

uniform circular motion: particle goes around a circle in time T with constant speed

Frequency: $f = \frac{1}{T}$

$$\vec{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$$



unit vectors define x- and y-axes!

$$= \left(R\cos\vartheta(t)\right)\hat{\imath} + \left(R\sin\vartheta(t)\right)\hat{\jmath}$$

NB: $\vartheta = 2\pi\frac{t}{T}$

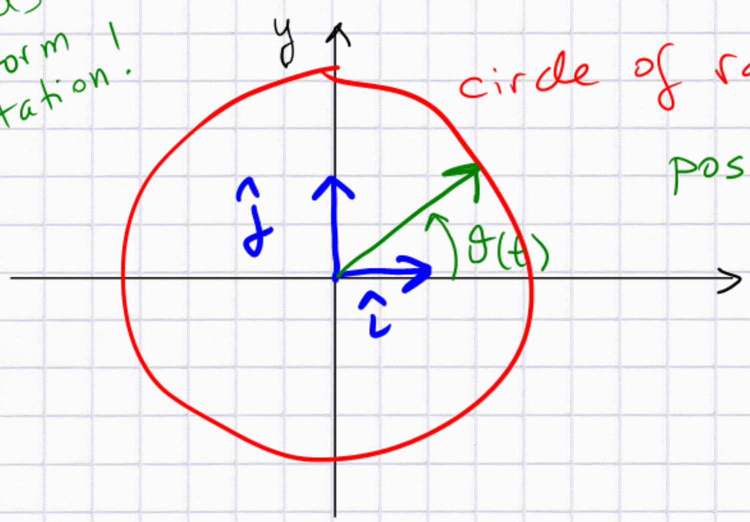
$$= R\left(\cos(2\pi f t)\,\hat{\imath} + \sin(2\pi f t)\,\hat{\jmath}\right)$$

$\Rightarrow \vartheta(T) = 2\pi$

$$= R\left(\cos(\omega t)\,\hat{\imath} + \sin(\omega t)\,\hat{\jmath}\right)$$

$\omega \equiv 2\pi f = \frac{2\pi}{T}$

models uniform rotation!



circle of radius R

position vector

$$\vec{r}(t) = [x(t), y(t)]$$

$$= x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$$

NOTE:

LHS = vector

RHS = vector

NEVER: $\cancel{\vec{v} = \text{scalar}}$

very common mistake

$$\cancel{\vec{F} = \frac{GmM}{r^2}}$$

Note:

$\cos(\underbrace{\omega t}_{})$ or other functions

argument is dimensionless!

$\sin(5\text{ sec})$ makes no sense.