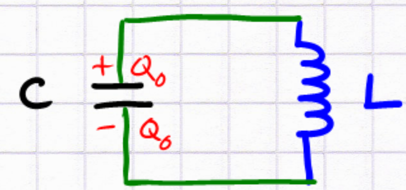
Electric oscillator and resonance

Consider an (idealized) circuit in which a capacitor C and an inductor L together form a loop. Assume an initial charge $\pm Q_0$ on the capacitor



At $t=0$ no current flows, but there is an $\vec{E}$ field, i.e., electrical energy in the capacitor. C starts discharging by attempting to pass a current through L.

→ The circuit is idealized, since we ignore the ohmic resistance R of the solenoid (and the wires)

There is no $\vec{B}$ field at $t=0$, as there is no current (yet)

Kirchhoff loop: $\Delta V_C = \frac{Q}{C}$; $\Delta V_L = -L \underbrace{\frac{dI}{dt}}_{<0}$

$$\Delta V_C + \Delta V_L = 0$$

$I = -\underbrace{\frac{dQ}{dt}}_{<0}$

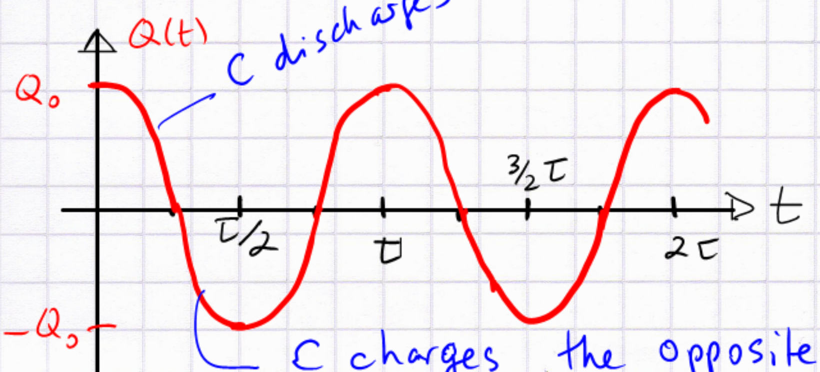
$$\frac{Q(t)}{C} - L\frac{dI}{dt} = 0$$

$$\frac{Q(t)}{C} + L\frac{d^2Q}{dt^2} = 0 \; ; \qquad \omega^2 \equiv \frac{1}{LC} \qquad \omega = \frac{2\pi}{\tau}$$

$$\frac{d^2Q}{dt^2} = -\omega^2 \, Q(t)$$

Q: what does this mean?

$$\therefore \; Q(t) = Q_0 \cos(\omega t)$$
$$= Q_0 \cos\left(2\pi \frac{t}{\tau}\right)$$



C charges the opposite way → why?

Observation 1 : 1^{st} discharge cycle $0 \le t < \frac{\tau}{4}$

- The current does not drop exponentially
- Initially, it remains constant → solenoid counter-EMF opposes current growth

$I(t) = -\frac{dQ}{dt}$ remains zero @ $t=0$

(slope in $Q(t) = 0$)

Slowly the counter-EMF is overcome. At $t = \tau/4$ the capacitor has $Q=0$, no charge on its plates.

Does the current stop? → No, $I = -\frac{dQ}{dt}$ is max at this point!

When $Q=0$ on the capacitor plates → no $\vec{E}$ field (no electrical energy)

Current $I = -\frac{dQ}{dt}$ is max. at this point, i.e., the solenoid has a max. $\vec{B}$ field at $t = \frac{\tau}{4}$.

Initially, all energy was in $\vec{E}$ (inside C), now all energy is in $\vec{B}$ (solenoid L).

The current continues due to the $\vec{B}$ inertia: the solenoid L now produces forward EMF to oppose the reduction of Φ_M. As the current continues to flow it charges up C, but in the opposite sense Q_0 (@ $t=0$) → 0 (@ $t=\frac{\tau}{4}$) → $-Q_0$ (@ $t=\tau/2$)

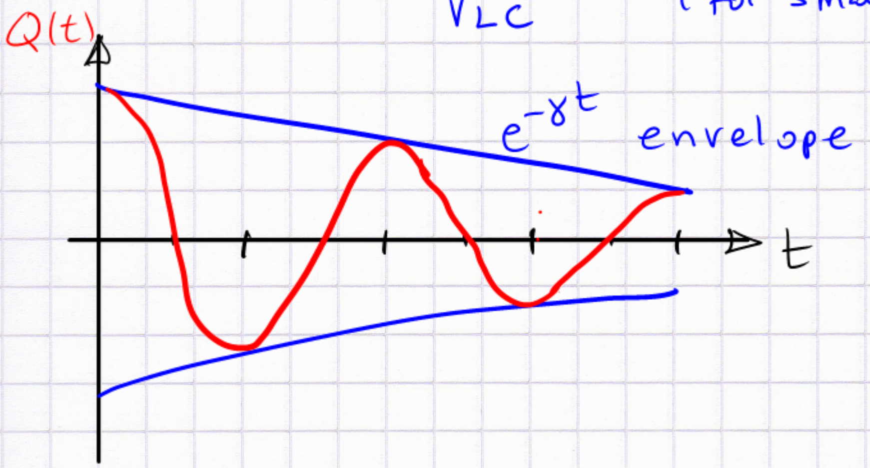
Realistic circuit: RLC includes damping $\sim R$ ③

