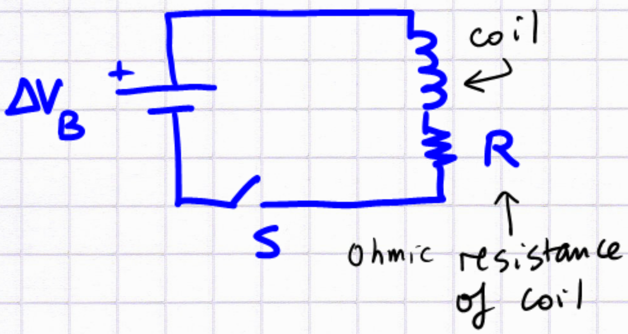
Inductance and RL circuit

We want to understand more about the inertia associated with magnetic fields, i.e., how they resist change.

Consider an N-turn solenoid (or coil) with no current through it at $t=0$. The coil has some ohmic resistance which is shown in series.



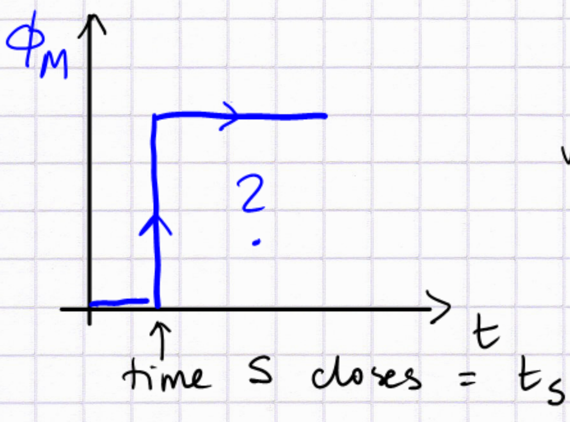
Assume there is no magnetic flux through the coil (earth's field and other magnets present in the lab are shielded)

Naively, we might think: close S, and $I = \frac{\Delta V_B}{R}$ is flowing instantly.

However, this would imply an instant turn-on of a magnetic field, i.e., also magnetic flux Φ_M.

Faraday's law $\mathcal{E} = -\frac{d\Phi_M}{dt}$ (with Lenz)

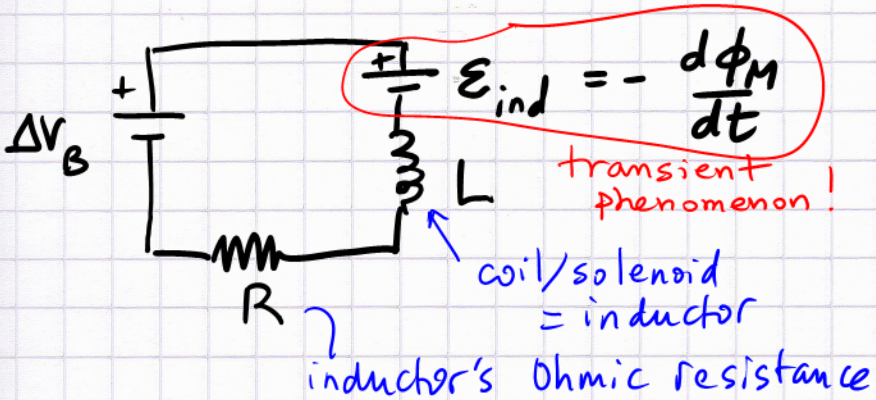
states that this is impossible:



would imply: $\left.\frac{d\Phi_M}{dt}\right|_{t_S} = \infty$!

There will be an induced EMF in the coil/solenoid to prevent an instant turn-on of the current!

So, what is actually going on? The coil (inductor) is trying to prevent an increase in magnetic flux ϕ_M from zero by countering the battery EMF with an induced EMF:



As with the Ohmic resistance of the coil we put the self-induced EMF in series with the coil

The induced EMF is a transient phenomenon:

As time passes the current grows, builds a magnetic field and the rate of change of ϕ_M decreases, i.e., the counter-EMF decreases.

Understand the induced EMF in detail:

Assume a solenoid (length >> radius; length = d)

$$B = \mu_0 \frac{NI}{d} \qquad (N = \text{\# of turns, for a coil: } d = \text{diameter})$$

The magnetic flux for a single turn: $\phi_M^{(1)} = BA = \mu_0 \frac{NI}{d} A$

The total magnetic flux: $\phi_M^{tot} = N\phi_M^{(1)} = \mu_0 \frac{N^2 I}{d} A$

Now insert into Faraday's law

$$EMF = -\frac{d\phi_M}{dt} = -\mu_0 \frac{N^2 A}{d} \frac{dI}{dt}$$

property of the solenoid (or coil):

$$L \equiv \frac{\mu_0 N^2 A}{d}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$ is the self-induced EMF

