Combined translational + rotational motion

1) baseball bat sweet spot problem (Giordano p. 269/70)

- model bat as a uniform rod (mass m, length L)

  Q: how realistic is that?

inertia about CM: $I_{CM} = \frac{1}{12} m L^2$ (thin rod)

(cf. table 8.2, p. 262)

$\uparrow$ geometric factor

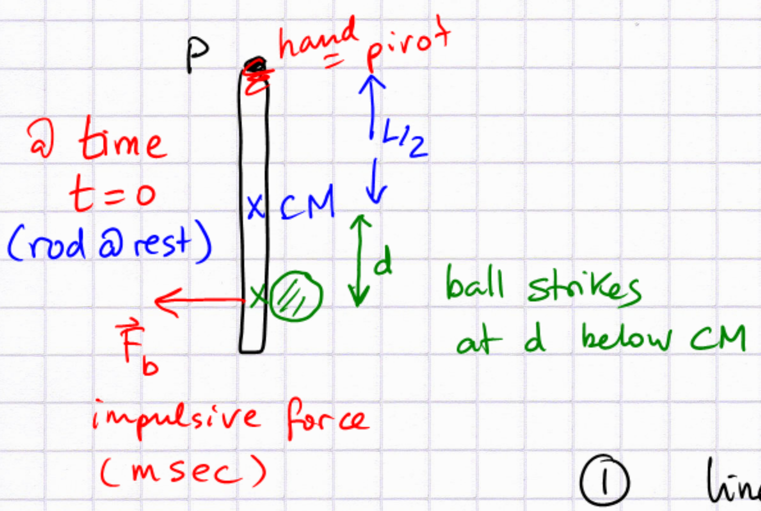
or $\int_{-L/2}^{L/2} x^2\, dm = \left(\frac{m}{L}\right) \int_{-L/2}^{L/2} x^2\, dx = \frac{m}{L} \frac{x^3}{3}\Big|_{-L/2}^{L/2} = \frac{m}{3L}\left(\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right) = \frac{mL^2}{12}$

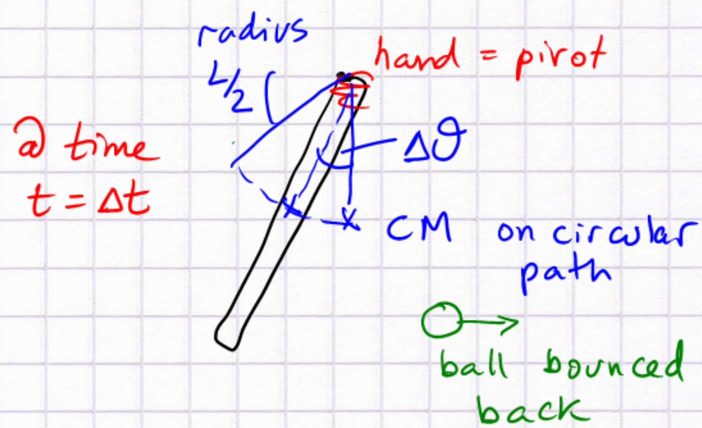
$\uparrow$ linear mass density

Idea: ball strikes rod such that it is put in a purely rotational motion about the handle end

$\therefore$ hand does not feel the hit (doesn't hurt)

Fig. 8.35 simplified





① linear motion: transfer $\vec{F}_b$ to CM

$$m\, a_{CM} = F_b \quad \text{(negative, to the left)}$$

② rotational motion about CM: $I_{CM}\,\alpha = \tau_{CM}$

$$\therefore \quad \frac{1}{12} m L^2 \alpha = F_b\, d$$

torque is into paper (<0)

$\therefore$ CW motion

③ link rotation with CM translation

$$v_{CM} = \frac{L}{2}\,\omega$$

circular motion, radius $L/2$

$$\therefore \quad a_{CM} = \frac{L}{2}\,\alpha$$

Combine 1 and 2 using 3 $\therefore$ $d_{ideal} = L/6$

Remarks:

(i) the video showing a suspended rod struck by a hammer shows that striking @ $d \approx L/6$ makes the rod swing without rocking motion.

(ii) Q: what happens if the ball strikes @ $d=0$ (@CM)?

A: rod just translates, doesn't rotate

(iii) true baseball batting:

a) bat is not uniform, less mass near handle

b) bat is driven forward, not a pure rotation

c) bat - ball collision $\rightarrow$ momentum conservation + approx. energy conservation?

2) Rolling (down an incline)

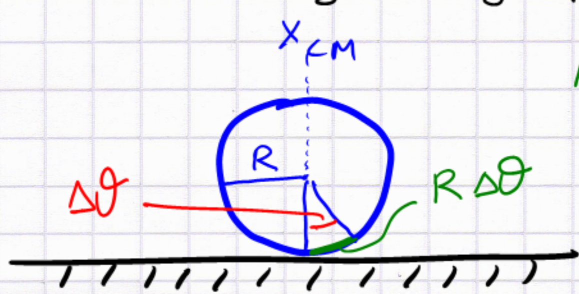
video compares arrival times of equal-size, equal-mass objects:

solid disk	vs.	ring	vs.	sphere
$I_{CM} = \frac{1}{2} mR^2$		$I_{CM} = mR^2$		$I_{CM} = \frac{2}{5} mR^2$

more inertia, same torque
less acceleration?

$2/5 < 1/2$
$0.4 \quad 0.5$

Idealized rolling: single point of contact, instantaneously at rest!