(cf. damped harmonic motion)

$$Q(t) = Q_0 \cos\left(2\pi \frac{t}{\tau}\right) \rightarrow Q_0 e^{-\gamma t} \cos\left(2\pi \frac{t}{\tau}\right)$$

$$= Q_0 e^{-\gamma t} \cos(\omega t)$$

$$\omega \approx \frac{1}{\sqrt{LC}}$$ (for small R, high-quality oscillator)



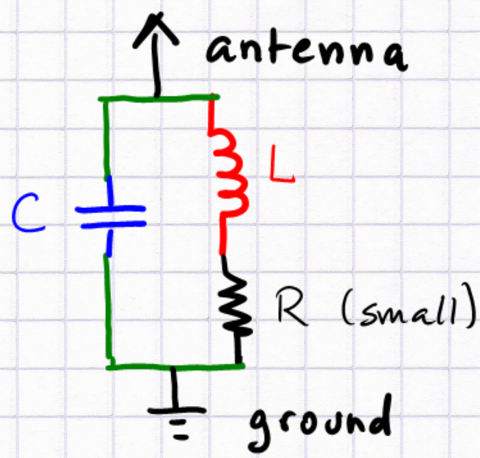
2 messages:

1) natural frequency $\omega \approx \omega_0$ (weak damping)

2) natural oscillations die out over time

Interesting problem:

High-frequency $\left(f_0 = \frac{\omega_0}{2\pi} > 200 \text{ kHz}\right)$ radio oscillator circuit has $f_0 \approx \frac{1}{2\pi\sqrt{LC}}$ fixed and picks up radiowaves of frequencies $\underbrace{f_T}_{\text{transmitter frequency}} \approx f_0$



What happens?

Antenna picks up some $\vec{E}$ field from the radio wave, charges C, then charge sloshes around in RLC circuit

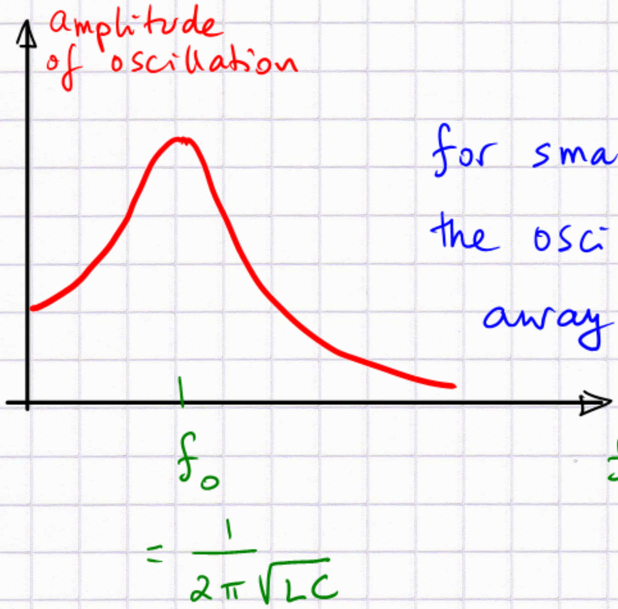
Suppose there is only one transmitter, fixed frequency f_T.

It causes the RLC oscillator to swing at f_0 (when the first wave front hits the antenna; but then this signal dies.

However, the wave keeps hitting the antenna with a signal at frequency f_T. Eventually, the RLC can only oscillate at frequency f_T (the f_0 part has died).

What can we look for?

How big is the steady-state amplitude of the f_T oscillations when f_T is <u>tuned</u> around f_0?



for small to moderate values of R the oscillator responds even if f_T is away from f_0.

This is not so desirable if we want to have many different transmitters and would like our circuit to pick out only one of the stations!

Fortunately, we can sharpen the response curve by reducing R. → higher-quality oscillators ($Q \sim \frac{\text{oscillation period}}{\text{damping time}}$) are <u>more selective</u>

For electronic watches / clocks a similar principle:

cheap watch (electromechanical transistor-based oscillator) (1970ies TIMEX; today's computer clock!!) → seconds / day error

quartz oscillator stabilized → seconds / month

"atomic clock" → national / international standards → internet time fraction of sec per year