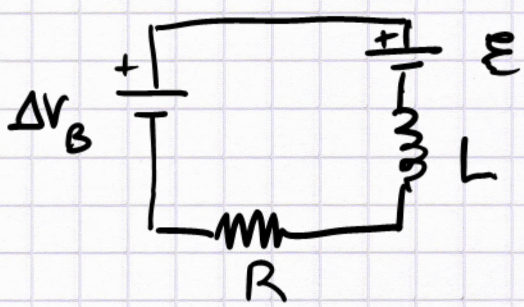
An inductor of inductance L (measured in Henry, $H = \frac{Vs}{A}$) opposes the turn-on of electrical current by a counter-EMF $\mathcal{E} = -L\frac{dI}{dt}$. Turning on a current ($\frac{dI}{dt} > 0$) it opposes the battery. Turning off a current ($\frac{dI}{dt} < 0$) by disconnecting the battery $\rightarrow$ current cannot stop immediately ($\frac{dI}{dt} \rightarrow -\infty$ is not allowed!), the inductor produces its own EMF to let the current continue for some time! (seems crazy, but it's true!)

Note: a simple, straight wire also has some inductance, as it surrounds itself with a $\vec{B}$ field if an electric current flows.

This self-induced EMF has something to do with energy storage: building up a $\vec{B}$ field by turning on an electric current implies energy storage analogous to energy storage in an $\vec{E}$ field in a capacitor.

Now use the Kirchhoff loop rule to derive the temporal shape of the current in an RL circuit:

$$\Delta V_B + \mathcal{E} - RI = 0$$



$$\Delta V_B - L\frac{dI}{dt} - RI = 0$$

We expect exponential behaviour for the current $I(t)$

At time $t=0$ the current I_0 should vanish (the EMF $\mathcal{E}$ opposes the flow of current to prevent the magnetic flux ϕ_M from changing)

$$\frac{\Delta V_B}{R} - \frac{L}{R}\frac{dI}{dt} - I = 0 \qquad \tau = \frac{L}{R} = \text{time constant}$$

$$I_{ss} - \tau \frac{dI}{dt} - I = 0 \qquad \frac{\Delta V_B}{R} = I_{ss} = \text{steady state current}$$

(when $\frac{dI}{dt} \to 0$)

This is solved by

$$I(t) = I_{ss}\left(1 - e^{-t/\tau}\right)$$

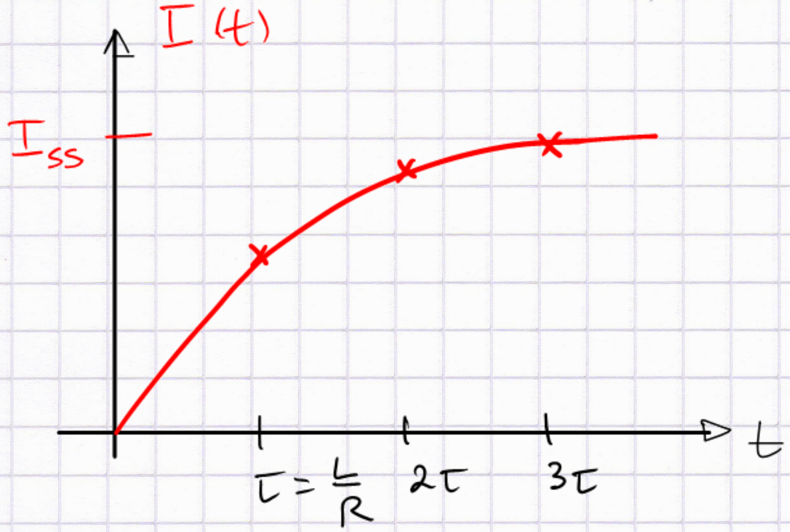
for $t \to \infty$: $I \to I_{ss}$

for $t \to 0$: $I = 0$

verify: $\frac{dI}{dt} = \frac{1}{\tau} I_{ss}\, e^{-t/\tau}$

$$\therefore \quad I_{ss} - \tau\left(\frac{1}{\tau} I_{ss}\, e^{-t/\tau}\right) - I_{ss}\left(1 - e^{-t/\tau}\right)$$

$$= I_{ss} - I_{ss}\, e^{-t/\tau} - I_{ss} + I_{ss}\, e^{-t/\tau} = 0 \quad \checkmark$$



When we close the switch in <u>any</u> circuit, especially in circuits with a coil or solenoid (electromagnet): the current cannot turn on instantly, it builds over time. The time constant is $\tau = L/R$.

Simple wires: self-inductance L is small $\Rightarrow$ time constant τ is short.

Often we need well-defined conditions: <u>coaxial cables</u>

$\rightarrow$ specify: $\frac{\text{resistance}}{\text{unit length}}$; $\frac{\text{capacitance}}{\text{unit length}}$; $\frac{\text{self inductance}}{\text{unit length}}$