Lenz rule

Faraday's law was derived for a special case: a metal bar was dragged through a perpendicular $\vec{B}$ field, Balancing the electric and magnetic forces led to a result for the potential difference

$$\Delta V = EMF$$ produced between the ends.

Then, we completed a loop (circuit) and found that the magnetic flux

$$\phi_M = \vec{B}\cdot\vec{A} = BA\cos\varphi \quad , \text{where } \varphi = \measuredangle\, \vec{B},\vec{A}$$

changes due to the change in permeated area, $A(t)$.

Faraday's law was then expressed as

$$\Delta V = -\frac{d}{dt}\phi_M$$

However, we can have $\phi_M(t)$ with $\frac{d\phi_M}{dt} \neq 0$ for other reason's, as well:

1) $B = B(t)$ magnetic field strength changes

2) $\varphi = \varphi(t)$ orientation of loop ($\vec{A}$) changes with respect to $\vec{B}$

The generation of EMF is not easily explained in these cases. Lenz studied EM induction some years after Faraday and came up with the following:

Given a current loop permeated by a $\vec{B}$ field, with the flux ϕ_M changing for whichever reason. A current I_{ind} will be induced with associated $\vec{B}_{ind}$ such that ϕ_M due to $\vec{B}+\vec{B}_{ind}$ tries to remain constant.

$\therefore$ if $\frac{d}{dt}\phi_M > 0$ then $\vec{B}_{ind}$ opposes $\vec{B}$, otherwise it supports it.

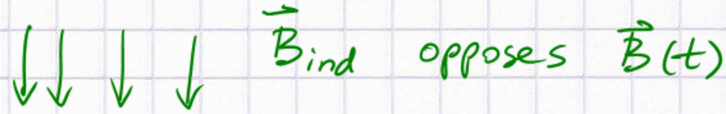
Note that the argument is via induced currents in the loop, but that we stick with Faraday by saying that an EMF was generated which produces $I_{ind.}$ and $B_{ind} \sim I_{ind}$.

This principle, that $\vec{B}$ fields have inertia, i.e., resist change will receive some support later (oscillator circuit, RLC)

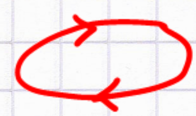
Lenz' law explains the "−" sign in Faraday's law:

induced magnetic fields <u>never boost</u> time-varying magnetic fields (otherwise there could be runaway solutions)

Example: $B(t)$ with $\frac{dB}{dt} > 0$ (field gets stronger, field lines are getting denser)

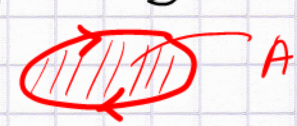


which implies



current is CW (clockwise)

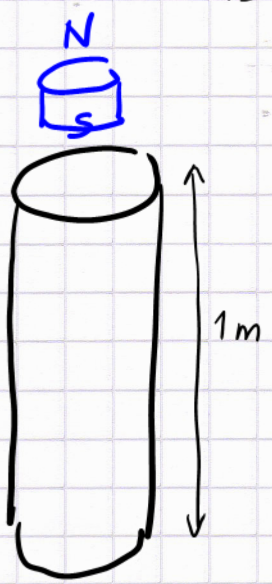
Note: what matters is the flux ϕ_M through the area surrounded by the loop!



Concept check: verify that for the bar dragged across the rails (with closed end) Lenz' rule gives the correct orientation of the current for both cases (area getting larger, $\frac{d\phi_M}{dt} > 0$) and (area getting smaller, $\frac{d\phi_M}{dt} < 0$)

Example copper tube (plumbing), strong magnet is dropped $\rightarrow$ slow terminal velocity is observed

copper half-inch tube



magnet appears after a few seconds

Q: which force opposes gravity?

Q: why is it a velocity-dependent "drag" force?

A: a segment of the copper tube = ring

a magnet approaching with some speed $\rightarrow$ $B(t)$

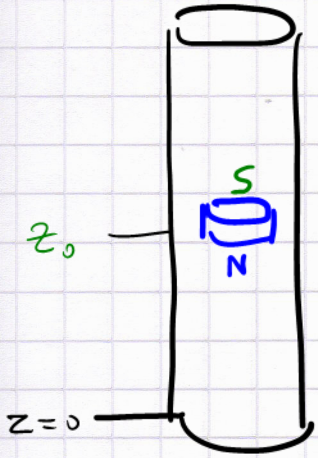
$\therefore$ flux change : $\Phi_M(t)$

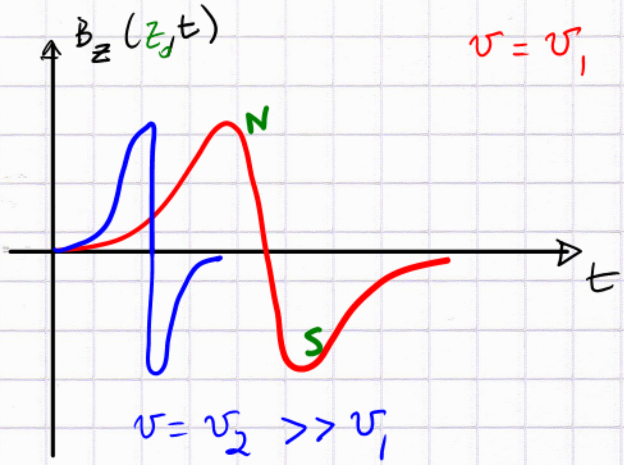
$\Rightarrow$ a current is induced below the magnet [also above, but in the opposite direction] $\therefore$ The current is stronger for a faster-moving magnet. $\vec{B}_{ind}$ opposes $\vec{B}_{magnet}$ for the tube segment below the magnet.

Thicker (higher-quality) copper tube with same diameter: magnet falls more slowly. Why? Same EMF for the same fall speed, but less resistance $\Rightarrow$ higher I_{ind}.

$$\Phi_M = \underset{\substack{\uparrow \\ \text{near the pole}}}{B_{magnet}} \cdot \underset{\substack{\uparrow \\ \text{cross section}}}{A}$$

so why $\Phi_M(t)$?





$$\frac{d\phi_M}{dt} = A\,\frac{dB_z}{dt} \Rightarrow \frac{d\phi_M}{dt} \sim v$$

Why is this so? The (somewhat sketchy) shape of the function $B_z(z_0,t)$, i.e., for fixed location z_0 we are asking what is the time pofile, can also be applied for fixed time t_0 as a function of z: below the N pole B_z is positive, above the S pole B_z is negative.

This shape describes the magnetic field as it travels down the copper tube according to

$$B_z(z,t) = \underbrace{f}_{\text{shape function shown in graph}}(z - vt) \qquad v = \text{terminal velocity}$$

When we calculate $\frac{d\phi_M}{dt}$ we use the chain rule, and the factor v arises for that reason.

Thus, gravity accelerates the magnet to such a velocity v until the EMF generated produces an induced current strong enough for $\vec{B}_{ind}$ to act on the magnet such that gravity is cancelled $\rightarrow$ velocity-dependent drag. The current loops travel!