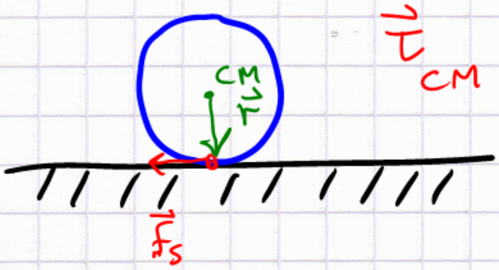


After time Δt angle increased (CW) by $\Delta\theta$;

CM advanced by $\Delta x = R\Delta\theta$

What causes rolling?

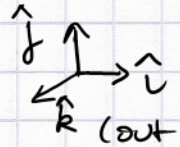
Torque about CM is provided by $\vec{f}_s$ = static friction



$\vec{\tau}_{CM} = \vec{r} \times \vec{f}_s$ is into page;

$\tau_z < 0$

CW rotation



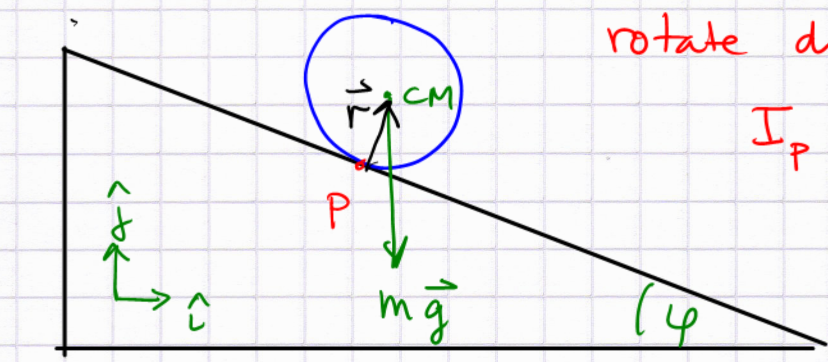
static friction ensures rolling as long as the required

$$f_s \le \mu_s N = \mu_s mg \, ;$$

Incline

simplest discussion (avoids f_s)

rotate disk about P (as pivot)

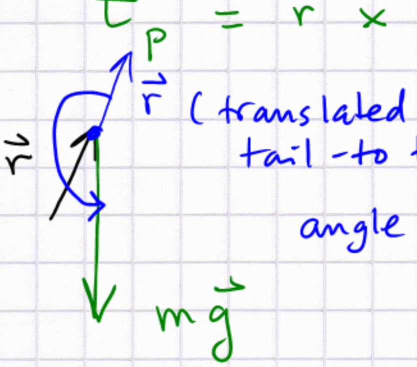


$$I_P = \frac{3}{2} mR^2 \qquad \left(= \tfrac{1}{2}mR^2 + mR^2\right)$$

motion of CM on circle of radius R

gravity provides rotational torque about P:

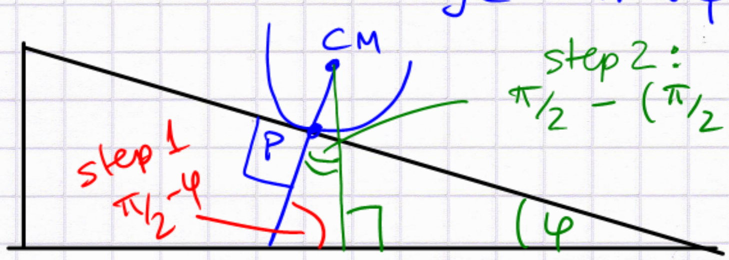
$$\vec{\tau}_P = \vec{r} \times (m\vec{g}) = R\,mg\,\sin\underbrace{(\measuredangle\, \vec{r}, m\vec{g})}_{\text{What is the angle?}}\,\hat{k}$$



angle $> \pi$ $\therefore$ sin (angle) is negative

$\therefore$ $\tau_{P,z} < 0$ $\therefore$ CW rotation

angle $= \pi + \varphi$ $(= 180^\circ + \varphi)$ why?



step 2:
$\pi/2 - (\pi/2 - \varphi) = \varphi$

$$\therefore |\tau_P| = R\,mg\,\sin\varphi$$

Newton - 2nd for rotation about P :

CW rotation
=> negative α
due to neg τ,
but work with magnitude

$$\frac{3}{2} m R^2 \alpha = \tau = R mg \sin\varphi$$

$$\frac{3}{2} R\alpha = g \sin\varphi$$

now use $a_{CM} = R\alpha$

$$\boxed{a_{CM} = \frac{2}{3} g \sin\varphi}$$

compare to slipping :

$$a_{CM}^{slip} = g_{\parallel} = g\sin\varphi$$

Q: why is the advancement of the CM slowed ?

A : CM is governed by : $\boxed{m\, a_{CM} = m g_{\parallel} - f_s}$

net force is :

$mg \sin\varphi - f_s$ $\qquad \therefore \quad f_s = \frac{1}{3} mg \sin\varphi$

- Static friction is responsible for rolling
- it slows the CM (we moved forces to CM, determined the net force, that gave (2nd law linear motion)
- we find how much f_s is required!

for large φ : $\frac{1}{3} mg \sin\varphi$ may exceed $\mu_s mg$,

then the roll goes over into a <u>skid</u>

Next lecture(s) : consider mechanical energy :

two parts : 1) $KE_{translation} = \frac{1}{2} m v_{CM}^2$

analogy

2) $KE_{rotation} = \frac{1}{2} I_{CM} \omega^2$

$$KE_{rot} = \frac{1}{2} \underbrace{\frac{1}{2} m R^2}_{disk} \left(\frac{v_{CM}}{R}\right)^2 = \frac{1}{4} m v_{CM}^2$$

rolling + translation energies are locked